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Generalized Solution Procedure for Slope Stability Analysis Using Genetic Algorithm

유전자 알고리즘을 이용한 사면안정해석의 일반화 해법

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요 지

이 논문에서는 사면 안정 해석시 경사 절편법을 이용하여 안전율을 구하며 유전자 알고리즘방법을 이용하여 한계파괴면을 결정하는 이론을 제안하였다. 해석방법에서는 한계 전단 파괴면과 안전율을 찾고 비선형 평형 방정식의 해를 구하기 위해 구속 최적화 문제로 간주하여 해석을 수행하였다. 유전자 알고리즘 방법의 효율성을 검증하기 위하여 예제를 논문에 포함하였다. 유전자 알고리즘 방법에 의하여 도출된 사면 안정 해석결과는 기존방법에 비하여 우수한 것으로 판명되었다.

Abstract

This paper pertains to the incorporation of a genetic algorithm methodology for determining the critical slip surface and the corresponding factor of safety of soil slopes using inclined slice method. The analysis is formulated as a constrained optimization problem to solve the nonlinear equilibrium equations and finding the factor of safety and the critical slip surface. The sensitivity of GA optimization method is presented in terms of development of failure surface. Example problem is presented to demonstrate the efficiencies of the genetic algorithm approach. The results obtained by this method are compared with other traditional optimization technique.

Keywords : Consolidation, CRS test, Strain rate, Incremental loading test, Preconsolidation pressure

1. Introduction

The stability of slopes has received wide attention due to its practical importance in the design of excavations, embankments, earth dams, and rock fill dams etc. Generally, limit equilibrium techniques are commonly used to assess the stability of slopes, as complex geological sub-soil profiles, seepage, and external loads can be easily dealt with. Most of these analytical approaches use either the vertical method of slices or the multiple-wedge methods. It has been recognized quite early that slope stability analysis is essentially a problem of optimization (Basudhar, 1976; Baker and Garber, 1977) namely the determination of the slip surface that yields the minimum factor of safety. Many methods of factor of safety computations for slopes using circular and noncircular slip surfaces

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have been developed over the years. Many slope stability softwares using the limit equilibrium analysis have been described in literature (Fredlund, 1984). Most of the programs provided an automated version of the existing methods of slope stability analysis. The need for auto-search led to the use of sophisticated optimization algorithms (Krugman and Krizek, 1973; Narayan and Ramamurthy, 1980). But such earlier attempts were based on the assumption of circular slip surfaces. Successful use of optimization techniques in slope stability analysis without any a ‑priori assumption regarding the shape of critical surface has been reported (Martin, J. B., 1982; Arai, K. and Tagyo, K., 1985; Bhattacharya, G. 1990). However, these analyses have been made by considering slices to be vertical and also traditional optimization algorithms have been used for automated search of critical slip surface and factor of safety.

The methods in use include a rectangular or trapezoidal grid search and simplex optimization. For noncircular slip surfaces, this is more complicated, as the number of variables to be optimized can be substantially larger. The traditional mathematical optimization methods that have been used include dynamic programming, conjugate-gradient, random search, and simplex optimization. The main shortcoming of these optimization techniques is the uncertainty as to the robustness of the algorithms to locate the global minimum factor of safety rather than the local minimum factor of safety for complicated and non-homogeneous geological subsoil conditions.

We proposed in this paper an alternative method of determining the critical slip surface using a genetic-based evolution technique called genetic algorithms (GAs). GAs have been found wide spread application in variety of problem domains because of their minimal requirement, ease of operation, global perspective. The GA is becoming increasingly popular in engineering optimization problems because it has been shown to be suitably robust for a wide variety of problems. The incorporation of genetic algorithms in the slope stability analysis will be described. Examples are presented to demonstrate the effectiveness of the proposed approach. The critical acceleration K_c required to bring the slope to a condition of limiting equilibrium is given by

$$K_c = \frac{AE + E_1 e_n e_{n-1} e_{n-2} \cdots - E_{n+1}}{PE} \tag{1}$$

Where,

$$AE = a_n + a_{n-1}e_n + a_{n-2}e_n e_{n-1} + \cdots\cdots + a_1 a_{n-1} e_n \ldots\ldots e_3 e_2 \tag{2}$$

