LMI-Based Synthesis of Robust Iterative Learning Controller with Current Feedback for Linear Uncertain Systems

Jianming Xu, Mingxuan Sun, and Li Yu

Abstract: This paper addresses the synthesis of an iterative learning controller for a class of linear systems with norm-bounded parameter uncertainties. We take into account an iterative learning algorithm with current cycle feedback in order to achieve both robust convergence and robust stability. The synthesis problem of the developed iterative learning control (ILC) system is reformulated as the γ-suboptimal H_∞ control problem via the linear fractional transformation (LFT). A sufficient convergence condition of the ILC system is presented in terms of linear matrix inequalities (LMIs). Furthermore, the ILC system with fast convergence rate is constructed using a convex optimization technique with LMI constraints. The simulation results demonstrate the effectiveness of the proposed method.

Keywords: H_∞ control, ILC, LFT, LMI.

1. INTRODUCTION

Iterative learning control (ILC) is a technique to control a system carrying out a task over a finite time interval repetitively such that the system output accurately tracks a specified reference trajectory. Motivated by human learning, the basic idea of iterative learning control is to use an iterative procedure to calculate the input signal from the previous operation data such that the tracking error is gradually reduced. Since the introduction of iterative learning control (ILC) methodology by Arimoto *et al.* [1], the general area of ILC has been the subject of intense research effort both in terms of the underlying theory and engineering practice [2]. Iterative learning control was initially developed as a feedforward action applied directly to the open-loop system. Although the pure feedforward learning control scheme is theoretically acceptable, it is unlikely to be applied to real systems without a feedback control. One reason is that it may generate harmful effects if the open-loop system is unstable or exists uncertainty. In addition, the tracking error may possibly grow quite large in the early stages of learning, though it eventually converges after a number of trials. Thus, in real environments, a current feedback control is commonly employed along with the iterative learning control for system robustness enhancement and better performance [3]. The combination of ILC with current feedback is typically done as a feedforward-feedback configuration where the current feedback controller ensures closed-loop stability and suppresses exogenous disturbances and the iterative learning controller provides improved tracking performance over a specific reference trajectory.

Most of the existing results have concentrated on deriving new algorithms and analyzing their convergence properties. Few results on ILC synthesis have been reported especially under model uncertainties in literature. Amann *et al.* proposed a two-step design procedure based on H_∞ optimization [4]. De Roover synthesized an iterative learning controller based on H_∞ control under unstructured uncertainties [5]. Moon and Doh [6,7] derived a sufficient condition for convergence of the iterative process in the presence of plant uncertainty. An iterative learning controller that satisfies the convergence condition can be obtained by μ-synthesis procedure called D-K iteration [8]. However, the D-K iteration cannot guarantee the global convergence, and usually leads to a high-order controller that is not easy to implement in the practical situation. It has recently been emphasized by Boyd *et al.* that many problems arising in system theory can be cast into the form of linear matrix inequalities (LMIs) [9], which belong to the group of convex problems, and thus one can efficiently find feasible and global solutions to them via interior-point methods. To the best of our knowledge, the ILC design problem for linear uncertain systems has not

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been investigated via LMI approaches.

In this paper, based on the frequency domain representation, we deal with the synthesis problem of the ILC algorithm with current feedback that assures convergence for a class of linear systems with norm-bounded time-varying parameter uncertainty. The problem is first reformulated as a γ-suboptimal H_∞ control problem via the linear fractional transformation (LFT). Then a sufficient convergence condition is established for the ILC system in terms of LMIs. The solutions of the LMIs can be used to construct a suitable ILC algorithm. Furthermore, a convex optimization problem with LMI constraints is formulated to design the ILC algorithm that achieves fast convergence speed of the resulting ILC system.

2. PROBLEM STATEMENT

Consider an iterative learning control system with current feedback shown in Fig. 1, where $y_d(t)$ is the desired output trajectory over a finite interval $t \in [0,T]$, $y_k(t)$, $u_k(t)$ and $e_k(t)$ are the system output, the control input and the tracking error at the k th iteration, respectively. $P(s)$ is the controlled plant, and can be described by the following state space model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ &= [A_0 + \Delta A]x(t) + [B_0 + \Delta B]u(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \Re^{n_1}$, $u(t) \in \Re^p$, $y(t) \in \Re^p$ denote the state, the control input and the system output, respectively, A_0, B_0, C are known real constant matrices with appropriate dimensions, ΔA, ΔB are matrix-valued functions representing time-varying parameter uncertainties in the system model, and are assumed to be of the following form:

$$[\Delta A \quad \Delta B] = DF(t)[E_1 \quad E_2], \tag{2}$$

where D, E_1, E_2 are known real constant matrices with appropriate dimensions, which represent the structure of uncertainties, and $F(t) \in \Re^{i \times j}$ is an unknown matrix function with Lebesgue measurable

