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Optimal Maintenance Scheduling in a Two Identical Component Parallel Redundant System Subject to Exponential Power Hazards

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Abstract. This paper presents equations, which can be used to evaluate the failure frequency and the failure rate of a two identical component parallel redundant system in which each component can operate in its wear out period, and the failure rate of each component is exponential power distribution. The optimum maintenance interval for a two identical component parallel redundant system can be obtained using these equations. The proposed approach is presented and illustrated using several numerical examples. The optimum maintenance interval for each component in a two identical parallel redundant system will depend on factors such as: failure rate, repair and maintenance times of each component in the parallel redundant systems.

Key words : *Failure frequency, exponential power distribution, two identical parallel redundant system, and optimum maintenance interval.*

1. INTRODUCTION

Component preventive maintenance is used to increase system availability and reduce the likelihood of failure. The criteria associated with a preventive maintenance plan depend on its cost, the type of system, and the reliability and availability requirements. The optimum preventive maintenance schedule of a component or system can be based on criteria such as minimizing the total cost with the desired or specified levels of operational safety and reliability, maximizing the availability of the component or system or optimizing some other specified objective.

The intended application in this paper is to elements of a system and the primary objective is to minimize the frequency of system failure. The concepts presented can be extended to include maintenance, repair and customer interruption costs [1-3].

Many power system components have a relatively short useful life [4]. These components can be made to remain within their useful life period for the bulk of their economically feasible life by regular and careful preventive maintenance. Useful life evaluation is invalid and could be extremely optimistic if the system contains components which are operating within their wear-out period. Preventive maintenance of a component is usually scheduled at regular intervals to prevent the component from entering the wear-out region .The regular maintenance schedule of a component may not, however, be the best maintenance schedule for the system.

A basic and common form of redundancy is to connect two identical components in parallel. This redundancy exists because the basic system design philosophy recognizes and, therefore, anticipates the possibility of component failure and the need to remove a component from service for preventive maintenance. A major cause of double contingency outages in a parallel redundant system is the occurrence of the component failure during the period when another component is out for maintenance. Component preventive maintenance is usually performed to limit the individual component failure rates. If the maintenance rate of a component is reduced in order to decrease the probability of occurrence of the double contingency noted above, then the individual component failure rate may increase significantly. A further possible cause of failure of parallel redundant configuration is due to common mode or common cause failures. This type of failure is not generally related to the maintenance associated with each individual component and does not influence the determination of an optimum maintenance schedule. This should, however, be examined as an important mode of system failure [5, 6].

In this paper, an assumption is made that each component in a two identical parallel redundant system operates initially in its useful life period, and can enter its wear-out period. Relationships between component failure rates and maintenance schedules are created and a set of equations developed to obtain the optimum maintenance interval in a two identical parallel redundant system. The optimum maintenance interval is determined based on the criterion that the total failure frequency of the parallel redundant system is minimized. Equations developed to determine the optimum maintenance interval based on the system failure rate are presented. The application of the proposed method is illustrated through using several numerical examples and the results obtained are compared.

2. EXPONENTIAL POWER DISTRIBUTION

In practice, exponential power distribution have been shown to be very flexible in modeling various types of lifetime distributions and they have been used to model any of the three parts in a bathtub-curve.

The hazard rate function of a component following exponential power distribution "as shown in Figure 1 " is given as [7]:

$$\lambda(t) = \alpha \beta t^{\beta-1} e^{\alpha t^{\beta}} \qquad ,t \geq 0, \alpha > 0, \beta > 0 \tag{2.1}$$

where α is a scale parameter, and β is the shape parameter.

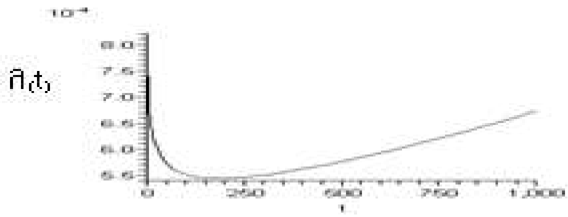

Figure 2.1. Exponential power hazard function ($\alpha = .000866$ and $\beta = .91$) against time.