$$PE = p_n + p_{n-1}e_n + p_{n-2}e_n e_{n-1} + \cdots\cdots + p_1 a_{n-1} e_n \ldots\ldots e_3 e_2 \tag{3}$$

$$p_i = \frac{W \cos(\phi_i' - \alpha_i)}{\cos\left(\phi_i' - \alpha_i + \overline{\phi_{i+1}'} - \delta_{i+1}\right)\sec\overline{\phi_{i+1}'}} \tag{4}$$

$$e_i = \frac{\cos\left(\phi_i' - \alpha_i + \overline{\phi_i'} - \delta_i\right)\sec\overline{\phi_i'}}{\cos\left(\phi_i' - \alpha_i + \overline{\phi_{i+1}'} - \delta_{i+1}\right)\sec\overline{\phi_{i+1}'}} \tag{5}$$

$$a_i = \begin{bmatrix} \left(W_i + (\Delta P_v)_i + q_i(\Delta l_q)_i\right)\sin\left(\overline{\phi_i'} - \alpha_i\right) + R_i \cos\overline{\phi_i'} \\ + (S_s)_{i+1} \cdot \sin\left(\overline{\phi_i'} - \alpha_i - \delta_{i+1}\right) - (S_s)_i \cdot \sin\left(\overline{\phi_i'} - \alpha_i - \delta_i\right) \end{bmatrix} \times \frac{\cos\overline{\phi_{i+1}'}}{\cos\left(\phi_i' - \alpha_i + \overline{\phi_{i+1}'} - \delta_{i+1}\right)} \tag{6}$$

Where, $R_i = c_i' b_i / \cos\alpha_i - U_i \tan\overline{\phi_i'}$ (7)

$$(S_s)_i = c_i' d_i - (P_w)_i \tan\overline{\phi_i'} \tag{8}$$

$(P_w)_i, (P_w)_{i+1}$ are the water pressures on the inclined inter slice faces.

U_i is the pore water pressure on the base of the slice.

The coefficient of critical acceleration (K_c) is calculated by using the equation 1. If for a slope K_c is not equal to zero, the static factor of safety is calculated by reducing the shear strength simultaneously on all sliding surfaces until the minimum K_c is obtained. This is achieved by the following substitutions in equations 4 to 8.

$$c_i'/F,\ \tan\phi_i'/F,\ \overline{c_i'}/f_L,\ \tan\overline{\phi_i'}/f_L,\ \overline{c_{i+1}'}/f_L,\ \tan\overline{\phi_{i+1}'}/f_L$$

Where, f_L = local factor of safety along inter slice faces.

f_L = F = average factor of safety along the surface, if F > 1.1; otherwise f_L = 1.1, have been adopted.

If there is no tension crack, then $E_1 = E_{n+1} = 0$. The forces acting on the sides and base of each slide are found by the progressive solution of the following equations, starting from the known condition that $E_1 = 0$.

$$E_{i+1} = a_i - p_i K_c + E_i e_i \tag{9}$$

$$X_i = \left(E_i - (P_w)_i\right)\tan\overline{\phi'_{i+1}} + \overline{c'_i} d_i \tag{10}$$

$$N_i = \begin{pmatrix} W_i + (\Delta P_v)_i + q_i(\Delta l_q)_i + X_{i+1}\cos\delta_{i+1} + X_i\cos\delta_i \\ - E_{i+1}\sin\delta_{i+1} + E_i\sin\delta_i + U_i\tan\phi'_i\sin\alpha_i - c'_i b_i\tan\alpha_i \end{pmatrix} \times \frac{\cos\phi'_i}{\cos(\phi'_i - \alpha_i)} \tag{11}$$

$$S_i = (N_i - U_i)\tan\phi'_i + c'_i b_i \sec\alpha_i \tag{12}$$

The normal stresses acting across the base and the sides of a slice are calculated as follows:

$$(\sigma'_b)_i = (N_i - U_i)\cos\alpha_i / b_i \tag{13}$$

$$(\sigma'_s)_i = \left(E_i - (P_w)_i\right)/d_i \tag{14}$$

1.1 Design Variables, Objective Function and Constraints

After the stability equations are derived it is necessary to identify the design variables and objective function, which control the analysis and are to be estimated. For this it is necessary to follow a set of iterative procedure to find the minimum value of the objective function and the corresponding values of the design variables at the optimal point. However, the search for the optimal values of the objective function and the corresponding design vector cannot be made unrestrictedly. Some design restrictions called constraints are to be imposed so that the obtained solution is physically meaningful. The design variables, objective function and constraints that are relevant to the present study are as follows:

1.2 Design Variables

The discretization model of the soil slope is shown in Fig 1. Referring to the figure the identification of design variables are made as follows:

The design vector D is,

$$D^T = \left(F, \alpha_1, \alpha_2, \ldots\ldots, \alpha_{n-1}, b_1, b_2, \ldots\ldots, b_n, x_s, x_T, z_T\right) \tag{15}$$

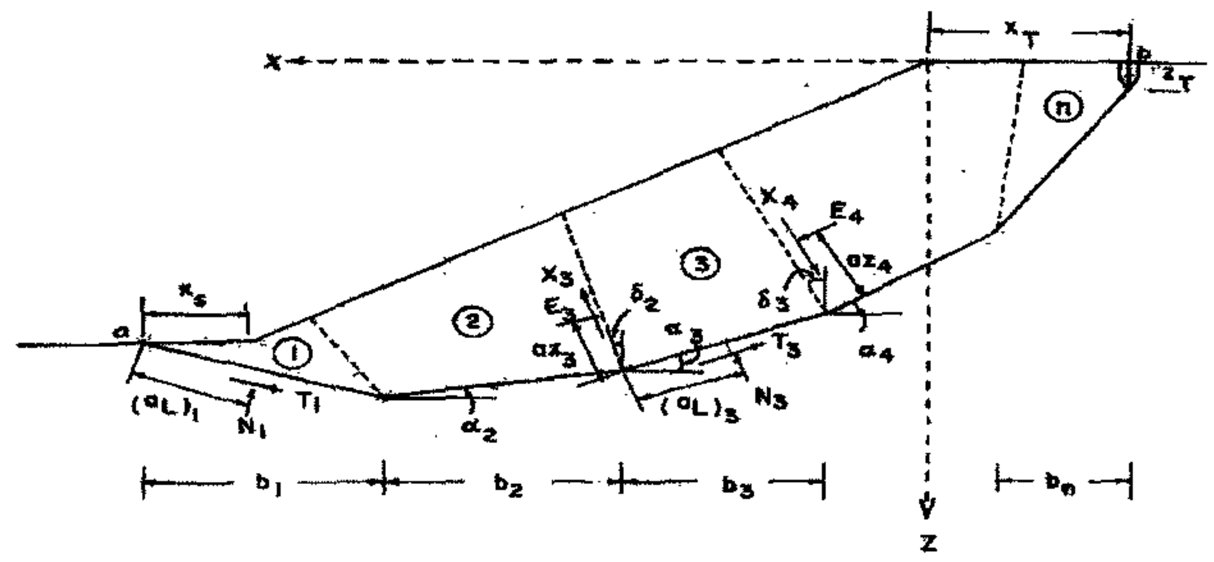

Fig. 1. Discretization model for with inclined slices

Where, F = average factor of safety along the slip surface.

$b_1, b_2, \ldots\ldots, bn$ = width of nth slices.

$\alpha_1, \alpha_2, \ldots\ldots, \alpha_{n-1}$ = base inclination of the slices with horizontal, positive in anti-clockwise direction.

$\delta_1, \delta_2, \ldots\ldots, \delta_{n-1}$ = angle of inclination of inclined faces with vertical, positive in clockwise direction.

x_s = distance of starting point of the slip surface from the bottom corner of the slope.

x_T, z_T = x and z coordinates of tension crack respectively.