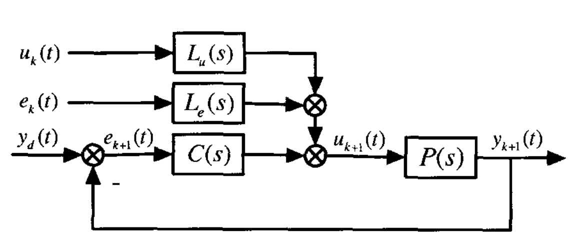

Fig. 1. An ILC system with current feedback.

elements and satisfies

$$F^T(t)F(t) \le I, \tag{3}$$

where I denotes the identity matrix of appropriate dimension.

Consider the ILC algorithm with current feedback given in the frequency domain as

$$\begin{aligned} U_{k+1}(s) = L_u(s)U_k(s) + L_e(s)E_k(s) \\ + C(s)E_{k+1}(s), \end{aligned} \tag{4}$$

where $L_u(s)$ and $L_e(s)$ are the learning controllers, $C(s)$ is the current feedback controller. To implement in the practical situations, all controllers should be in RH_∞ space.

In the formulation of the synthesis problem of the ILC system, we make the following assumptions.

Assumption 1: The initial state of the plant $P(s)$ is invariant with respect to k, so $Y_k^0(s) = Y^0(s)$ for $k = 1, 2, \ldots$, where $Y_k^0(s)$ denotes the Laplace transform of $y_k(0)$.

Assumption 2: Let $L_u(s)$ be $l_u(s)I$ where $l_u(s)$ is a type of low-pass filter, with a cut-off frequency ω_c being higher than the tracking bandwidth, such that $\| l_u(j\omega) \| = 1$, $\forall \omega \in [0, \omega_c]$, and $\| l_u(j\omega) \| < 1$, $\forall \omega > \omega_c$.

Lemma 1 [7]: Suppose that Assumptions 1 and 2 are satisfied and $Y_d(s), Y^0(s) \in H_2$. Then the ILC system in Fig. 1 is L_2 convergent with remaining error if the H_∞ norm of stable $L(s)$ is less than 1, where

$$L(s) = (1 + P(s)C(s))^{-1}(L_u(s) - P(s)L_e(s)). \tag{5}$$

Moreover, the remaining error $E_\infty(s)$ is

$$\begin{aligned} E_\infty(s) &= \lim_{k \to \infty} (Y_d(s) - Y_k(s)) \\ &= (1 - L(s))^{-1} D_d(s)(Y_d(s) - Y^0(s)) \in H_2, \end{aligned} \tag{6}$$

where

$$D_d(s) = (1 + P(s)C(s))^{-1}(I - L_u(s)). \tag{7}$$

For given $L_u(s)$, the synthesis problem of the proposed ILC system is converted into solving $C(s)$ and $L_e(s)$ from the following (sub)optimal H_∞ synthesis problem:

$$\| (I + P(s)C(s))^{-1}(L_u(s) - P(s)L_e(s)) \|_\infty = \gamma < 1. \tag{8}$$

Note that the smaller is γ, the faster is the

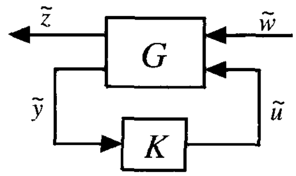

Fig. 2. Diagram representation of $\mathbf{F}_l(G,K)$.

convergence rate of the error, because

$$\| (E_{k+1}(s) - E_\infty(s) \|_2 \le \| L(s) \|_\infty \| E_k(s) - E_\infty(s) \|_2 \quad (9)$$

γ can be used to indicate the convergence rate of the ILC system. To solve the problem described in (8), for practical situations, we adopt the approach suggested in Zhou *et al.* [8]. Therefore, the ILC synthesis problem is reformulated in the standard plant format, depicted in Fig. 2.

