Analytical Models to Predict Power Harvesting with Piezoelectric Transducer

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Abstract

Advances in low power design open the possibility to harvest energy from the environment to power electronic circuits. Electrical energy can be harvested from piezoelectric transducer. Piezoelectric materials can be used as mechanisms to transfer mechanical energy usually vibrating system into electrical energy that can be stored and used to power other devices. Micro- to milli-watts power can be generated from vibrating system. We developed definitive and analytical models to predict the power generated from a cantilever beam attached with piezoelectric transducer. Analytical models are pin-force method, enhanced pin-force method and Euler-Bernoulli method. Harmonic oscillations and random noise will be the two different forcing functions used to drive each system. It has been selected the best model for generating electric power based upon the analytical results obtained.

Key words : Vibration-to-Electric Energy Conversion, Self-Powered Sensors, PZT, Cantilever Beam, Power Harvesting, Harmonic Driving Force.

I. Introduction

In today's world, we are unmistakably moving towards a technological way of life. More and more people are using portable electronic devices than ever before during travel. These devices require very less power but at the same time offer versatility in communication and problem solving. But as the technology for portables has grown tremendously, battery and energy storage technology has not kept up. New technology allows for these portables to become smaller, but battery size remains the same. Perhaps, sometimes the battery must be larger in order to accommodate the greater power demands by a portable device. An alternative for batteries is to create energy while on the go. By employing piezoelectric transducer, it is possible to harvest power from vibrating systems. Piezoceramic materials can convert mechanical strain energy to electrical energy.

Using low power design trends, the possibility of using ambient energy to power future digital systems was discussed[1],[2]. At low power(10's to 100's of μW) levels, an interesting question arises: can we use ambient energy sources to power electronic systems? A circuit powered by ambient sources has a potentially infinite lifetime, as long as the source persists. In long-lived sensor embedded systems where battery replacement is difficult, generating power from ambient sources becomes imperative. Various schemes have been proposed to eliminate the need for batteries in a portable digital system among these. Table 1 lists potential power output for a wide variety of energy sources. The power is available from directly converting the energy of footsteps by inserting a piezoelectric transducer in the heel of a shoe[3]. A direct transduction technique like this has the potential to generate large amounts of power, of the order of 5 W.

Research in the piezoceramic field generally concerns actuation and control or self-sensing technology[4]. An up-and-coming field of research is power harvesting with piezoelectric materials. In 1984, researchers implanted polyvinylidene fluoride(PVDF) patch onto the rib cage of a mongrel dog to harvest energy during inspiration[5]. Other experiments followed, and many were successful in harvesting several micro- to milli-watts of usable power. One area in piezoceramic research that has been neglected is the modeling of piezoelectric generators on cantilever beams and plates, as well as the optimization of piezoceramic models in the use of power harvesting. Some of the examples of ambient energy sources are given in Table 1. This paper will explore the theoretical values and predicted values of power and also selects the best analytical model for fabricating a structure and performing measurements using a test setup.

II. Modeling the PZT Sensor

Piezoelectric materials such as lead zirconate-titanates

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Table 1. Examples of ambient energy sources.

Energy source	Transducer	Power
Walking (Direct conversion)	Piezoelectric	5 W
Solar	Photovoltaic cell	20 mW
Magnetic field	Coil	1.5 mW
Walking (Vibration)	Discrete moving coil	400 μW
High frequency vibration	MEMS moving coil	100 μW
RF field	Antenna	5 μW

(PZTs) are much suitable for power harvesting[6],[7]. PZTs are solid solutions of lead zirconate and lead titanate. These manufactured ceramics have much better properties than the naturally occurring piezoelectrics. A PZT bender sensor is a thin wafer of PZT material. To achieve a more useful bender, two thin wafers are adhered together and connected in parallel. PZT benders produce an electric field and thus a voltage in the 1-direction when subjected to transverse deflections caused by bending force strains in the 3-direction. For modeling of a unimorph beam with a single wafer mounted on its surface, the three most common methods used are: (1) pin-force method, (2) enhanced pin-force method, and (3) Euler-Bernoulli method. The pin-force model is the most primitive of the three methods. The enhanced pin-force model expands upon the pin-force concept. The Euler-Bernoulli model is the most complex of the three.