1.3 Objective Function

Once the design variables are identified, the function, which is to be optimized, called objective function and denoted by F (D) should be developed. In this case, by taking only the force equilibrium, minimization of factor of safety subjected to the condition that the value of K_c should be zero is the objective. In this case the solution is achieved by putting the value of K_c as a constraint.

Here, F (D) = F (16)

1.4 Design Constraints

To ensure that the obtained solution is physically meaningful, the following design constraints need to be imposed.

The critical surface should be concave when looked from the top.

As the soil cannot take tension, the developed normal stress at the base of the slice should be positive to avoid generation of tension in the soil and inconsistent direction of shear.

Normal stresses generated on the inclined inter slice faces should be positive to avoid development of tension there.

Since the value of Kc should be very small, the following constraint has been put on K_c. Minimization of the objective function should result in values of K_c tending to zero.

The last point (b) of the critical slip surface should not intersect the sloping portion.

2. Genetic Algorithm Tool

Genetic algorithm is a search technique based on the principle and mechanism of natural selection and evaluation where the stronger individuals are likely to survive in a competing environment. The GA operates on an iterative procedure on a set (population) of candidate solution of the problem to be optimized. Each candidate (chromosome) of this set is a concatenated version of the binary substrings representing design variables of the problems to be optimized. Initially these candidate solutions are generated randomly which are then altered probabilistically and carried forward for next iteration (or generation) guided by three basic processes namely selection, crossover and mutation. The fitness in a GA technology is nothing but the value of objective function. Thus each solution string is associated with a fitness value. (1) selecting, according to the fitness value, some of the solution strings of the present generation and also the resulting combination and (2) rejecting others so as to keep the population size constant form a new generation of solution. While selection operation makes more copies of better string, the crossover parameter controls over the creation of new string by exchanging information among the strings. In order to preserve some of the good strings that are present in the population, selection of strings for the crossover is done with a probability. Mutation operator acts as a switch when the population becomes homogeneous due to iterative use of cross over and mutation operations.

The actual optimization process requires the values of some GA parameters such as string length of each decision variable, population size, crossover parameters. Based on the desired accuracy, the string length for each decision variable is taken.

2.1 Methodology

In order to apply GA, the slope stability problem is defined in terms of certain design variables. The function to be optimized (objective function) and the guiding rules (constraints) are expressed in terms of these design variables.

2.2 GA Based Problem Formulation

In the context of genetic algorithm, the present problem has been put into the mathematical framework as follows:

Find $(x_1,x_2,\cdots\cdots..,x_m)$ to minimize F(x) subject to $g_j(x) \geq 0$ for j = 1,2,·········m.

Where, $x_1,x_2,\cdots\cdots..,x_m$ represents the design variables corresponding to the base inclination of the slices, base widths, locus of start point, and position of tension crack. The terms $g_j(x)$ are set of j constraints. F(x) denotes either objective function or the sum of objective function and penalty term as discussed later.

The objective function is taken as the factor of safety of the slope.

2.3 Fitness Function

Fitness of any string is the value returned to GA, based on which GA operators modify the population. In present problem Fit(x) is used as fitness function which denotes the factor of safety with penalties after applying transformation to convert maximization problem to the minimum one. GA operators minimize F(X) which in turn reduces the penalties and factor of safety of the given slope.

Transformation:

As GAs are basically maximization search techniques, to convert the minimization problem to maximization one, many types of transformations are available. In the present formulation the following transformation is used.

$$Fit(x) = \frac{1}{1+F(x)} \tag{17}$$

Where, Fit(x) is the Fitness function and F(x) is the objective function.

2.4 Constraint Handling

Various physical and behavior constraints are used to solve this class of problems. To take care of constraints, the reproduction operator may be modified so that if the solution is feasible (one or more constraints are violated), the string is not copied to the mating pool. The problem of finding a feasible solution is as difficult as the finding of the optimal solution, especially when the number of design variables and the design constraints are more. This penalty terms are usually used to take care of these constraints. So, in constrained optimization case, instead of using the objective function as fitness, each constraint's violation is added to the objective function. As in the present problem the constraints are taken in the normalized form, and a single penalty coefficient is taken here. Hence the composite function reduces to

$$F_{comp}(x) = F(x) + r_k \sum_{j=1}^{n_{con}} G_j(x) \quad (18)$$

Where, $F_{comp}(x)$ is the composite function, $G_j(x)$ is the constraint term applied to each variable, n_{con}= total number of constraints and r_k is the penalty parameter.

