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Reliability Equivalence Factors of a Series - Parallel System in Weibull Distribution

M. A. El-Damcese
Department of Mathematics
Faculty of Science, Tanta University, EGYPT

M. M. Khalifa*
Department of Mathematics
Faculty of Education, Kafr El-Sheikh University 33516, EGYPT

Abstract – This paper discusses the reliability equivalences of a series-parallel system. The system components are assumed to be independent and identical. The failure rates of the system components are functions of time and follow Weibull distribution. Three different methods are used to improve the given system reliability. The reliability equivalence factor is obtained using the reliability function. The fractiles of the original and improved systems are also obtained. Numerical example is presented to interpret how to utilize the obtained results.

Key Words: *Reliability, equivalence factors, series- parallel system, Weibull distribution.*

1. INTRODUCTION

In reliability theory, one way to improve the performance of a system is to use the redundancy method. There are two main such methods:

1. Hot duplication method: in this case, it is assumed that some of the system components are duplicated in parallel.
2. Cold duplication method: in this case, it is assumed that some of the system components are duplicated in parallel via a perfect switch.

Unfortunately, for many different reasons, such as space limitation, high cost, etc, it is not always possible to improve a system by duplicating some or all of its components. For example, satellites and space aircrafts have limited space which doesn't allow component duplication. Also, some microchips are so expansive that manufacturers

* Corresponding Author.
E-mail address: meldamcese@yahoo.com

cannot afford to duplicate them. In such cases where duplication is not possible, the engineer turns to another well-known method in reliability theory, the so-called reduction method. In this method, it is assumed that the failure rates of some of the system components are reduced by a factor ρ , $0 < \rho < 1$. Now, once the reduction method is adopted, the main problem facing the engineer is to decide to what degree the failure rate should be decreased in order to improve the system. To solve this problem, one can make equivalence between the reduction method and the duplication method based on some reliability measures. In other words, the design of the system improved by the reduction method should be equivalent to the design of the system improved by one of the duplication methods. The comparison of the designs produces the so-called reliability equivalence factors, see sarhan (2008).

The concept of the reliability equivalence factors was introduced by Rade (1989) and applied to various reliability systems; see Rade (1990, 1991). Rade (1993) applied this concept for the two-component parallel and series systems with independent and identical components whose lifetimes follow the exponential distribution. Sarhan (2000, 2002, 2004, 2005, 2009) derived the reliability equivalence factors of other more general systems. The systems studied by Sarhan are the series system (2000), a basic series-parallel system (2002), a bridge network system (2005), the parallel system (2005), a parallel-series system (2008), and a general series-parallel system (2009). All these systems have independent and identical exponential components.

In this paper we assume that the failure rate of the system components follow Weibull distribution. Unlike the constant failure rate of exponential distribution, Weibull distribution has time varying failure rate.

In the current study, we consider a general series-parallel system and assume that all components are independent and identically Weibull distributed. First, we computed the reliability function of the system. Second, we computed the same reliability measure when the system is improved using the reduction method. Third, we computed the same measure when the system is improved using the hot and cold duplication methods. Finally, we equate the reliability function of the system improved by duplication with the reliability function of the system improved by reduction to get the survival reliability equivalence factors. These factors can be used by the engineer to decide to what degree the failure rate of some of the system components should be decreased in order to improve the performance of the system without duplicating any component.

2. SERIES - PARALLEL SYSTEM

The system considered here consists of m subsystems connected in parallel, with subsystem i consisting of n_i components in series for $i = 1, 2, \ldots, m.$ Figure (2.1) shows the diagram of a series–parallel system.