The reliability function for such a case is given by:

$$R(t) = \exp[-\int_0^t \lambda(\mathrm{t})\mathrm{dt}] = \exp[1 - \mathrm{e}^{\alpha \mathrm{t}^{\beta}}] \quad (2.2)$$

The failure density function shown as Figure 2 is obtained by:

$$f(t) = -\frac{dR(t)}{dt} = \alpha\, \beta\, t^{\beta-1}\, \mathrm{e}^{\alpha \mathrm{t}^{\beta}} \exp[1 - \mathrm{e}^{\alpha \mathrm{t}^{\beta}}] \quad (2.3)$$

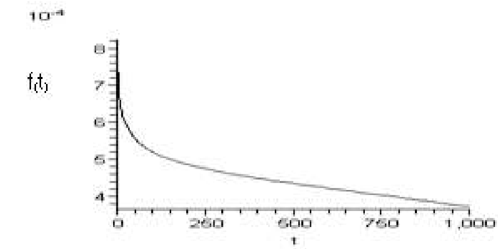

Figure 2.2. Exponential power density function with ($\alpha = .000866$ and $\beta = .91$) against time

The distribution takes the shape of bathtub with a minimum value of the hazard function, $\lambda(t)$ at $t_{\min}$, when $\frac{d\lambda(t)}{dt} = 0$, and then we have

$$t_{\min} = \left(\frac{1-\beta}{\alpha\beta}\right)^{1/\beta} \quad (2.4)$$

After $t_{\min}$, the hazard rate function continuous to increase with time, so one can evaluate the average value of $\lambda(t)$ during an operational period starting from $t_{\min}$ and ending by $t_{\max}$ as follows:

$$\hat{\lambda} = \bar{\lambda}(t) = \frac{1}{t_{max} - t_{min}} \int_{t_{min}}^{t_{max}} \lambda(t)\, dt$$

$$\hat{\lambda} = \frac{1}{t_{max} - t_{min}} \left\{ \exp\left[t_{max}^{\beta} \left(\frac{1-\beta}{\beta\ t_{min}^{\beta}} \right) \right] - \exp\left[\frac{1-\beta}{\beta} \right] \right\} \tag{2.5}$$

Since from equation (2.4), one can obtain: $\alpha = \frac{1-\beta}{\beta\ t_{min}^{\beta}}$

Assuming that the parameters t_{min} , t_{max} and $\hat{\lambda}$ are known, equation (2.5) can be solved for β by a simple iterative procedure. Then, equation (2.4) is used to obtain α .

3. TIME-DEPENDENT FAILURE RATE

The useful life phase is characterized by a constant hazard rate (λ) and is associated with an exponential distribution as shown in Figure 3.1 The wear-out phase is characterized by a rapidly increasing hazard rate and exponential power as shown in Figure 2.1. This distribution is used in this chapter to illustrate the proposed procedure.

The modified distribution shown in Figure 3.2 was obtained by combining Figure 2.2 and 3.1. Then, we can say that area in Figure 2.2 must, therefore, be equal to area in Figure 3.1 or:

$$\int_0^{t_u} \lambda\, e^{-\lambda t}\, dt = \int_0^{t_u} \alpha\, \beta\, t^{\beta-1}\, e^{\alpha t^{\beta}}\, e^{(1-e^{\alpha t^{\beta}})} dt$$

From integration both sides, we have

$$T = \frac{[\, \ln(1+\lambda\, t_u)\,]^{1/\beta}}{\alpha^{1/\beta}} \tag{3.1}$$

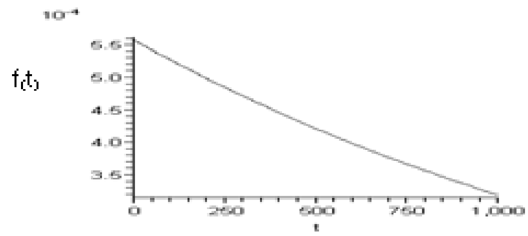

Figure 3.1. Exponential density function with ($\lambda = .000557$) against time

The point in time t_u at which the component leaves the useful life period and begins to wear-out.