3. Results

The present methodology is validated with the help of a published example.

3.1 Example Problem

A problem (Spencer, 1967, Figure 2) is considered for the validation of the optimization formulation of predicting the minimum factor of safety and corresponding critical slip surface. The same problem has been reanalyzed and reported by Bhattacharya (1990) in order to validate his proposed direct and indirect optimization formulation of stability computations using vertical slices as reported in literature. Here, by using the vertical slices, the same problem has been reanalyzed and reported. The results obtained are compared with that of Bhattacharya (1990) and some other solutions reported in the literature.

This problem has been solved in conjunction with genetic algorithms by considering the failed soil mass bounded by the failure surface and the free ground surface to be made up of a number of vertical slices. The following GA parameters have been found suitable for solution of this class of problem after successive numerical experimentations as shown in Table 1.

In the present method by taking interslice faces vertical and by taking 4, 6, 8, 10 and 12 number of slices, the factor of safety and the critical slip surface is obtained. In the present analysis no tension crack is taken and the starting point of the slip surface is taken at the toe of the slope.

The effect of number of slices on critical slip surface and the factor of safety of the slope are critically examined by using genetic algorithms. The results are shown in Table 2, and Figure 3. The critical accelerations associated with the critical slip surfaces corresponding to different number of slices are also indicated in the Table 2. From the Table 2 it is seen that with the increase in the number of slices from 4 to 8, the obtained factor of safety decreases. After 8 number of slices the factor of safety increases. However the change in factor of safety with the change in number of slices is marginal. Also, from Figure 3, it is shown that the critical slip surfaces obtained with 4, 6, 8, 10 and 12 number of slices fall in a narrow

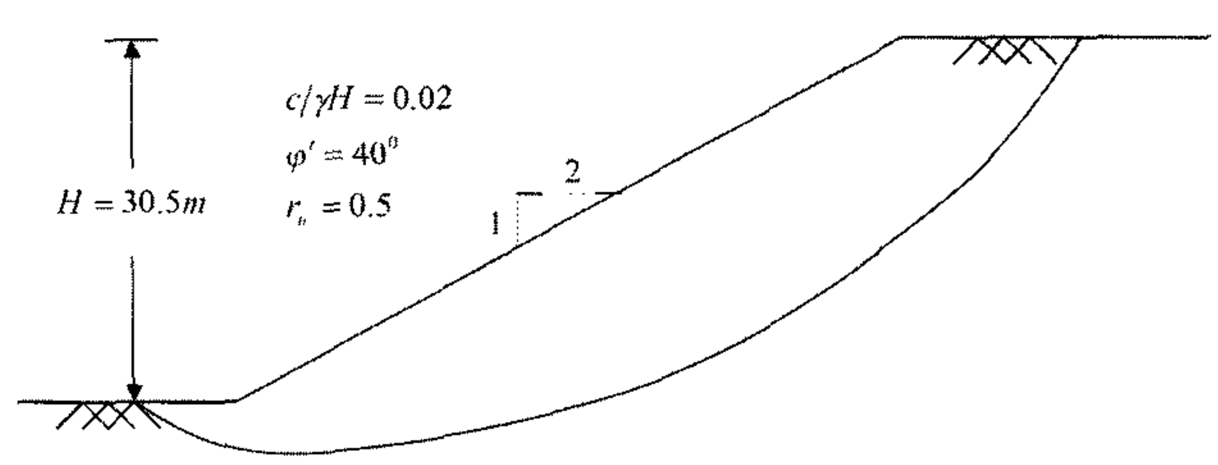

Fig. 2. Spencer's problem

Table 1. GA parameters

Population size	20
Probability of crossover (P_c)	0.8
Probability of mutation (P_m)	0.1
Total string length	16

Table 2. Factor of safety and critical acceleration factor for different numbers of vertical slices