2-1 Pin-Force Method

In this method the mechanical interaction between the piezoelectric(PZT) patch and the substrate elements are connected at the extreme ends of the PZT. Perfect bonding between the two elements is implied and the adhesive layer is infinitely stiff. Shear stress in the piezoelectric is concentrated only in a small area at the pin ends. The strain in the beam is assumed to follow Euler-Bernoulli beam theory where the strain increases linearly through the thickness. The strain is assumed to be constant through the thickness of the piezoceramic, because of this constant strain the pin-force method does not take into consideration the bending stiffness of the piezoelectric patches. This method is limited when the stiffness of the substrate becomes approximately five times larger than the piezo stiffness. The power generated using this method is higher than the enhanced pin-force and Euler-Bernoulli method but the accuracy is decreased because as the adhesive layer becomes thicker, or the bonding material becomes less stiff.

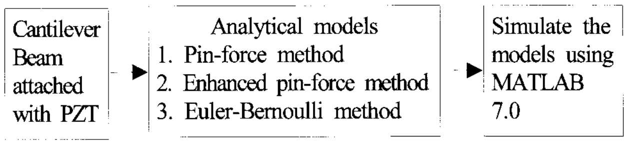

Fig. 1. Piezoelectric power generating model.

2-2 Enhanced Pin-Force Method

The enhanced pin-force method expands upon the pin-force method by taking under consideration the PZT bending stiffness. The strain does not remain constant as in the pin-force method but increases linearly through the PZT thickness. The drawback still exists when using this method the PZT is assumed to bend on its own neutral axis. This assumption basically treats the PZT and substrate as two separate structures connected only by the end pins.

2-3 Euler-Bernoulli Method

In this method piezopatch(PZT) and substrate both bend about a common neutral axis which is no longer the neutral axis of the beam. Perfect bonding is assumed, and the PZT is considered to be a layer of the beam, because of this we can conclude that Euler-Bernoulli method is the best method. This neutral axis is calculated by a modulus-weighted algorithm.

Now, power harvesting with piezoelectric transducer can be modeled using various analytical methods for both sinusoidal and random forcing functions. Fig. 1 shows piezoelectric power generating models.

III. Analytical Estimation of Power Generation from a PZT

The power harvesting of a PZT will be calculated analytically. The substrate will first be modeled as a cantilever beam. The three modeling techniques of the PZT will be used and their results are compared. A clamped-free beam will be used as the substrate. Two different driving functions will be applied to the cantilever beam. A point-force, harmonic function will be applied to the PZT-substrate system. With a driving frequency close to the first resonant frequency of the substrate, the harmonic function will produce maximum displacement. The second driving function will be a random noise generator which will simulate the random vibrations.

3-1 Cantilever Beam Model

In the present work, the assumed dimensions and pro-

Table 2. Assumed dimensions and properties of the beam and PZT.

Item description	Parameter	Value	Units
Cantilever beam	Length	0.558	m
	Width	0.05	m
	Thickness	0.004	m
	Density	2,715	kg/m^3
	Young's modulus	71×10^{9}	Pa
PZT	Length	0.073	m
	Width	0.05	m
	Thickness	0.508×10^{-3}	m
	Young's modulus	62×10^{9}	Pa
	Dielectric constant	-320×10^{-12}	m/V
	Voltage constant	-9.5×10^{-3}	Vm/N
	Internal resistance	330	kΩ

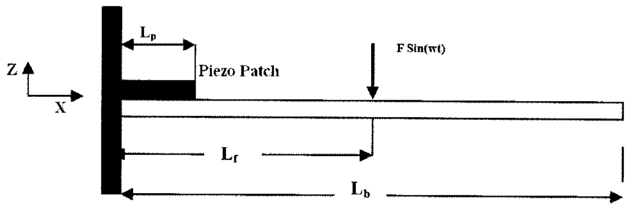

Fig. 2. Cantilever beam model.

perties of the beam and PZT are shown in the Table 2. The setup for the cantilever beam model along with PZT is shown in the Fig. 2. The PZT is attached to the beam near the clamped edge for maximum strain. For the estimated power that a PZT can produce from beam vibrations to be calculated, the moment that the PZT experiences must first be determined. This moment can be evaluated by solving for the deflection of the beam and then estimating the experienced moment as a function of the beam's curvature. Following the Euler-Bernoulli definition of a beam, the length is over ten times larger than the beam width.