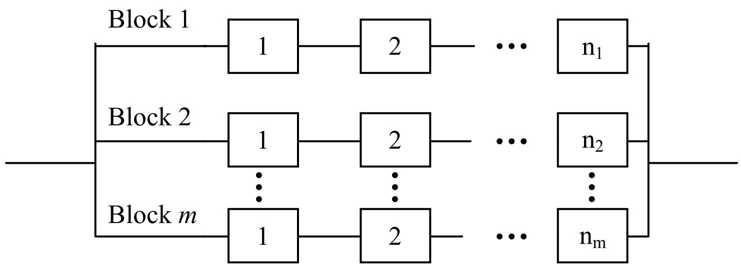

Figure 2.1. a Series-parallel system.

Let $r_i(t)$ be the reliability of subsystem i and $r_{ij}(t)$ be the reliability of component j, $1 \le j \le n_i$, in subsystem i, $i = 1, 2, \ldots, m$. Then

$$r_i(t) = \prod_{j=1}^{n_i} r_{ij}(t) \tag{2.1}$$

The system reliability, $R(t)$, is given by

$$R(t) = 1 - \prod_{i=1}^{m} (1 - r_i(t))$$

Using (2.1) the system reliability is given by:

$$R(t) = 1 - \prod_{i=1}^{m} (1 - \prod_{j=1}^{n_i} r_{ij}(t)) \tag{2.2}$$

Assume that the system components are independent and identical. The lifetime of each component is Weibull distributed with failure rate $z(t) = \lambda^{\beta} \beta\, t^{\beta-1}; \lambda, \beta > 0$. That is $r_{ij}(t) = \exp(-(\lambda t)^{\beta})$, for $1 \le j \le n_i$ and $i = 1, 2, \ldots, m$.

Thus the system reliability becomes

$$R(t) = 1 - \prod_{i=1}^{m} (1 - \exp(-n_i (\lambda t)^{\beta})) \tag{2.3}$$

3. THE IMPROVED SYSTEMS

The reliability of the system can be improved according to one of the following two different methods:

1- Reduction method.
2- Standby redundancy method:
(a) Hot standby redundancy
(b) Cold standby redundancy

In the following sections, we will derive the reliability functions of the systems improved according to the methods mentioned above.

3.1. The Reduction Method

It is assumed in this method that the system can be improved by reducing the failure rates of a set A of system components by a factor s, $0 < s < 1$. Here, we consider that reducing the failure rate by reducing only the scale parameter λ of the set A of system components by a factor ρ.

Assuming that the set A consists of k components; $k \le n$, where n denotes the total number of the system components and the components belonging to A can be distributed into the m subsystems of the system such that k_i components of the subsystem i belong to the set A where $0 \le k_i \le n_i$, $i = 1, 2, \ldots, m$. Such a set is denoted by either $A_{|A|}^{(|A_1|,|A_2|,\ldots,|A_m|)}$ or $A_{|k|}^{(|k_1|,|k_2|,\ldots,|k_m|)}$.

Let $R_{A,\rho}(t)$ denotes the reliability function of the system improved by reducing the scale parameter λ of a set A of its components by a factor ρ. The reliability function of component j in the subsystem i after reducing its scale parameter λ by a factor ρ is given by

$$r_{ij,\rho}(t) = \exp(-(\rho\lambda t)^{\beta}) \quad (3.1)$$

Thus $R_{A,\rho}(t)$ is given as follows:

$$R_{A,\rho}(t) = 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{k_i} r_{ij,\rho}(t) \prod_{j=1}^{n_i - k_i} r_{ij}(t))$$

Hence

$$R_{A,\rho}(t) = 1 - \prod_{i=1}^{m}(1 - \exp(-[n_i + k_i(\rho^{\beta} - 1)](\lambda t)^{\beta})) \quad (3.2)$$

3.2. The Hot Duplication Method

In the hot duplication method each component of a set B is improved by assuming a hot duplication of another identical one. Suppose that the set B consists of h components, $1 \leq h \leq n$. Thus the set B can be written as a union of m disjoint subsets $B_1, \ldots, B_m$ such that the subset B_i contains those components belonging to the subsystem i; $i = 1, 2, \ldots, m$. That is $0 \leq h_i \leq n_i$ such a set is denoted by either $B_{|B|}^{(|B_1|,|B_2|,\ldots,|B_m|)}$ or $B_{|h|}^{(|h_1|,|h_2|,\ldots,|h_m|)}$.