Within this framework, tools are available for computing a stabilizing K that minimizes $\| T_{\tilde{z}\tilde{w}} \|_\infty$ (with $T_{\tilde{z}\tilde{w}}$ being the transfer function from $\tilde{w}$ to $\tilde{z}$). To formulate the ILC synthesis in this standard plant framework, this transfer function from $\tilde{w}$ to $\tilde{z}$ is considered, given by the lower linear fractional transformation (LFT) of G and K. If G is partitioned as:

$$G := \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad (10)$$

then the lower LFT of G and K, denoted $\mathbf{F}_l(G,K)$, is defined as:

$$\mathbf{F}_l(G,K) := G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} = T_{\tilde{z}\tilde{w}}. \quad (11)$$

Clearly, $L(s)$ can be described in this form, by taking:

$$G := \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} L_u(s) & P(s) \\ \begin{bmatrix} -L_u(s) \\ -I \end{bmatrix} & \begin{bmatrix} -P(s) \\ 0 \end{bmatrix} \end{bmatrix} \quad (12)$$

and

$$K := \begin{bmatrix} C(s) & L_e(s) \end{bmatrix}. \quad (13)$$

It should be noted that for solution of the synthesis problem using the standard plant format, if the learning controller $L_u(s)$ is described by the following state space equation:

$$\begin{aligned} \dot{x}_u &= A_u x_u + B_u u_u, \\ y_u &= C_u x_u + D_u u_u, \end{aligned} \quad (14)$$

where $x_u \in \Re^{n_2}$, $u_u \in \Re^p$, $y_u \in \Re^p$ denote the state, the control input and measure output, respectively. Then the plant G is described in state space coordinates:

$$\begin{aligned} \dot{\tilde{x}}_u &= A_1\tilde{x} + B_1\tilde{w} + B_2\tilde{u}, \\ \tilde{z} &= C_1\tilde{x} + D_{11}\tilde{w}, \\ \tilde{y} &= C_2\tilde{x} + D_{21}\tilde{w}, \end{aligned} \quad (15)$$

where $\tilde{x} = [x^T \ x_u^T]^T \in \Re^n$ $(n = n_1 + n_2)$, $\tilde{u} = u \in \Re^p$, $\tilde{w} = u_u \in \Re^p$, $\tilde{z} \in \Re^p$ and $\tilde{y} \in \Re^{2p}$ denote the state, the control input, the disturbance input, the controlled output and the measure output of the system G, respectively; and

$$A_1 = \begin{bmatrix} A & 0 \\ 0 & A_u \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ B_u \end{bmatrix}, \ B_2 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ C_1 = \begin{bmatrix} C & C_u \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -C & -C_u \\ 0 & 0 \end{bmatrix}, \ D_{11} = D_u, \ D_{21} = \begin{bmatrix} -D_u \\ -I \end{bmatrix}.$$

As a consequence, the synthesis problem of the ILC system is reduced to a γ-suboptimal H_∞ control problem, i.e., to design an output controller (13) such that $\| T_{\tilde{z}\tilde{w}} \|_\infty = \gamma < 1$ for the system (15). In this paper, we solve the problem via an LMI approach.

In the proof the main results, we will need the following lemmas.

Lemma 2 (the Bounded Real Lemma) [10]: Consider a continuous time transfer function $T(s)$ of realization $T(s) = D + C(sI - A)^{-1}B$, the following statements are equivalent:

(i) $\| D + C(sI - A)^{-1}B \|_\infty < \gamma$ and A is stable;

(ii) There exists a symmetric positive definite solution X to the LMI:

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0.$$

Lemma 3 [10]: Given matrices M, N and symmetric matrix S of appropriate dimensions, then

$$S + MFN + N^T F^T M^T < 0,$$

for any uncertain matrix F satisfying $F^T F \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$S + \varepsilon MM^T + \varepsilon^{-1} N^T N < 0.$$

Lemma 4 [11]: Given a symmetric matrix $\Pi \in \Re^{m \times n}$ and two matrices Ψ, Φ of column dimension m, consider the problem of finding some matrix Θ of compatible dimension such that

$$\Pi + \Psi^T \Theta \Phi + \Phi^T \Psi < 0.$$

Denoted by N_Ψ, N_Φ any matrices whose columns form bases of the null space of Ψ and Φ, respectively. Then the above inequality is solvable for Θ if and only if

$$N_\Psi^T \Pi N_\Psi < 0, \quad N_\Phi^T \Pi N_\Phi < 0.$$

Lemma 5 [12]: Given symmetric positive definite matrices $X, Y \in \Re^{n\times n}$, then there exist matrices $M, N \in \Re^{n\times p}$ and symmetric matrices $Z, W \in \Re^{p\times p}$ satisfying

$$\begin{bmatrix} X & M \\ M^T & Z \end{bmatrix} > 0, \quad \begin{bmatrix} X & M \\ M^T & Z \end{bmatrix}^{-1} = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}$$

if and only if

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \text{ and } \operatorname{rank}(I - XY) \le p.$$