The modified failure density function as shown in fig 4:

$$f(t)=\begin{cases}\lambda\, e^{-\lambda t}, & t\le t_u\\ \alpha\,\beta\,(t+T-t_u)^{\beta-1}\, e^{\alpha\,(t+T-t_u)^{\beta}}\ \exp[1-e^{\alpha\,(t+T-t_u)^{\beta}}], & t\ge t_u\end{cases}$$

(3.2)

The survivor function is

$$R(t)=\int_0^{\infty} f(t)\,dt=\begin{cases}e^{-\lambda t}, & t_u\le t\\ \exp[1-e^{\alpha\,(t+T-t_u)^{\beta}}], & t_u\ge t\end{cases} \tag{3.3}$$

The hazard rate is:

$$\lambda(t)=\frac{f(t)}{R(t)}=\begin{cases}\lambda\, e^{-\lambda t}, & t\le t_u\\ \alpha\,\beta\,(t+T-t_u)^{\beta-1}\, e^{\alpha\,(t+T-t_u)^{\beta}}, & t\ge t_u\end{cases} \tag{3.4}$$

If $t=t_u$ in the last equation, from equation (3.1), we find:

$$\lambda=\alpha\,\beta\ T^{\beta-1}\, e^{\alpha\, T^{\beta}} \tag{3.5}$$

Using equation (3.1),

$$\alpha\,\beta\ T^{\beta-1}\, e^{\alpha\, T^{\beta}}=\beta\,\alpha^{1/\beta}\,[\ln(1+\lambda\, t_u)]^{1-1/\beta}\,[1+\lambda\, t_u]=\lambda \tag{3.6}$$

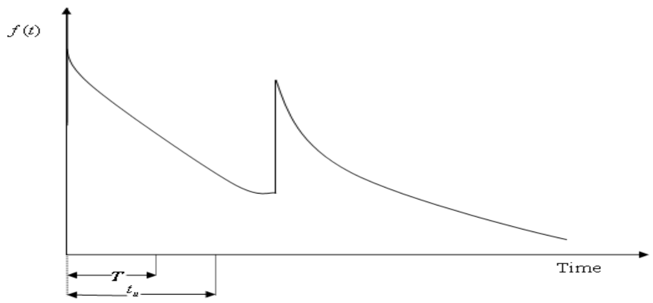

Figure 3.2. The modified failure density function of each component

The conditional probability of failure $Q_c(x)$

$$Q_c(\mathrm{x}) = \frac{P(t \prec \mathrm{T} \prec \mathrm{t+x})}{R(t)} = \frac{\int_{\mathrm{t}}^{\mathrm{t+x}} \mathrm{f(t)\,dt}}{\mathrm{R(t)}}$$

Since $\int_t^{t+x} f(t)\,dt = \int_t^{t+x} \lambda\, \mathrm{e}^{-\lambda \mathrm{t}} dt = \mathrm{e}^{-\lambda \mathrm{t}} - \mathrm{e}^{-\lambda (\mathrm{t+x})}$, $\mathrm{t} \le \mathrm{t_u}$,

$$\int_{t+T-t_u}^{t+T-t_u+x} f(t)\,\mathrm{dt} = \int_{t+T-t_u}^{t+T-t_u+x} \alpha\, \beta\, t^{\beta\text{-}1}\, \mathrm{e}^{\alpha\, t^{\beta}} \exp[1-\mathrm{e}^{\alpha\, t^{\beta}}]$$

$$= \exp[1 - \mathrm{e}^{\alpha\,(t+T-t_u)^{\beta}}] - \exp[1 - \mathrm{e}^{\alpha\,(t+T-t_u+x)^{\beta}}], \quad \mathrm{t} \ge \mathrm{t_u}$$