Let $r_{ij}^{H}(t)$ denote the reliability function of the component j in the subsystem i, when it is improved according to hot duplication method. Thus,

$$r_{ij}^{H}(t) = (2 - \exp(-(\lambda t)^{\beta}))\exp(-(\lambda t)^{\beta}). \tag{3.3}$$

Let $R_B^H(t)$ denotes the reliability function of the design obtained by improving the components belonging to the set B according to the hot duplication method. Thus, the function $R_B^H(t)$ can be derived as follows:

$$\begin{aligned} R_B^H(t) &= 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{h_i} r_{ij}^{H}(t) \prod_{j=1}^{n_i - h_i} r_{ij}(t)) \\ &= 1 - \prod_{i=1}^{m}(1 - [2 - \exp(-(\lambda t)^{\beta})]^{h_i} \exp(-n_i(\lambda t)^{\beta})) \end{aligned} \tag{3.4}$$

3.3. The Cold Duplication Method

In the cold duplication method, it is assumed that each component of a set B is connected with an identical component via a perfect switch. Assume that the set B consists of c components, $1 \leq c \leq \mathrm{n}$. Thus, the set B can be written as a union of m disjoint subsets B_1, …, B_m such that the subset B_i contains those components belonging to the subsystem i. That is, $0 \leq c_{\mathrm{i}} \leq \mathrm{n}_{\mathrm{i}}$. We denote such a set by either $B_{|B|}^{(|B_1|,|B_2|,\ldots,|B_m|)}$ or $B_{|c|}^{(|c_1|,|c_2|,\ldots,|c_m|)}$.

Let $r_{ij}^{C}(t)$ denote the reliability function of the component j in the subsystem i; $1 \leq j \leq n_i$, when it is improved according to the cold duplication method. Thus,

$$r_{ij}^{C}(t) = (1 + (\lambda t)^{\beta})\exp(-(\lambda t)^{\beta}) \tag{3.5}$$

Let $R_B^C(t)$ denotes the reliability function of the design obtained by improving the components belonging to the set B according to the cold duplication method. The function $R_B^C(t)$ can be derived as follows:

$$\begin{aligned} R_B^C(t) &= 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{c_i} r_{ij}^{C}(t)\prod_{j=1}^{n_i - c_i} r_{ij}(t)) \\ &= 1 - \prod_{i=1}^{m}(1 - (1 + (\lambda t)^{\beta})^{c_i} \exp(-n_i(\lambda t)^{\beta})) \end{aligned} \tag{3.6}$$

4. RELIABILITY EQUIVALENCE FACTORS

Xia and Zhang (2007) defined the reliability equivalence factor as a factor by which a characteristic of components of a system design has to be multiplied in order to reach equality of a characteristic of this design and a different design regarded as a standard.

As mention above, the reliability equivalence factor is defined as the factor by which the failure rates of some of the system's components should be reduced in order to reach equality of the reliability of another better system. Unlike the constant failure rate of exponential distribution, the failure rate of Weibull distribution is time varying accordingly, the method used to obtain the reliability equivalence factors in the case of using Weibull distribution is different than the method used in the exponential case.

For convenience of calculation, while time varying failure rate is reduced by factor s, we consider that the scale parameter of Weibull distribution is reduced from λ to $\rho\lambda$. From the failure rate of Weibull distribution

$$z(t) = \lambda^{\beta}\beta\, t^{\beta - 1},$$

Then

$$s\, z(t) = (\rho\lambda)^{\beta}\beta\, t^{\beta - 1} \tag{4.1}$$

Obviously, s will increase as ρ increases, and they fall also in interval $(0,1)$. In what follows, we will present how to calculate ρ and we obtain s by taking ρ in equation (4.1). Next, we present some of reliability equivalence factors of the improved series-parallel system studied here.

