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Forward Backward PAST (Projection Approximation Subspace Tracking) Algorithm for the Better Subspace Estimation Accuracy

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Abstract

The projection approximation subspace tracking (PAST) is one of the attractive subspace tracking algorithms, because it estimatesthe signal subspace adaptively and continuously. Furthermore, the computational complexity is relatively low. However, thealgorithm still has room for improvement in the subspace estimation accuracy. In this paper, we propose a new algorithm to improve the subspace estimation accuracy using a normally ordered input vector and a reversely ordered input vector simultaneously.
Keywords: Subspace Estimation, Projection Approximation Subspace Tracking, Forward and Backward Covariance Matrix

I. Introduction

In recent years, subspace-tracking algorithms have been intensively studied and widely applied to reduce the computational complexity of subspace estimation. Instead of updating the whole eigen-structure, subspace-tracking algorithm only works with the signal or noise subspace. This makes subspace-tracking algorithm more efficient than conventional methods using eigenvalue decomposition (ED) or singular value decomposition (SVD). One very attractive subspace-tracking algorithm is the projection approximation subspace-tracking (PAST) algorithm [1]. The idea of the PAST is to make the expectation of the squared difference between the input vector and the projected vector minimum. With proper projection approximation, the PAST derives a recursive least squares (RLS) algorithm for tracking the signal subspace. However, the PAST still has room for the improvement in the subspace estimation accuracy. To improve the PAST algorithm, many different algorithms have been proposed. For example, Jung-Lang Yu developed a correlation-based projec approximation subspace tracking (COPAST), to improve the convergence property of the sub-space tracking [2].

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In addition, Lim et al. proposed an algorithm to control the estimation window size automatically to handle time-varying signals [3]. Abed-Meraim et al. proposed an orthonormal version of the projection approximation and subspace tracking (PAST) algorithm for fast estimation and tracking of principal subspace or/and principal components of a vector sequence. The orthonormal PAST (OPAST) algorithm guarantees the orthonormality of the weight matrix at each iteration [4].

In this paper, we propose a new algorithm to improve the subspace estimation accuracy in the PAST algorithm. The proposed algorithm utilizes the normal forward ordered input vector and the reversal ordered input vector simultaneously. This forward and backward input vectors can build up the better sample covariance. Therefore, we expect the improved PAST algorithm with the better subspace estimation accuracy. In the simulation section, we show the performance comparison with other PAST –style algorithms. For the comparison, we compare the proposed algorithm with the conventional PAST and the COPAST.

II. PAST Algorithm

PAST (Projection Approximation Subspace Tracking) algorithm is based on the minimum property of the unconstrained cost function.

$$J(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i} \|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{W}(t)\mathbf{x}(i)\|^2 , \quad (1)$$

where $x=[x_1 \cdots x_N]^T$ is the inputs to the neural networks and $W(t)$ is a $N \times r$ matrix representing the signal subspace. To derive recursive update of $W(t)$ from $W(t-1)$, Yang in [1] approximate $W^H(t)x(i)$ by the expression $y(i)=W^H(i-1)x(i)$ which can be calculated for $1 \le i \le t$ at the time instant t. Then (1) results in a modified cost function,

$$J'(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i} \|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\|^2 . \quad (2)$$

Table 1. Summary of PAST Algorithm.

$\mathbf{W}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \vdots \end{bmatrix}, [L \times r]$ $\mathbf{P}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, [r \times r]$ Do t = 1, ⋯ $\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$ $\mathbf{h}(t) = \mathbf{P}(t-1)\mathbf{y}(t)$ $\mathbf{g}(t) = \mathbf{h}(t)/(\lambda \mathbf{I} + \mathbf{y}^H(t)\mathbf{h}(t))$ $\mathbf{P}(t) = \frac{1}{\lambda}[\mathbf{P}(t-1) - \mathbf{g}(t)\mathbf{h}^H(y)]$ $\boldsymbol{\varepsilon}(t) = \mathbf{x}(t) - \mathbf{W}(t-1)\mathbf{y}(t)$ $\mathbf{W}(t) = \mathbf{W}(t-1) + \boldsymbol{\varepsilon}(t)\mathbf{g}^H(t)$ END
where $x(t) = [x_1(t), x_2(t), \cdots, x_L(t)]^T$ and $x_i(t)$ is i-th sensor signal at t-thsnapshot. The ith principal vector is the ith column of $W(t)$. The dimension of $W(t)$ is $[L \times r]$. r is the dimension of signal subspace.