Then

$$Q_c(x) = \begin{cases} 1 - \mathrm{e}^{\alpha\, x^{\beta}}, & \mathrm{t} \le \mathrm{t_u} \\ \exp[1 - \mathrm{e}^{\alpha\,(t+T-t_u)^{\beta}}] - \exp[1 - \mathrm{e}^{\alpha\,(t+T-t_u+x)^{\beta}}], & \mathrm{t} \ge \mathrm{t_u} \end{cases} \quad (3.7)$$

4. CALCULATION OF SYSTEM FAILURE FREQUENCY

The parameters are defined as follows:
t_m : is the maintenance interval of each component.
$f_{m1}(t_m)$, $f_{m2}(t_m)$: are the maintenance outage frequencies of components 1 and 2 respectively.
$f_{f1}(t_m)$, $f_{f2}(t_m)$: are the failure frequencies of components1 and 2 respectively.
$f_{pp}(t_m)$: is the failure frequency due to a forced outage overlapping a forced outage.
$f_{pm}(t_m)$: is the failure frequency due to a forced outage overlapping a maintenance outage.
$f_T(t_m)$: is the total failure frequency of the two parallel redundant components.

The equations for the total failure rate of a two identical component parallel redundant system are:

$f_{pp}(t_m) = f_{f1}(t_m)$ * [Probability that 2 fails during the repair time of 1]

$+ f_{f2}(t_m)$ * [Probability that 1 fails during the repair time of 2]

$$f_{pp}(t_m) = f_{f1}(t_m) * \mathrm{P}[\mathrm{t_m} \le T_2 \le \mathrm{t_m} + r | T_2 \succ \mathrm{t_m}]$$
$$+ f_{f2}(t_m) * \mathrm{P}[\mathrm{t_m} \le T_1 \le \mathrm{t_m} + r | T_1 \succ \mathrm{t_m}]$$

Thus $f_{pp}(t_m)$ is:

$$f_{pp}(t_m) = f_{f1}(t_m) * Q_c(x = r)\Big|_{\mathrm{t=t_m}} + f_{f2}(t_m) * Q_c(x = r)\Big|_{\mathrm{t=t_m}} \tag{4.1}$$

$+ \ f_{m2}(t_m)$ * [Probability that 1 fails during the maintenance of 2]

Thus $f_{pm}(t_m)$ is:

$$f_{pm}(t_m) = f_{m1}(t_m) * Q_c(x = r'')\Big|_{\mathrm{t=t_m}} + f_{m2}(t_m) * Q_c(x = r'')\Big|_{\mathrm{t=t_m}} \tag{4.2}$$

and $f_{\mathrm{T}}(\mathrm{t_m}) = f_{\mathrm{pp}}(\mathrm{t_m}) + f_{pm}(t_m)$

the average repair time r = r_1 = r_2, and the average maintenance time $r'' = r_1'' = r_2''$.

also , $f_{f1}(\mathrm{t_m}) = f_{f2}(\mathrm{t_m}) = \lambda_a(t_m)$

Where $\lambda_a(t_m)$ is the average failure rate of a component during t_m is given as follows:

$$\lambda_a(t_m) = \frac{1}{\mathrm{t_m}} \int_0^{t_m} \lambda(t)\,\mathrm{dt}$$
$$= \frac{1}{\mathrm{t_m}} [\int_0^{t_u} \lambda\,\mathrm{dt} + \int_{t_u}^{t_m} \alpha\,\beta\,(\mathrm{t + T - t_u})^{\beta-1}\ \mathrm{e}^{\,(\mathrm{t+T-t_u})^{\beta}}\,\mathrm{dt}\,]$$
$$= \frac{1}{\mathrm{t_m}} [\int_0^{t_u} \lambda\,\mathrm{dt} + \int_{t_u}^{t_m} \alpha\,\beta\,(\mathrm{t + T - t_u})^{\beta-1}\ \mathrm{e}^{\,(\mathrm{t+T-t_u})^{\beta}}\,\mathrm{dt}\,] \tag{4.3}$$

and $f_m(t_m) = f_{m1}(t_m) = f_{m2}(t_m) = 1/t_m$ (Occ./hour)