4.1. Hot Reliability Equivalence Factor

The hot reliability equivalence factor, $s^{H}_{A,B}(\alpha)$, is defined as a factor by which the failure rate of a set A components should be reduced so that one could obtain a design of the system components with a reliability function that equals the reliability function of a design obtained from the original system by assuming hot duplications of a set B of system components.

As mentioned before, the failure rate reduced by $s^{H}_{A,B}(\alpha)$ is equal to the scale parameter reduced from λ to $\rho^{H}_{A,B}(\alpha)\lambda$. That is, $\rho^{H}_{A,B}(\alpha)$ is the solution of the following system of two equations

$$R_{A,\rho}(t)=\alpha, \quad R^{H}_{B}(t)=\alpha \tag{4.2}$$

Therefore, from equations (3.2), (3.4) and (4.2), $\rho^{H}_{A,B}(\alpha)$ is the solution of the following non-linear system of equations with respect to $x=\exp(-(\lambda\, t)^{\beta})$ and ρ for a given α

$$\alpha = 1-\prod_{i=1}^{m}(1-x^{n_i+k_i(\rho^{\beta}-1)}) \tag{4.3}$$

$$\alpha = 1-\prod_{i=1}^{m}(1-(2-x)^{h_i}x^{n_i}) \tag{4.4}$$

The above system of non-linear equations has no closed-form solution. So, a numerical technique method is used to get the solution of such a system. So we have $\rho=\rho^{H}_{A,B}(\alpha)$. Hence the hot reliability equivalence factor $s^{H}_{A,B}(\alpha)$ is obtained from equation (4.1).

4.2. Cold Reliability Equivalence Factor

The cold reliability equivalence factor, $s^{C}_{A,B}(\alpha)$, is defined as a factor by which the failure rate of a set A of system components should be reduced so that one could obtain a design of the system components with a reliability function that equals the reliability function of a design obtained from the original system by assuming cold duplications of a set B of system components.

As mentioned before, the failure rate reduced by $s^C_{A,B}(\alpha)$ is equal to the scale parameter reduced from λ to $\rho^C_{A,B}(\alpha)\lambda$. That is, $\rho^C_{A,B}(\alpha)$ is the solution of the following system of two equations

$$R_{A,\rho}(t)=\alpha, \qquad R^C_B(t)=\alpha \tag{4.5}$$

Therefore, from equations (3.2), (3.6) and (4.5), $\rho^C_{A,B}(\alpha)$ is the solution of the following non-linear system of equations with respect to $x=\exp(-(\lambda\, t)^{\beta})$ and ρ for a given α

$$\alpha = 1-\prod_{i=1}^{m}(1-x^{n_i+k_i(\rho^{\beta}-1)}) \tag{4.6}$$

$$\alpha = 1-\prod_{i=1}^{m}(1-(1+\ln(1/x))^{c_i}x^{n_i}) \tag{4.7}$$

As it seems, the above system of non-linear equations has no closed-form solution. So, a numerical technique method is used to get the solution of such a system. So we have $\rho=\rho^C_{A,B}(\alpha)$. Hence the cold reliability equivalence factor $s^C_{A,B}(\alpha)$ is obtained from equation (4.1).

5. α-FRACTILES

In this section, we deduce the α-fractiles of the original design and the improved designs which are a popular measure of reliability in mechanical industry.

Let $L(\beta,\alpha)$ be the α-fractile of the original system and $L^D_B(\beta,\alpha)$ be the α-fractile of the design obtained by improving the set B components according to hot (D = H) or cold (D = C) duplication method.