3. MAIN RESULTS

Theorem 1: Consider the system (15), the continuous time γ-suboptimal H_∞ problem is solvable if and only if there exist a scalar $\varepsilon > 0$, symmetric matrices R, S satisfying

$$\Omega^T \begin{bmatrix} \tilde{A}_0 R + R\tilde{A}_0^T + \varepsilon \tilde{D}\tilde{D}^T & R\tilde{E}_1^T & RC_1^T & B_1 \\ \tilde{E}_1 R & -\varepsilon I & 0 & 0 \\ C_1 R & 0 & -\gamma I & D_{11} \\ B_1^T & 0 & D_{11}^T & -\gamma I \end{bmatrix} \Omega < 0, \quad (16)$$

$$\Xi^T \begin{bmatrix} S\tilde{A}_0 + \tilde{A}_0^T S + \varepsilon^{-1}\tilde{E}_1^T \tilde{E}_1 & SB_1 & C_1^T & S\tilde{D} \\ B_1^T S & -\gamma I & D_{11}^T & 0 \\ C_1 & D_{11} & -\gamma I & 0 \\ \tilde{D}^T S & 0 & 0 & -\varepsilon^{-1} I \end{bmatrix} \Xi < 0, \quad (17)$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \ge 0, \quad (18)$$

where

$$\tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & A_u \end{bmatrix}, \ \tilde{B}_2 = \begin{bmatrix} B_0 \\ 0 \end{bmatrix}, \ \tilde{D} = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{E}_1 = \begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{E}_2 = \begin{bmatrix} E_2 \\ 0 \end{bmatrix},$$

$\Omega = diag\{N_R, I\}$, $\Xi = diag\{N_S, I\}$, N_R and N_S denote bases of the null spaces of $[\tilde{B}_2^T \tilde{E}_2^T]$ and $[C_2 D_{21}]$, respectively.

In addition, there exist γ-suboptimal controllers of order $h < n$ (reduced order) if and only if (16)-(18) hold for a scalar $\varepsilon > 0$ and some R, S, which further satisfy:

$$rank(I - RS) \le h. \quad (19)$$

Proof: Given any proper real-rational controller K of realization

$$K = D_K + C_K(sI - A_K)^{-1} B_K \quad (A_K \in \Re^{h\times h}) \quad (20)$$

a realization of the closed-loop transfer function from $\tilde{w}$ to $\tilde{z}$ is obtained as:

$$\mathbf{F}_l(G, K) = D_{cl} + C_{cl}(sI - A_{cl})^{-1} B_{cl}, \quad (21)$$

where

$$A_{cl} = \begin{bmatrix} A_1 + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix}, \ C_{cl} = \begin{bmatrix} C_1 & 0 \end{bmatrix},$$

$$B_{cl} = \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix}, \ D_{cl} = D_{11}.$$

Gathering all controller parameters into the single variable

$$\Theta := \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \quad (22)$$

and introducing the shorthand:

$$\bar{A} = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \ \bar{B}_2 = \begin{bmatrix} 0 & B_2 \\ I_h & 0 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 0 & I_h \\ C & 0 \end{bmatrix}, \ \bar{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}$$

the closed-loop matrices A_{cl}, B_{cl} can be written as:

$$A_{cl} = \bar{A} + \bar{B}_2 \Theta \bar{C}, \ B_{cl} = \bar{B}_1 + \bar{B}_2 \Theta \bar{D}_{21}. \quad (23)$$

From the Bounded Real Lemma, (20) is a h order γ-suboptimal controller if and only if the LMI

$$\begin{bmatrix} \begin{pmatrix} (\bar{A} + \bar{B}_2\Theta\bar{C})^T X_{cl} \\ +X_{cl}(\bar{A}+\bar{B}_2\Theta\bar{C}) \end{pmatrix} & X_{cl}(\bar{B}_1 + \bar{B}_2\Theta\bar{D}_{21}) & C_{cl}^T \\ (\bar{B}_1+\bar{B}_2\Theta\bar{D}_{21})^T X_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (24)$$

holds for some $X_{cl} > 0$ in $\Re^{(n+h)\times(n+h)}$.