Surface	Number of slices	Factor of safety	Critical acceleration factor (K_c)
1	4	1.06063	0.000072
2	6	1.04593	0.000049
3	8	1.03815	−0.000063
4	10	1.09061	−0.000048
5	12	1.10861	0.000903

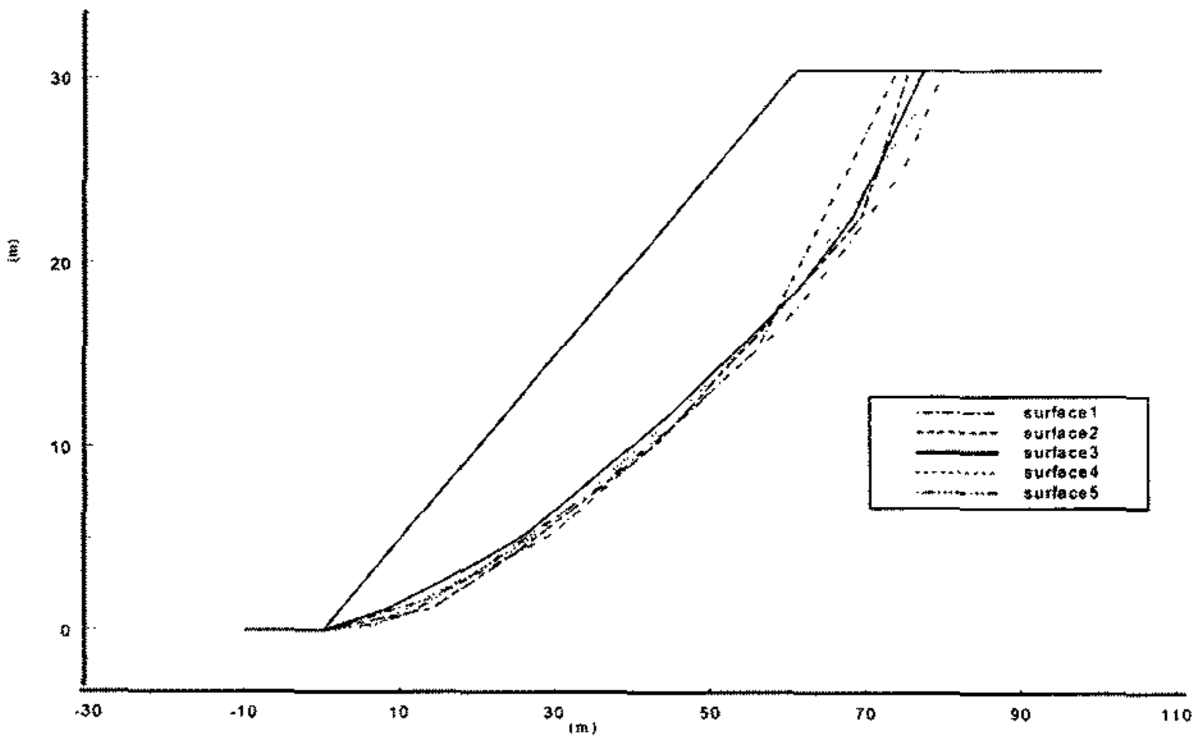

Fig. 3. Effect of number of vertical slices on the critical slip surface (problem 2)