Using equations (3.7), (4.1), (4.2) and (4.3) we have:

$$f_{pp}(t_m) = \frac{2}{\mathrm{t_m}} [\lambda\,\mathrm{t_u} + \mathrm{e}^{\alpha\,(t_m+T-t_u)^{\beta}} - \mathrm{e}^{\alpha\,T^{\beta}}\,] * [1 - \exp(\mathrm{e}^{\alpha\,(t_m+T-t_u)^{\beta}} - \mathrm{e}^{\alpha\,(t_m+T-t_u+r)^{\beta}}\ )] \tag{4.4}$$

and

$$f_{pm}(t_m) = \frac{2}{\mathrm{t_m}}[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})] \tag{4.5}$$

Then $f_T(t_m) = \frac{2}{\mathrm{t_m}}[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$

$$+ \frac{2}{\mathrm{t_m}}[\lambda\,\mathrm{t_u} + \mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha\mathrm{T}^\beta}] * [1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta})] \tag{4.6}$$

The optimum maintenance interval for a two component parallel redundant system $t_{m.\mathrm{opt}}$ can be obtained by setting $df_T(t_m)/dt_m$ to zero in the period of time (t_u, ∞), when $df_T(t_m)/dt_m$ is not zero in the period of time (t_u, ∞), then t_u is the optimum maintenance interval for the two component parallel redundant system.

Letting $df_T(t_m)/dt_m = 0$

$$\begin{aligned}
df_T(t_m)/dt_m = & -\frac{2}{\mathrm{t_m^2}}[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})] \\
& -\frac{2}{\mathrm{t_m}}[\exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})] \\
& *[\alpha\beta(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \alpha\beta(t_m+T+r''-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta}] \\
& +\frac{2}{\mathrm{t_m}}[\lambda\,\mathrm{t_u} + \mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha\mathrm{T}^\beta}] * [\exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta})] \\
& *[\alpha\beta(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \alpha\beta(t_m+T+r-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta}] \\
& -\frac{2}{\mathrm{t_m^2}}[\lambda\,\mathrm{t_u} + \mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha\mathrm{T}^\beta}] * [1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta})] \\
& +\frac{2}{t_m}[\alpha\beta(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta}] * [1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta})] \\
= & \, 0
\end{aligned} \tag{4.7}$$

5. CALCULATION OF SYSTEM FAILURE RATE

The parameters are defined as follows:

t_m : is the maintenance interval of each component.

$\lambda_1''(t_m)$, $\lambda_2''(t_m)$: are the maintenance outage rates of components 1 and 2 respectively.
$\lambda_1(t_m)$, $\lambda_2(t_m)$: are the failure rates of components1 and 2 respectively.
$\lambda_{pp}(t_m)$: is the system failure rate due to a forced outage overlapping a forced outage.
$\lambda_{pm}(t_m)$: is the system failure rate due to a forced outage overlapping a maintenance outage.
$\lambda_T(t_m)$: is the total system failure rate of the two parallel redundant components.