Since there exist time-varying parameter uncertain matrices in the system model $P(s)$, then the matrices A, B_2 can be written into:

$$\bar{A} = \bar{A}_0 + \bar{D}\bar{F}\bar{E}_1, \quad \bar{B}_2 = \bar{B}_{20} + \bar{D}\bar{F}\bar{E}_2, \tag{25}$$

where

$$\bar{A}_0 = \begin{bmatrix} \tilde{A}_0 & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{B}_{20} = \begin{bmatrix} 0 & \tilde{B}_2 \\ I & 0 \end{bmatrix}, \ \bar{D} = \begin{bmatrix} \tilde{D} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{F} = diag\{F, F\,F\}, \ \bar{E}_1 = \begin{bmatrix} \tilde{E}_1 & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{E}_2 = \begin{bmatrix} 0 & \tilde{E}_2 \\ 0 & 0 \end{bmatrix}.$$

Substituting (25) into (24), using Lemma 3 and the Schur complement argument, then the LMI (24) holds for all F satisfying $F^T F \le I$ if and only if there exist a scalar $\varepsilon > 0$, Θ and X_{cl} such that

$$\Pi + \Psi_{X_{cl}}^T \Theta \Phi + \Phi^T \Theta^T \Psi_{X_{cl}} < 0, \tag{26}$$

where

$$\Pi = \begin{bmatrix} \bar{A}_0^T X_{cl} + X_{cl}\bar{A}_0 & X_{cl}\bar{B}_1 & C_{cl}^T & X_{cl}\bar{D} & \bar{E}_1^T \\ \bar{B}_1^T X_{cl} & -\gamma I & D_{cl}^T & 0 & 0 \\ C_{cl} & D_{cl} & -\gamma I & 0 & 0 \\ \bar{D}^T X_{cl} & 0 & 0 & -\varepsilon^{-1} I & 0 \\ \bar{E}_1 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix},$$

$$\Psi_{X_{cl}} = \begin{bmatrix} \bar{B}_{20} X_{cl} & 0 & 0 & 0 & \bar{E}_2^T \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \bar{C} & \bar{D}_{21} & 0 & 0 & 0 \end{bmatrix}.$$

We can now invoke Lemma 4 to eliminate Θ and obtain solvability conditions depending only on ε, X_{cl} and the plant parameters. Specifically, let $N_{\Psi_{X_{cl}}}$ and N_{Φ} denote matrices whose columns form bases of the null space of $\Psi_{X_{cl}}$ and Φ, respectively. Then by Lemma 4 (26) holds for some Θ if and only if

$$N_{\Psi_{cl}}^T \Pi N_{\Psi_{cl}} < 0, \quad N_{\Phi}^T \Pi N_{\Phi} < 0. \tag{27}$$

Now, if we define $\Psi = [\bar{B}_{20} \ \ 0 \ \ 0 \ \ 0 \ \ \bar{E}_2^T]$ and $Y = diag\{X_{cl}, I, I, I, I\}$, then $\Psi_{X_{cl}} = \Psi Y$. Hence $N_{\Psi_{X_{cl}}}^T = Y^{-1} N_{\Psi}$ is a base of the null space of $\Psi_{X_{cl}}$ whenever N_{Ψ} is a base of the null space of Ψ. Consequently,

$$N_{\Psi_{X_{cl}}}^T \Pi N_{\Psi_{X_{cl}}} < 0$$

is equivalent to

$$N_{\Psi}^T \Lambda N_{\Psi} < 0,$$

where

$$\Lambda = (Y^{-1})^T \Pi (Y^{-1})$$

$$= \begin{bmatrix} X_{cl}^{-1}\bar{A}_0^T + \bar{A}_0 X_{cl}^{-1} & \bar{B}_1 & X_{cl}^{-1} C_{cl}^T & \bar{D} & X_{cl}^{-1}\bar{E}_1^T \\ \bar{B}_1^T & -\gamma I & D_{cl}^T & 0 & 0 \\ C_{cl} X_{cl}^{-1} & D_{cl} & -\gamma I & 0 & 0 \\ \bar{D}^T & 0 & 0 & -\varepsilon^{-1} I & 0 \\ \bar{E}_1 X_{cl}^{-1} & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}.$$

Furthermore, (27) is equivalent to

$$N_{\Psi}^T \Lambda N_{\Psi} < 0, \quad N_{\Phi}^T \Pi N_{\Phi} < 0. \tag{28}$$

Finding a positive definite matrix X_{cl} satisfying (28) is awkward since it involves both X_{cl} and its inverse simultaneously. This can be done by partitioning X_{cl} and X_{cl}^{-1} as

$$X_{cl} := \begin{bmatrix} S & N \\ N^T & * \end{bmatrix}, \ X_{cl}^{-1} := \begin{bmatrix} R & M \\ M^T & * \end{bmatrix}, \tag{29}$$

where $R, S \in \Re^{n \times n}$ and $M, N \in \Re^{n \times h}$.