The Euler-Bernoulli method is used to model the cantilever beam. The governing equation of the beam is

$$\rho A\frac{\partial^4\omega(x,t)}{\partial t^4}+E_bI_b\frac{\partial^4\omega(x,t)}{\partial x^4}=F(t) \tag{1}$$

where x is the displacement of the beam, ω is a driving frequency, ρ is the density of the beam, A is the cross-sectional area, and $F(t)$ is the external force applied to the beam. E_b is the electric field and I_b is the moment of inertia of the beam. For the beam with harmonic driving force the beam is driven harmonically as

$$\frac{\partial^4\omega(x,t)}{\partial t^4}+c^2\frac{\partial^4\omega(x,t)}{\partial x^4}=\frac{F_0}{\rho A}\sin(\omega t)\delta(x-L_f) \tag{2}$$

where L_f is the position of the applied force from the clamped edge of the beam, F_0 is the maximum force applied, and

$$c^2=\frac{E_bI_b}{\rho A}. \tag{3}$$

The driving frequency will be equal to the beam's first natural frequency because the largest deflections occur at the first natural frequency. The solution will be of the form

$$\omega(x,t)=\sum_{i=1}^{3}q_i(t)X_i(x) \tag{4}$$

where q_i is the i-th modal coordinate equation of the beam and X_i is the i-th mode shape of the beam. For consistency, only the first three mode shapes will be used. The general mode shape equation for a cantilever beam is given as[3]

$$X_i(x)=\frac{[\cosh(\beta_ix)-\cos(\beta_ix)]-[\sinh(\beta_iL_b)-\sin(\beta_iL_b)]}{[\cosh(\beta_iL_b)-\cos(\beta_iL_b)][\sinh(\beta_ix)-\sin(\beta_ix)]} \tag{5}$$

where L_b is a beam length. Also, $\beta_i^4=\omega_{ni}^4/c^2$ and ω_{ni} is the i-th natural frequency which is found from characteristic equation

$$\cos(\beta_iL_b)\cosh(\beta_iL_b)=-1. \tag{6}$$

The first five beam natural frequencies are shown in Table 3. Using the orthogonality, the external force can be simplified to the expression

$$F_i(t)=\frac{F_0}{\rho A}\sin(\omega t)X_i(L_f) \tag{7}$$

and F_0 will be set to 1.

Table 3. The first five beam natural frequencies.

Natural frequency	rad/s	Hz
1	66.68	10.612
2	417.87	66.506
3	1170.1	186.22
4	2294	365.1
5	3790.1	603.22

In the present work, the analytical power values are compared in order to determine predictive methods for estimating power from vibration systems. Power harvesting from mobile digital systems is from vibrations and most suitable driving force for simulation is the random driving force. The process for calculating power from random vibrations remains the same except that the forcing function is changed. Let us consider the external driving force to have random vibration content instead of sinusoidal or any other definitive content. A function $F(t)$ is often characterized as being random if the value of $F(t)$ for a given value of t is known only statistically[8]. The random vibration function is given by

$$F(t)=\sum_{i=0}^{n} F_0 \sin\left(\omega_{rand} t-\theta_{rand}\right) \tag{8}$$

where F_0 value will be set to 1, ω_{rand} is any possible frequency within an arbitrary frequency range, θ_{rand} is any possible phase shift with values between 0 and 2π and n is the arbitrary number of iterations that create a sufficiently random function.

MATLAB can generate a random signal by the summation of random sine waves. We assumed fifty sine functions with random frequencies and phases were chosen to be summed together to produce random vibration. The random vibration signal is simulated by using MATLAB's random number generator and the equation as follows

$$F(t)=\sum_{i=1}^{50} A \sin\left(R_{rand} 2\pi f_{arb} t-R_{rand} 2\pi\right) \tag{9}$$

Also, R_{rand} is a random number between 0 and 1 generated by MATLAB and f_{arb} is any frequency that falls within an arbitrarily set range of frequencies.