The equations for the total failure rate of a two identical component parallel redundant system are:

$\lambda_{pp}(t_m) = \lambda_1(t_m)*$ [Probability that 2 fails during the repair time of 1]
$+ \lambda_2(t_m)$ *[Probability that 1 fails during the repair time of 2]

Thus $\lambda_{pm}(t_m)$ is:

$$\lambda_{pp}(t_m) = \lambda_1(t_m) * Q_c(x=r)\Big|_{t=t_m} + \lambda_2(t_m) * Q_c(x=r)\Big|_{t=t_m} \tag{5.1}$$

$\lambda_{pm}(t_m) =$ $\lambda_1''(t_m)$ *[Probability that 2 fails during the maintenance of 1]
$+ \lambda_2''(t_m)$ *[Probability that 1 fails during the maintenance of 2]

Thus $\lambda_{pm}(t_m)$ is:

$$\lambda_{pm}(t_m) = \lambda_1''(t_m) * Q_c(x=r'')\Big|_{t=t_m} + \lambda_2''(t_m) * Q_c(x=r'')\Big|_{t=t_m} \tag{5.2}$$

and $\lambda_{\mathrm{T}}(\mathrm{t_m}) = \lambda_{\mathrm{pp}}(\mathrm{t_m}) + \lambda_{pm}(t_m)$

for $\lambda_1(t_m) = \lambda_2(t_m) = \alpha\,\beta\,(\mathrm{t_m} + \mathrm{T} - \mathrm{t_u})^{\beta - 1}\, \mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u})^{\beta}}$

and $\lambda_1''(t_m) = \lambda_2''(t_m) = 1/\ t_m$ (Occ. /hour)

Moreover, using the equations (12), (20) and (21) we have:

$$\lambda_{pp}(t_m) = 2\alpha\,\beta\,(\mathrm{t_m} + \mathrm{T} - \mathrm{t_u})^{\beta-1}\, \mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u})^{\beta}} \\ *[1 - \exp(\mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u})^{\beta}} - \mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u}+r)^{\beta}})] \tag{5.3}$$

Also $$\lambda_{pm}(t_m) = \frac{2}{\mathrm{t_m}}[1 - \exp(\mathrm{e}^{\alpha\,(t_m+T-t_u)^{\beta}} - \mathrm{e}^{\alpha\,(t_m+T-t_u+r'')^{\beta}})] \tag{5.4}$$

Then

$$\lambda_T(t_m) = \frac{2}{\mathrm{t_m}}[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})] + 2\,\alpha\,\beta$$
$$*(\mathrm{t_m} + \mathrm{T} - \mathrm{t_u})^{\beta-1}\,\mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u})^\beta} *[1 - \exp(\mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u})^\beta} - \mathrm{e}^{\alpha\,(\mathrm{t_m}+\mathrm{T}-\mathrm{t_u}+r)^\beta})] \tag{5.5}$$

The optimum maintenance schedule for each component in a two identical parallel redundant system can be obtained by setting $d\lambda_T(t_m)/dt_m$ to zero.

Letting $d\lambda_T(t_m)/dt_m$=0

$$d\lambda_T(t_m)/dt_m = -\frac{2}{\mathrm{t_m^2}}[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$$
$$-\frac{2}{\mathrm{t_m}}[\exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$$
$$*[\alpha\,\beta\,(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \alpha\,\beta\,(t_m+T+r''-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta}]$$
$$+2\alpha\,\beta\,(\beta-1)\,(t_m+T-t_u)^{\beta-2}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta}\,[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$$
$$-2\,\alpha\,\beta\,(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta}\,[\exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$$
$$*[\alpha\,\beta\,(t_m+T-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \alpha\,\beta\,(t_m+T+r-t_u)^{\beta-1}\,\mathrm{e}^{\alpha(t_m+T-t_u+r)^\beta}]$$
$$+2\alpha^2\,\beta^2\,(t_m+T-t_u)^{2\beta-2}\,\mathrm{e}^{\alpha(t_m+T-t_u)^\beta}\,[1 - \exp(\mathrm{e}^{\alpha(t_m+T-t_u)^\beta} - \mathrm{e}^{\alpha(t_m+T-t_u+r'')^\beta})]$$
$$= 0 \tag{5.6}$$

6. NUMERICAL EXAMPLE

The system data is as follows: r = 8(hours), r'' = 20(hours), and the parameters $\beta, \alpha, \lambda, t_u, T, t_m, f_{pp}(t_m), f_{pm}(t_m)$ *and* $f_T(t_m)$ of each component was obtained by using equations (3.1), (3.6), (4.7) and (4.4)- (4.6)and are given in Table 6.1.