This becomes the exponentially weighted least squares cost function which is well studied in adaptive filtering. $J'(W(t))$ is minimized if

$$\begin{aligned} \mathbf{W}(t) &= \mathbf{C}_{xy}(t)\mathbf{C}_{yy}^{-1}(t), \\ \mathbf{C}_{xy}(t) &= \sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{xy}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t), \\ \mathbf{C}_{yy}(t) &= \sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{yy}(t-1) + \mathbf{y}(t)\mathbf{y}^H(t). \end{aligned} \quad (3)$$

III. Forward-Backward PAST Algorithm

In (3), $C_{yy}(t)$consists of the sample covariance matrix $\hat{\mathbf{R}}_{xx}$ $\hat{R}$and the principal matrix $W(t)$ as (4).

$$\mathbf{C}_{yy}(t) = \mathbf{W}^H(t)\hat{\mathbf{R}}_{xx}(t)\mathbf{W}(t), \quad (4)$$

where $\hat{\mathbf{R}}_{xx}(t) = \beta\hat{\mathbf{R}}_{xx}(t-1) + \mathbf{x}(t)\mathbf{x}^H(t)$. Therefore, PAST algorithm is used the sample covariance matrix $\hat{\mathbf{R}}_{xx}$ which is,

$$\hat{\mathbf{R}}_{xx}(t) = \beta\hat{\mathbf{R}}_{xx}(t-1) + \begin{bmatrix} x_1(t) \\ \vdots \\ x_L(t) \end{bmatrix} [x_1^*(t) \ \cdots \ x_L^*(t)] . \quad (5)$$

Generally, the theoretical covariance matrix R_{xx} is Toeplitz and persymmetric, [5]. However, the estimated covariance matrix $\hat{\mathbf{R}}_{xx}(t)$ in (5) does not guaranteeall the properties of the theoretical covariance matrix, because it does not satisfy the persymmetric property such as $\mathbf{J}(\hat{\mathbf{R}}_{xx}(t))^T\mathbf{J} \neq \hat{\mathbf{R}}_{xx}(t)$, where $\mathbf{J} = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}$ is the so-called reversal matrix.

We can make the sample covariance matrix Toeplitz and persymmetric by some modification of (5). The modified sample covariance matrix $\tilde{\mathbf{R}}_{\mathbf{xx}}$ is,

$$\tilde{\mathbf{R}}_{\mathbf{xx}}(t) = \frac{1}{2}\left(\hat{\mathbf{R}}_{\mathbf{xx}}(t) + \mathbf{J}\hat{\mathbf{R}}^{T}_{\mathbf{xx}}(t)\mathbf{J}\right), \quad (6)$$

where $\mathbf{J} = \begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix}$ is the so-called reversal matrix.

The second term of (6) has the following detailed form,

$$\mathbf{J}\hat{\mathbf{R}}^{T}_{\mathbf{xx}}(t)\mathbf{J} = \beta\mathbf{J}\hat{\mathbf{R}}^{T}_{\mathbf{xx}}(t-1)\mathbf{J} + \begin{bmatrix} x_L^*(t) \\ \vdots \\ x_1^*(t) \end{bmatrix} \begin{bmatrix} x_L(t) & \cdots & x_1(t) \end{bmatrix}. \quad (7)$$