The magnitude of the signal will also vary randomly according to frequencies and phase shifts generated at every time step. It can be expected that maximum power calculated will be less than the power calculated from the harmonically driven beam because of this random assignment of driving frequencies. As the arbitrary frequency range f_{arb} increases, the power produced will decrease.

The convolution integral for any arbitrary input to evaluate q_i is in the form

$$q_i(t)=\frac{1}{\omega_{di}} e^{-\zeta\omega_n t} \int_0^t F_i(\tau) e^{\zeta\omega_n t} \sin\left[\omega_{di}(t-\tau)\right] d\tau \tag{10}$$

where ω_n is a natural frequency, ω_{di} is a damped natural frequency, and ζ is a damping ratio. The most common damping ratio values fall between 0.01 and 0.05[8]. For simplicity, the damping ratio will be assumed to be the average of this range 0.03. The curvature of the beam can be estimated as

$$K(x,t)=\frac{\partial^2 \omega(x,t)}{\partial x^2}. \tag{11}$$

To eliminate the dependence of length from the expression, the average curvature was evaluated as

$$\bar{K}(t)=\frac{1}{L_p} \int_0^{L_p} K(x,t)\, dx. \tag{12}$$

Here, the limits of integration are the lengths along the beam where the PZT starts and ends. Finally, the applied moment acting on the beam is

$$M(t)=E_b I_b \bar{K}(t). \tag{13}$$

3-2 Maximum Voltage and Power

The equations for the voltage generated for the pin-force method, V_p, enhanced pin-force method, V_{ep}, and Euler-Bernoulli method, V_{eb}, are as follows, respectively[8],

$$V_p=\frac{6g_{31}M}{bt_b(3-\psi)} \tag{14}$$

$$V_{ep}=\frac{6g_{31}TM}{bt_a\left(3T^2-1-\psi T^2\right)}. \tag{15}$$

$$V_{eb}=\frac{6g_{31}M\psi(1+T)}{bt_a\left[1+\psi^2T^2+2\psi\left(2+3T+2T^2\right)\right]}. \tag{16}$$

In (14)~(16), g_{31} is a PZT voltage constant, $\Psi=(E_b t_b)/(E_a t_a)$. t_a and t_b are the thickness of the PZT and beam, E_a and E_b are the electric field of the PZT and beam, respectively. b is the position of PZT from clamped edge of the beam. In (15) and (16), $T=t_b/t_a$. Now, substituting (13) into (14), (15) and (16) the time dependent PZT voltages are calculated for three different methods. The equation used to calculate power from an AC voltage signal is

$$P=\sum_{i=0}^{n} \frac{(V_s)^2 t_i R_L}{(R_L+R_s)^2 n} \tag{17}$$

where V_s is the source voltage, and n is a number of time steps. R_L and R_s the load and source resistances, respectively.

Fig. 3 shows the three signals with a phase shift of 90 degrees, voltages calculated from the pin-force and

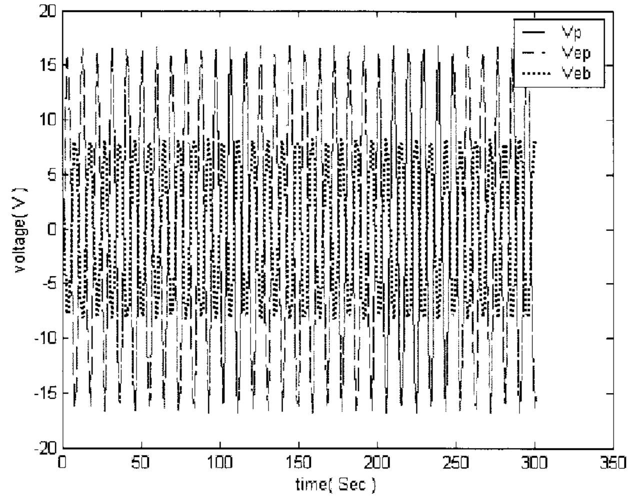

Fig. 3. Voltage generated by harmonic driving force applied at L_f=0.2 m from clamped end of the beam.

enhanced pin-force method closely match each other. Though the pin-force method assumes that the strain in the PZT remains constant and the enhanced pin-force method considers an increasing linear strain through the PZT, the PZT is so thin that both methods produce strain values relatively close to one another. The Euler-Bernoulli method produces a voltage that is more than half the other two voltages because it does not erroneously assume that the PZT bends on its on neutral axis.