Table 6.1. Test values of various parameters

β	α	λ	t_u	T	t_m	$f_{pp}(t_m)$	$f_{pm}(t_m)$	$f_T(t_m)$
0.91	8.7×10^{-4}	5.6×10^{-4}	252.76	250	1800	9×10^{-6}	2×10^{-5}	2.9×10^{-5}
0.97	2×10^{-5}	1.6×10^{-5}	823.67	800	53200	1×10^{-8}	2.3×10^{-8}	3.3×10^{-8}

The effect of component failure rate $\lambda\ (t_m)$ on the optimum maintenance schedule for each component in a two identical component parallel redundant system can be obtained using equation (5.6) is given in Table 6.2.

Table 6.2. Test values of parameters for the example

β	α	λ	t_u	T	t_m	$\lambda_{pp}(t_m)$	$\lambda_{pm}(t_m)$	$\lambda_T(t_m)$
0.91	8.7×10^{-4}	5.6×10^{-4}	252.76	250	1600	1.9×10^{-5}	2.1×10^{-5}	4×10^{-5}
0.97	2×10^{-5}	1.6×10^{-5}	823.67	800	46000	1.2×10^{-8}	3.5×10^{-8}	4.7×10^{-8}

The general relationship between the maintenance interval t_m and the system failure frequency is shown in Figure 6.1 for Table 6.1.

The characteristics of $f_{pp}(t_m)$, $f_{pm}(t_m)$, $f_T(t_m)$ can be plotted as shown in Figure 6.2 for Table 6.1.

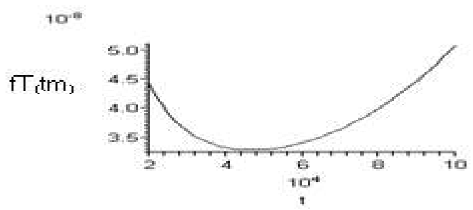

Figure 6.1. Relationship between maintenance interval and system failure frequency

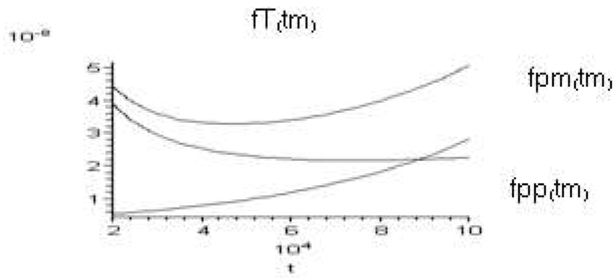

Figure 6.2. Relationship between maintenance interval and failure frequency due to [preventive maintenance and forced outage, preventive maintenance, and forced outage] respectively.

7. CONCLUSION

Equations for the total failure frequency or the total failure rate of a two identical component parallel redundant system have been developed. Optimum preventive maintenance scheduling of each component in a two identical parallel redundant system was obtained by minimizing the total failure frequency of the system. The maintenance interval of each component in a two identical parallel redundant system will depend on factors such as failure rate, repair rate and maintenance times of each component in the system.

It should be clearly appreciated that a component failure can overlap a failure or a planned maintenance activity on the other component but a planned maintenance action would not be initiated if the other component were failed or undergoing maintenance.

In general, the maintenance interval (schedule) increases as the component failure rate decreases, the maintenance interval (schedule) also increases as the parameter β increases and α decreases, the total failure frequency (rate) decreases as the parameters t_u , T and t_m increases. In conclusion, the proposed analytical method is a feasible technique for analyzing the effect of preventive maintenance scheduling in two identical parallel redundant systems.

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