The modified sample covariance matrix $\tilde{\mathbf{R}}_{\mathbf{xx}}(t)$ has the sample covariance matrix of the reverse ordered vector as well as that of the normal forward ordered vector. Therefore, we can call the $\tilde{\mathbf{R}}_{\mathbf{xx}}(t)$ as the forward-backward covariance matrix. The forward-backward covariance matrix, $\tilde{\mathbf{R}}_{\mathbf{xx}}(t)$, is invariant to the transform, $\mathbf{J}(\bullet)^T\mathbf{J}$,

$$\mathbf{J}(\tilde{\mathbf{R}}_{\mathbf{xx}})^T\mathbf{J} = \tilde{\mathbf{R}}_{\mathbf{xx}}. \quad (8)$$

In turn, we can expect that the estimated principal components derived from $\tilde{\mathbf{R}}_{\mathbf{xx}}(t)$ are likely to be more accurate than those obtained from $\tilde{\mathbf{R}}_{\mathbf{xx}}(t)$. Magnus Jansson in [6] also performed a direct comparative study of the relative accuracy of the two sample covariances, forward-only sample covariance and forward-backward sample covariance, respectively, and showed quantitatively the gain of using the forward-backward estimate compared to the forward-only estimate.

To apply the forward-backward covariance matrix to the PAST, we should modify (6) into a recursive form. The recursive forward-backward covariance matrix is as follows.

$$\tilde{\mathbf{R}}_{\mathbf{xx}}(t) = \beta\tilde{\mathbf{R}}_{\mathbf{xx}}(t-1) + \begin{bmatrix} x_1(t) & x_L^*(t) \\ \vdots & \vdots \\ x_L(t) & x_1^*(t) \end{bmatrix} \begin{bmatrix} x_1^*(t) & \cdots & x_L^*(t) \\ x_L(t) & \cdots & x_1(t) \end{bmatrix}. \quad (9)$$

Comparing (9) with (6), (9) needs a scaling factor, 1/2. However, the scaling factor does not affect the subspace so that we dismiss the factor. Applying the matrix inversion lemma in [1] to (9), we can derive a new PAST algorithm with the forward-backward covariancematrix. In Table 2, we summarized the proposed forward backward PAST algorithm.

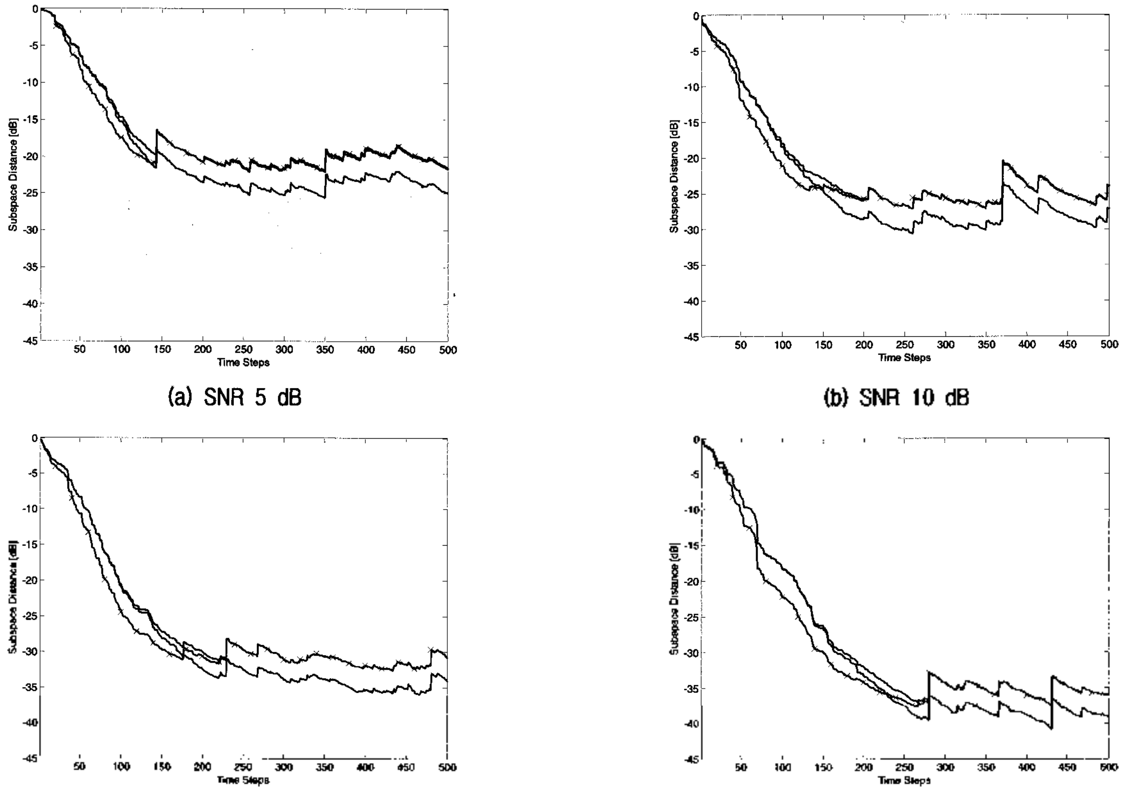
Table 2. Summary of the proposed forward-backward PAST (FB-PAST).