Consider the first constraint $N_{\Psi}^T \Lambda N_{\Psi} < 0$. With the partition (29), Λ can be written into

$$\Lambda = \begin{bmatrix} \tilde{A}_0 R + R\tilde{A}_0^T & \tilde{A}_0 M & B_1 & RC_1^T & \tilde{D} & 0 & R\tilde{E}_1^T & 0 \\ M^T \tilde{A}_0^T & 0 & 0 & M^T C_1^T & 0 & 0 & M^T \tilde{E}_1^T & 0 \\ B_1^T & 0 & -\gamma I & D_{11}^T & 0 & 0 & 0 & 0 \\ C_1 R & C_1 M & D_{11} & -\gamma I & 0 & 0 & 0 & 0 \\ \tilde{D}^T & 0 & 0 & 0 & -\varepsilon^{-1} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon^{-1} I & 0 & 0 \\ \tilde{E}_1 R & \tilde{E}_1 M & 0 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}. \tag{30}$$

Meanwhile, from

$$\Psi = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{B}_2^T & 0 & 0 & 0 & 0 & 0 & \tilde{E}_2^T & 0 \end{bmatrix} \tag{31}$$

it follows that base of the null space of Ψ are of the form

$$N_\Psi = \begin{bmatrix} W_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ W_2 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{32}$$

where $N_R := \begin{bmatrix} W_1^T & W_2^T \end{bmatrix}^T$ is any base of the null space of $\begin{bmatrix} \tilde{B}_2^T & \tilde{E}_2^T \end{bmatrix}$.

Observing that the second row of N_Ψ is identically zero, and invoking Schur complement argument again, the condition $N_\Psi^T \Lambda N_\Psi < 0$ can be reduced to (16). Similarly, the condition $N_\Phi^T \Lambda N_\Phi < 0$ is equivalent to (17). Hence X_{cl} satisfies (28) if and only if R,S satisfy (16)-(17). Moreover, by Lemma 5 $X_{cl} > 0$ is equivalent to R, S satisfying (18)-(19). □

Theorem 2: Suppose that Assumptions 1 and 2 are satisfied and $Y_d(s), Y^0(s) \in H_2$. Then the ILC system in Fig. 1 is robustly ℓ_2 convergent with remaining error and robustly stable for any uncertain matrix F satisfying $F^T F \le I$, if there exist scalars $0 < \gamma < 1$, $\varepsilon > 0$ and symmetric matrices R, S, satisfying (16)-(19).

Proof: The constraint condition of γ ($0 < \gamma < 1$) guaranteed the convergence of the ILC via Lemma 1. Furthermore, by using Lemma 1 and Theorem 1, we can directly obtain the results of the Theorem 2. □

Now we propose the following design procedure for the controller (13).

Theorem 3: Consider the ILC system in Fig. 1 and given $\varepsilon > 0$, if the following optimization problem

$$\begin{aligned} &\min_{R,S} \ \gamma \\ &s.t. \ (i) \quad (16),(17),(18) \\ &\quad\ \ (ii) \quad 0 < \gamma < 1 \end{aligned} \tag{33}$$

has an optimal solution γ^*, R, S, then the parameter matrix Θ of the controller (13) of order $h = n$ is derived in terms of the solvability to the LMI (26).

Proof: Since $R, S \in \Re^{n \times n}$ and rank $(I - RS) \le n$, the controller (13) of order $h = n$ satisfy the rank constraint (19). On the bases of Theorem 2, the optimal convergence rate γ^* and matrices R, S can be obtained by solving the optimization problem (33). Furthermore, computing two matrices $M, N \in \Re^{n \times h}$ such that

$$MN^T = I - RS, \tag{34}$$

an adequate X_{cl} is then obtained as solution of the linear equation:

$$\begin{bmatrix} S & I \\ N^T & 0 \end{bmatrix} = X_{cl} \begin{bmatrix} I & R \\ 0 & M^T \end{bmatrix}, \tag{35}$$

and the existence of a solution Θ to the inequality (26) can be guaranteed in virtue of Theorem 1. □

Remark 1: Since the matrix inequalities (16) and (17) contain ε and ε^{-1}, respectively, the optimization problem (33) can be solved by two steps as follows:

Step 1: To solve the following optimization problem

$$\begin{aligned} &\min_{R,\varepsilon} \ \gamma \\ &s.t.(i) \quad (16) \\ &\quad\ (ii) \quad 0 < \gamma < 1, \end{aligned}$$

an optimal solution γ^*, R^* and ε^* can be obtained.