Ⅳ. Results

The voltage and power for each of the three methods is calculated. To generate the maximum power, the load impedance is set to equal the internal PZT impedance of 330 kΩ. The Fig. 4 shows the powers P_p, P_{ep}, and P_{eb} generated for the pin-force method, enhanced pin-force method and Euler-Bernoulli methods respectively for harmonic driving force applied at L_f=0.2 m. Table 4 shows the voltage and power values calculated from each method for both sinusoidal and random driving forces. The power produced using the Euler-Bernoulli method is lower than the other two values of power. This is because of the key factor of the Euler-Bernoulli method correctly assuming that the PZT does not bend on its own neutral axis but bends on another shifted neutral axis. These three power values are the maximum possible powers for the factors and dimensions given.

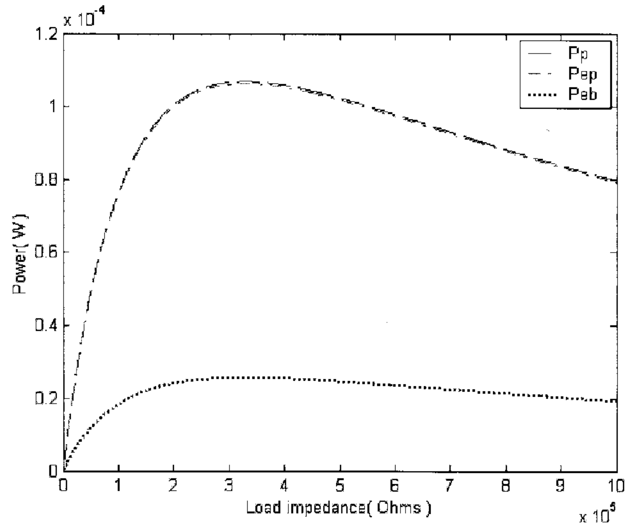

Fig. 4. Power generated by harmonic driving force applied at L_f=0.2 m from clamped end of the beam.

Table 4. Voltage and power for beam excited by harmonic and random force.

Analytical method	Harmonic driving force		Random driving force	
	Voltage (V)	Power (μW)	Voltage (V)	Power (μW)
Pin-force	16.8225	106.70	20.0265	34.740
Enhanced pin-force	16.7750	106.13	19.9730	34.554
Euler-Bernoulli	8.2404	25.59	10.2863	8.33

The analysis of the three piezoelectric modeling methods shows that the Euler-Bernoulli method better estimates the behavior of a piezoelectric element used for designing a power harvesting system. Though the pin-force and enhanced pin-force method are simpler in nature, they fail to consider the constant bonding between the piezoelectric element and the substrate.

Ⅴ. Conclusions

The model using Euler-Bernoulli method has been developed to estimate the characteristics of piezoelectric elements. By optimizing certain variables in the beam analytical model, the power produced by the PZT is increased. The PZT location was found to be best when the PZT was attached next to clamped end of the beam. The PZT length was optimized at a length of 0.3 m. This model accurately predicts deflection, voltage and power generated from a beam that is excited by a harmonic oscillations and random noise as forcing functions. Random excitations reduce the potential power that can be produced from the structure vibrations. As the bandwidth of possible frequencies excited decreases, the potential power output increases.

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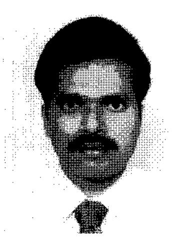

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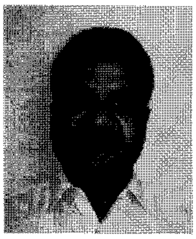

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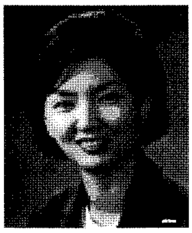

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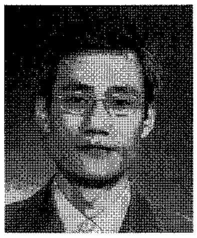

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