The α-fractile of a system having reliability function $R(t)$, $L(\beta,\alpha)$, is defined as the solution of the following equation with respect to L:

$$R(L(\beta,\alpha)/\lambda)=\alpha \tag{5.1}$$

Using equations (2.3) and (5.1), one can obtain L of the original system, by solving the following equation with respect to L:

$$\alpha = 1 - \prod_{i=1}^{m}(1 - \exp(-n_i L^{\beta})) \tag{5.2}$$

Also, the α-fractile of the improved system that has the reliability function $R_B^D(t)$, $L_B^D(\beta,\alpha)$, can be obtained by solving the following equation with respect to L:

$$R_B^D(L(\beta,\alpha)/\lambda) = \alpha, \ D = H,C \tag{5.3}$$

Thus, according to (3.4) and (5.3), one can find $L_B^H(\beta,\alpha)$ by solving the following equation with respect to L:

$$\alpha = 1 - \prod_{i=1}^{m}(1 - (2 - \exp(-L^{\beta}))^{h_i} \exp(-n_i L^{\beta})) \tag{5.4}$$

Finally, using (3.6) and (5.3), one can compute $L_B^C(\beta,\alpha)$ by solving the following equation with respect to L:

$$\alpha = 1 - \prod_{i=1}^{m}(1 - (1 + L^{\beta})^{c_i} \exp(-n_i L^{\beta})) \tag{5.5}$$

Equations (5.2), (5.4) and (5.5) have no closed-form solutions in L, so a numerical technique method is used to get the values of α-fractiles.

6. A NUMERICAL RESULTS

Some numerical results are given in this section to illustrate how to interpret the theoretical results previously obtained.

In the following example, we assume a series-parallel system with $n = 5$, $m = 2$, $n_1 = 2$, $n_2 = 3$ and $\beta = 3$. The components are independent and identical. For such system $\rho_{A,B}^D(\alpha)$; D=H(C) for different sets $A = A_{|A|}^{(|A_1|,|A_2|)}$, $B = B_{|B|}^{(|B_1|,|B_2|)}$ when α = 0.1, 0.5, 0.9 are computed. Tables 6.1 and 6.2 give $\rho_{A,B}^H(\alpha)$ and $\rho_{A,B}^C(\alpha)$, respectively.

The negative value of $\rho_{A,B}^D(\alpha)$ means that it is not possible to reduce the failure rate of the set A components in order to improve the design of system to be equivalent with that design of the system which can be obtained by improving the set B components according to the redundancy methods. Table 6.3 gives the α-fractiles of the original system for α = 0.1, 0.5, 0.9. Table 6.4 gives the α-fractiles of the systems improved according to hot and cold duplication methods for α = 0.1, 0.5, 0.9.