Step 2: To solve the feasible problem about the matrix inequalities (17) and (18) where the variables R and ε are respectively substituted by R^* and ε^* obtained in Step 1, a feasible solution S can be given.

4. AN ILLUSTRATIVE EXAMPLE

To illustrate an application of the ILC technique, onsider an annealing process model [13]. The thermal processing setup is illustrated schematically in Fig. 3. A controlled heater lamp heats the part in the furnace and the furnace chamber.

For such thermal processing has two states: part temperature T_P and furnace temperature T_F. This process is a nonlinear system. For the application of the learning algorithm, this process is linearized around the stationary point $T_P = 500^\circ C$ and $T_F = 500^\circ C$, leading a continuous-time linear uncertain model described as (1), in which

$$A_0 = \begin{bmatrix} -1.2960 & 1.2960 \\ 0.0950 & -0.8950 \end{bmatrix}, \ B_0 = \begin{bmatrix} 0 \\ 1000 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ D = \begin{bmatrix} -1 \\ 0.09 \end{bmatrix}, \ E_1 = \begin{bmatrix} -1 & 1 \end{bmatrix}, \ E_2 = 3$$

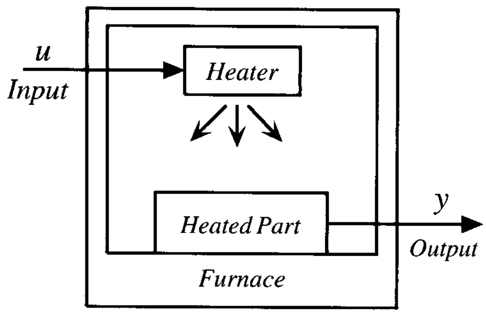

Fig. 3. Thermal process overview.

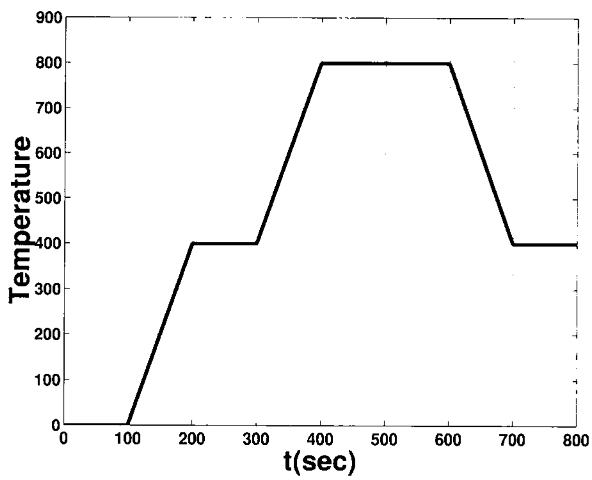

Fig. 4. Reference temperature trajectory.

and the system output $y = T_P$ is the part temperature that is assumed to be directly measured. In this simulation, the control goal is to ramp the part temperature from $0^\circ C$ to $400^\circ C$ in 100sec, maintain it at $400^\circ C$ for 100sec, then ramp the temperature up to $800^\circ C$ in 100sec, stay there for 200sec, and finally, ramp the temperature down to $400^\circ C$ in 100sec and stay there for 100sec, as Fig. 4.

$l_u(s)$ is chosen as a first order low-pass filter and $L_u(s)$ is described by the state space equation (14), in which

$$A_u = -50, B_u = 1, C_u = 50, D_u = 0. \quad (36)$$

By applying Theorem 3 and solving the corresponding optimization problem (33), we obtain $\gamma = 0.6762$,

$$\Theta = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} = \begin{bmatrix} -5849.4 & 27376 & -28547 & 194870 & 3.3497 \\ 1818.2 & -9252.6 & -6431.9 & -71146 & -221.05 \\ 15531 & -78530 & -53022 & -606444 & -338.03 \\ -23.651 & 110.7 & -115.44 & 788.03 & 0.013454 \end{bmatrix}.$$

Thus, it follows from the above equation, (5), (13) and (20) that

$$C(s) = \frac{788s^3 + 1.112\times10^8 s^2 + 5.639\times10^9 s + 4.324\times10^9}{s^3 + 6.812\times10^4 s^2 + 7.434\times10^8 s + 3.694\times10^{10}} \quad (37)$$

$$L_e(s) = \frac{0.01345s^3 + 15390s^2 - 2.7\times10^9 s - 2.103\times10^9}{s^3 + 6.812\times10^4 s^2 + 7.434\times10^8 s + 3.694\times10^{10}} \quad (38)$$

and the Bode Diagram of $L(s)$ as Fig. 5.