$\tilde{\mathbf{W}}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \vdots \end{bmatrix}$, [L×r]
$\tilde{\mathbf{P}}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$, [r×r]
Do t = 1, …
$\tilde{\mathbf{x}}(t) = [\mathbf{x}_f(t) \vdots \mathbf{x}_b^*(t)]$
$\tilde{\mathbf{y}}(t) = \tilde{\mathbf{W}}^H(t-1)\tilde{\mathbf{x}}(t)$
$\tilde{\mathbf{h}}(t) = \tilde{\mathbf{P}}(t-1)\tilde{\mathbf{y}}(t)$
$\tilde{\mathbf{g}}(t) = \tilde{\mathbf{h}}(t)/(\lambda\mathbf{I}_{[2\times2]} + \tilde{\mathbf{y}}^H(t)\tilde{\mathbf{h}}(t))$
$\tilde{\mathbf{P}}(t) = \frac{1}{\lambda}[\tilde{\mathbf{P}}(t-1) - \tilde{\mathbf{g}}(t)\tilde{\mathbf{h}}^H(t)]$
$\tilde{\mathbf{e}}(t) = \tilde{\mathbf{x}}(t) - \tilde{\mathbf{W}}(t-1)\tilde{\mathbf{y}}(t)$
$\tilde{\mathbf{W}}(t) = \tilde{\mathbf{W}}(t-1) + \tilde{\mathbf{e}}(t)\tilde{\mathbf{g}}^H(t)$
END
Where $\mathbf{x}(t) = \mathbf{x}_f(t) = [x_1(t), x_2(t), \cdots, x_L(t)]^T$, $\mathbf{x}_b(t) = [x_L(t), x_{L-1}(t), \cdots, x_1(t)]^T$ and $x_i(t)$ is i-th sensor signal at t-th snapshot. The i-th principal vector is the i-th column of $\tilde{\mathbf{W}}(t)$. The dimension of $\tilde{\mathbf{W}}(t)$ is [L×r]. r is the dimension of signal subspace.

IV. Simulation Results

In this section, we demonstrate the applicability of the proposed algorithm to the subspace estimation. We assume the signal subspacecomes from a narrow band far-field source by using a linear uniform array with 8 sensors. For the experimental purpose, we set the scenario that the angle of arrival of the signal comes from $-30°$.

In Fig. 1, we compare the estimation accuracy of the proposed algorithm with the conventional PAST algorithm and COPAST algorithm in the fixed forgetting factors of 0.98 under the four different SNR cases of 5 dB, 10 dB, 15 dB and 20 dB, respectively. Especially we select COPAST for the comparison



(a) SNR 5 dB (b) SNR 10 dB

(c) SNR 15 dB (d) SNR 20 dB

Fig. 1. The subspace estimation accuracy comparisons (solid line : the proposed algorithm, -x- : COPAST in [2], dotted line : the conventional PAST).

because we can put the COPAST and the proposed algorithm into the same category, in that COPAST and the proposed algorithm modify the data matrix itself in PAST to improve the performance.

To compare the quality of the estimated subspace, we show the distance between the estimated sub-space and the true subspace, which is defined in [7].