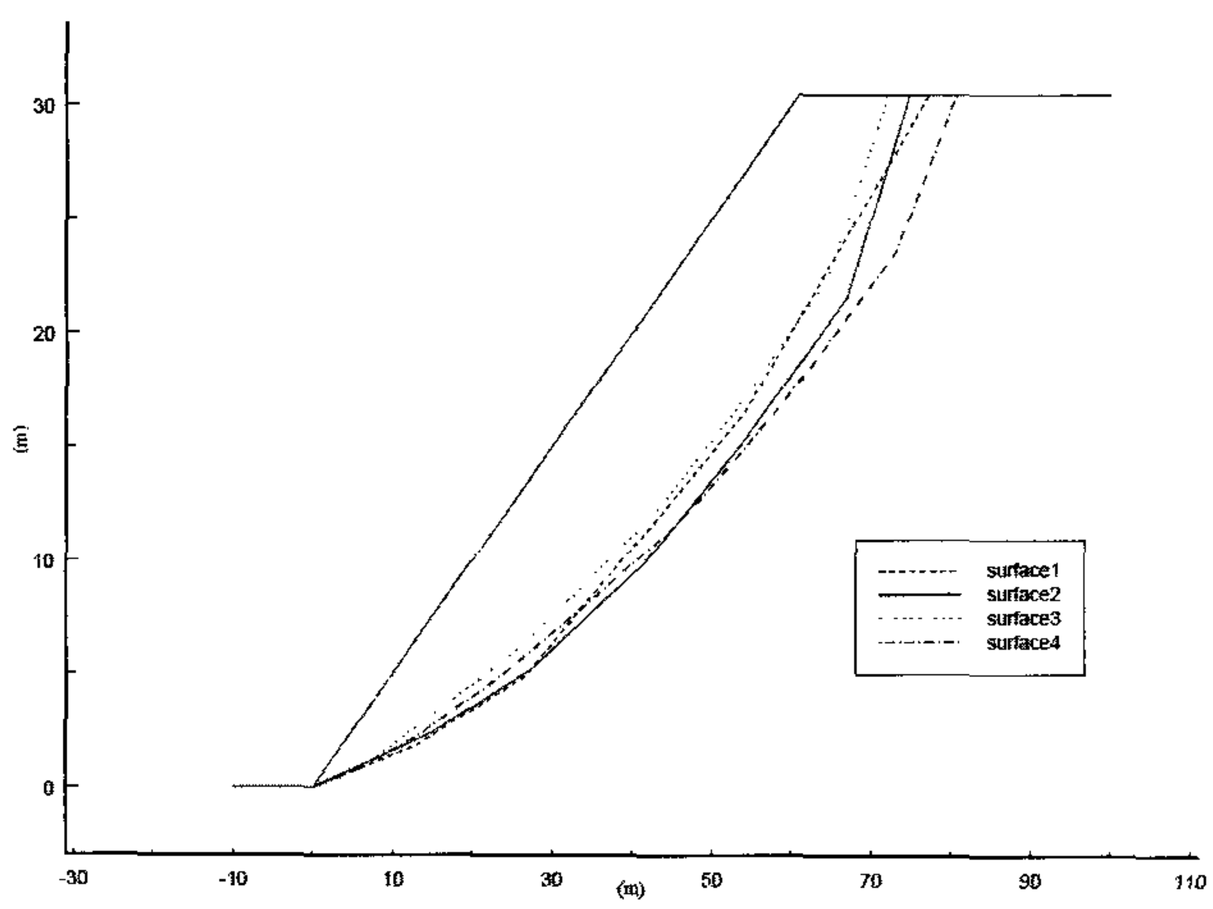

Fig. 4. Comparison of critical slip surfaces by different methods

zone which is also observed by Janbu (1979). The critical accelerations obtained in all these cases are sufficiently small to be taken to be zero as shown in Figure 3. Thus it is prudent to take 8 numbers of slices if vertical slices are considered in the analysis.

In Figure 4 the critical slip surface obtained by 8 number of slices has been compared with the solution reported by Bhattacharya (1990) using Janbu's method with non-linear programming and Patra et al. (2003) using Sarma's method with non-linear programming. From the figure it is seen that the solutions obtained are in close agreement with

Table 3. Comparison of Solutions with other investigators

Surface	Investigator	Factor of Safety
1	Patra et al. (2003)	1.04
2	Bhattacharya (1990)	1.00
3	Spencer (1967)	1.07
4	Present Method	1.04

the known solutions reported in the literature. All these critical surfaces fall in a zone rather than a well-defined failure surface.

4. Concluding Remarks

It has been shown that the GA method that uses probabilistic transition rules rather than deterministic rules means that the search is normally not trapped in local optima unlike other traditional methods. This method is capable of obtaining the optimal solution starting from broad range of domain of design variables.

The critical slip surfaces obtained through GAs crowd over a zone instead of single well-defined surface. The factor of safety obtained by this method is not very much sensitive to the number of slices.

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