Table 6.1. $\rho^{H}_{A,B}(\alpha)$

	α	$B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(0,2)}$	$B_2^{(1,1)}$	$B_2^{(2,0)}$	$B_3^{(1,2)}$	$B_3^{(0,3)}$	$B_3^{(2,1)}$	$B_4^{(1,3)}$	$B_4^{(2,2)}$	$B_5^{(2,3)}$
$A_1^{(0,1)}$	0.1	-ve	0.836	0.589	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.5	-ve	0.715	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	0.537	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_1^{(1,0)}$	0.1	0.851	0.956	0.897	0.828	0.684	0.793	0.824	0.673	0.743	0.654	0.623
	0.5	0.733	0.876	0.705	0.628	-ve	0.438	0.385	-ve	-ve	-ve	-ve
	0.9	0.556	0.779	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_2^{(1,1)}$	0.1	0.874	0.964	0.915	0.854	0.721	0.822	0.850	0.711	0.776	0.694	0.665
	0.5	0.815	0.915	0.796	0.744	0.459	0.635	0.612	0.360	0.427	-ve	-ve
	0.9	0.744	0.865	0.622	0.577	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_2^{(2,0)}$	0.1	0.931	0.978	0.951	0.922	0.871	0.908	0.920	0.867	0.890	0.862	0.853
	0.5	0.886	0.942	0.877	0.854	0.775	0.815	0.809	0.762	0.770	0.744	0.721
	0.9	0.837	0.903	0.787	0.772	0.633	0.693	0.649	0.607	0.599	0.573	0.528
$A_2^{(0,2)}$	0.1	0.788	0.925	0.844	0.762	0.616	0.724	0.758	0.606	0.673	0.588	0.561
	0.5	0.758	0.881	0.736	0.679	0.401	0.567	0.544	0.313	0.373	-ve	-ve
	0.9	0.699	0.833	0.572	0.528	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_3^{(2,1)}$	0.1	0.936	0.980	0.955	0.927	0.875	0.913	0.925	0.872	0.895	0.866	0.857
	0.5	0.902	0.952	0.894	0.872	0.794	0.834	0.828	0.780	0.789	0.762	0.737
	0.9	0.866	0.923	0.821	0.808	0.672	0.733	0.689	0.645	0.637	0.609	0.560
$A_3^{(1,2)}$	0.1	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	-ve	-ve	0.680	0.758	0.746	0.651	0.669	0.607	0.538
	0.9	0.831	0.906	0.765	0.744	0.441	0.607	0.495	0.307	0.227	-ve	-ve
$A_3^{(0,3)}$	0.1	0.871	0.952	0.902	0.856	0.788	0.837	0.854	0.784	0.813	0.777	0.767
	0.5	0.854	0.924	0.843	0.815	0.722	0.769	0.761	0.707	0.716	0.687	0.661
	0.9	0.825	0.896	0.771	0.756	0.609	0.672	0.626	0.581	0.573	0.546	0.498
$A_4^{(2,2)}$	0.1	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	-ve	-ve	0.818	0.856	0.850	0.806	0.814	0.788	0.764
	0.9	0.891	0.937	0.853	0.842	0.722	0.777	0.737	0.697	0.689	0.663	0.614
$A_4^{(1,3)}$	0.1	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	-ve	-ve	0.782	0.827	0.819	0.768	0.777	0.747	0.718
	0.9	0.875	0.929	0.833	0.819	0.685	0.747	0.702	0.658	0.649	0.620	0.568
$A_5^{(2,3)}$	0.1	0.947	0.984	0.963	0.939	0.893	0.927	0.937	0.889	0.911	0.885	0.877
	0.5	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve