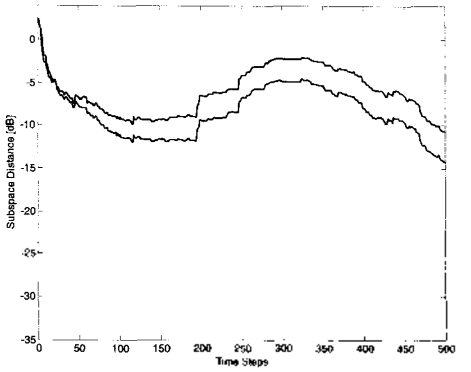
$$\sin\theta(\mathbf{S},\tilde{\mathbf{S}}) = \left\|(\mathbf{I}-\mathbf{P})\tilde{\mathbf{P}}\right\|, \quad (9)$$

where S is the true subspace, $\tilde{\mathbf{S}}$ is the estimated subspace, P is the projector onto S and $\tilde{\mathbf{P}}$ is the projector onto $\tilde{\mathbf{S}}$. The results in Fig.1 show that the proposed algorithm estimated the subspace more accurately than the conventional PAST algorithm and the COPAST in steady state in all SNR cases.

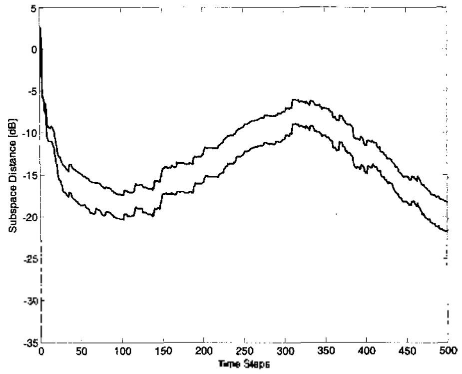
Fig. 2 shows the simulation results for the non-stationary signal environment. For the experimental purpose, we set the scenario that the angle of arrival of a signal changes from -0.2 rad to -0.3 rad as well as that of another signal changes from -0.3 rad to -0.2 rad. We compare the estimation accuracy of the proposed algorithm with the conventional PAST algorithm in the fixed forgetting factors of 0.98. Fig. 2 also shows that the proposed algorithm estimated the subspace more accurately than the PAST al-gorithm even in the nonstationary environments.

V. Conclusion

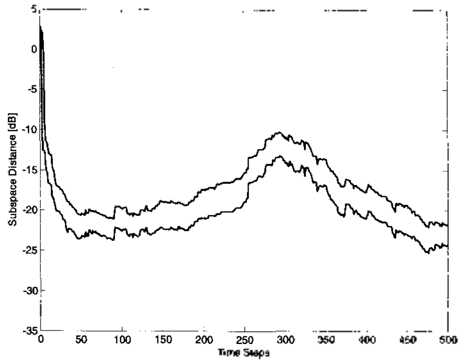
In this paper, we have proposed the forward back-ward PAST (F/B PAST) algorithm to estimate the signal subspace. The F/B PAST applies the forward -backward covariance matrix to the conventional PAST. It improves the property of the estimated covariance matrix to get closer to the ideal covariance matrix. From the simulation results, we can see the



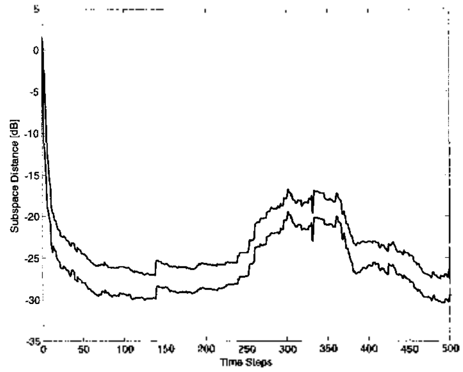
(a) SNR 5 dB



(b) SNR 10 dB



(a) SNR 15 dB



(b) SNR 20 dB

Fig. 2. The subspace estimation accuracy comparisons in slowly varying and crossing signals (solid line : the proposed algorithm, dotted line : the conventional PAST).

proposed F/B PAST outperforms the conventional PAST in the estimation accuracy.

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[Profile]

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