Table 6.2. $\rho^{C}_{A,B}(\alpha)$

	α	$B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(0,2)}$	$B_2^{(1,1)}$	$B_2^{(2,0)}$	$B_3^{(1,2)}$	$B_3^{(0,3)}$	$B_3^{(2,1)}$	$B_4^{(1,3)}$	$B_4^{(2,2)}$	$B_5^{(2,3)}$
$A_1^{(0,1)}$	0.1	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_1^{(1,0)}$	0.1	-ve	-ve	0.804	-ve	-ve	-ve	0.604	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	0.548	0.437	-ve	-ve	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_2^{(1,1)}$	0.1	-ve	-ve	0.832	-ve	-ve	-ve	0.647	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	0.694	0.634	-ve	0.375	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	-ve	0.520	0.476	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_2^{(2,0)}$	0.1	0.402	0.314	0.913	0.286	0.197	0.289	0.848	0.287	0.290	0.781	0.769
	0.5	0.323	0.257	0.835	0.815	0.162	0.763	0.729	0.244	0.695	0.665	0.637
	0.9	0.186	0.149	0.757	0.747	0.554	0.654	0.572	0.532	0.527	0.501	0.452
$A_2^{(0,2)}$	0.1	-ve	-ve	0.736	-ve	-ve	-ve	0.544	-ve	-ve	-ve	-ve
	0.5	-ve	-ve	0.627	0.566	-ve	0.326	-ve	-ve	-ve	-ve	-ve
	0.9	-ve	-ve	0.473	0.431	-ve	-ve	-ve	-ve	-ve	-ve	-ve
$A_3^{(2,1)}$	0.1	0.510	0.425	0.918	0.397	0.301	0.400	0.852	0.397	0.401	0.783	0.771
	0.5	0.422	0.355	0.854	0.834	0.250	0.782	0.746	0.340	0.709	0.677	0.646
	0.9	0.251	0.211	0.794	0.784	0.588	0.693	0.608	0.565	0.559	0.530	0.474
$A_3^{(1,2)}$	0.1	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	0.539	0.493
	0.5	-ve	-ve	0.792	0.758	-ve	0.655	0.565	-ve	0.438	0.136	-ve
	0.9	-ve	-ve	0.720	0.704	-ve	0.509	-ve	-ve	-ve	-ve	-ve
$A_3^{(0,3)}$	0.1	0.463	0.381	0.843	0.354	0.266	0.358	0.761	0.355	0.358	0.689	0.678
	0.5	0.378	0.315	0.792	0.769	0.220	0.709	0.670	0.301	0.632	0.599	0.570
	0.9	0.220	0.185	0.739	0.729	0.525	0.630	0.544	0.503	0.498	0.469	0.418
$A_4^{(2,2)}$	0.1	0.621	0.552	-ve	0.528	0.446	0.532	-ve	0.529	0.532	0.789	0.777
	0.5	0.542	0.483	0.874	0.856	0.386	0.807	0.773	0.469	0.736	0.703	0.670
	0.9	0.354	0.312	0.829	0.821	0.642	0.741	0.662	0.619	0.614	0.583	0.523
$A_4^{(1,3)}$	0.1	0.592	0.519	-ve	0.495	0.411	0.499	-ve	0.496	0.499	0.720	0.706
	0.5	0.507	0.447	0.848	0.827	0.349	0.769	0.729	0.433	0.687	0.649	0.613
	0.9	0.317	0.278	0.806	0.797	0.598	0.707	0.619	0.574	0.568	0.536	0.474
$A_5^{(2,3)}$	0.1	-ve	-ve	0.931	-ve	-ve	-ve	0.872	-ve	-ve	0.806	0.794
	0.5	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	0.775	0.747	0.718
	0.9	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve

Table 6.3. the α-fractiles $L(\beta,\alpha)$

α	0.1	0.5	0.9
$L(\beta,\alpha)$	1.082	0.798	0.538

Table 6.4. the α-fractiles $L_B^D(\beta,\alpha)$

	α	$B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(2,0)}$	$B_2^{(0,2)}$	$B_3^{(2,1)}$	$B_3^{(1,2)}$	$B_3^{(0,3)}$	$B_4^{(2,2)}$	$B_4^{(1,3)}$	$B_5^{(2,3)}$
L_B^H	0.1	1.143	1.099	1.153	1.212	1.123	1.216	1.167	1.154	1.223	1.188	1.234
	0.5	0.859	0.827	0.881	0.945	0.866	0.957	0.910	0.915	0.974	0.949	0.999
	0.9	0.592	0.568	0.619	0.696	0.613	0.714	0.659	0.685	0.741	0.720	0.782
L_B^C	0.1	1.187	1.112	1.200	1.333	1.162	1.335	1.227	1.241	1.343	1.279	1.362
	0.5	0.882	0.836	0.910	1.037	0.895	1.048	0.955	0.990	1.069	1.029	1.112
	0.9	0.599	0.571	0.631	0.758	0.626	0.778	0.683	0.742	0.811	0.782	0.874

7. CONCLUSION

In this paper we discussed the reliability equivalence of a series-parallel system with identical and independent components. It is assumed that the components of the system had time varying failure rates. Three ways namely the reduction, hold duplication and cold duplication methods are used to improve the system reliability. A reliability equivalence factor was derived. A numerical example is used to illustrate how the results obtained can be applied. In the future we hope that we will be able to study the reliability equivalence of more complicated systems with non identical components.

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