Fig. 6 shows tracking errors at 1st iteration (only current feedback controller), 2nd iteration, 3rd iteration, and 10th iteration. By adding the learning controllers, the tracking errors are diminished as the iteration number increases. More quantitative information can be obtained from Fig. 7 showing the root mean square (RMS) values of the tracking errors, respectively. In addition, in the Fig. 7 the solid line, dash line and dot line denote the corresponding results about the nominal plant model, the plant model for $F(t) = 1$ and the plant model for $F(t) = -1$,

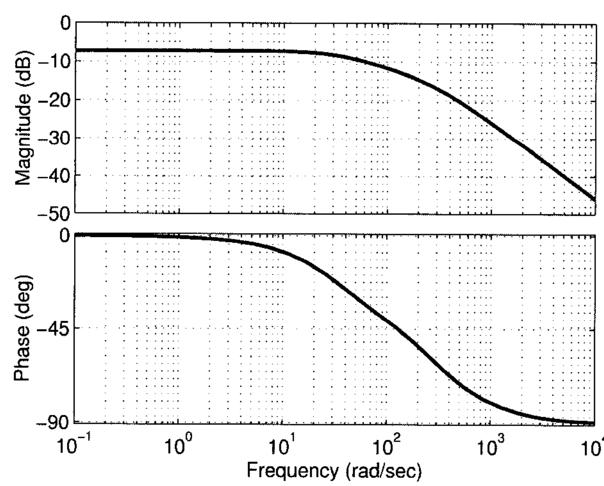

Fig. 5. The bode diagram of $L(s)$.

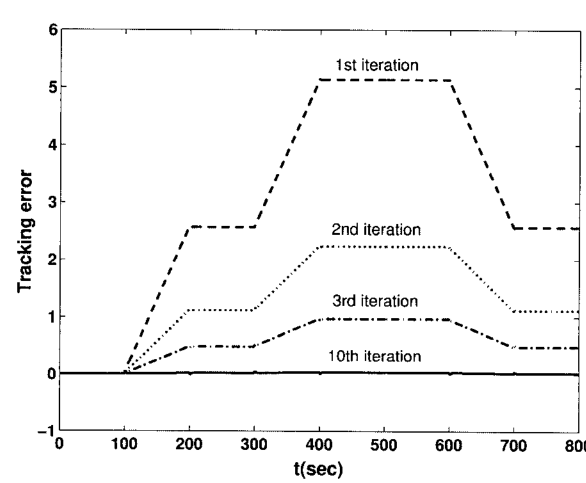

Fig. 6. Tracking errors at 1st (dash), 2nd (dot), 3rd (dash-dot), and 10th (solid) iteration.

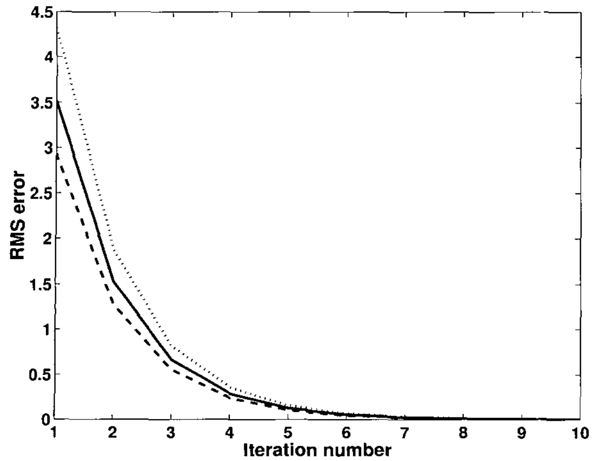

Fig. 7. Root mean square values of tracking errors versus iteration number.

respectively. These simulations show that the resulting ILC system with the current feedback controller (37) and the learning controllers (36) and (38) is robustly ℓ_2 convergent and robustly stable.

5. CONCLUSIONS

It has been shown in this paper that the design of the iterative learning control algorithm can be generalized to the design of a γ-suboptimal H_∞ controller, by choosing an appropriate complex matrix G, and reformulating the ILC synthesis problem in the standard plant format. A sufficient convergence condition for the proposed ILC process in the presence of plant uncertainty is given in terms of linear matrix inequalities. Based on the derived condition, we showed that the iterative learning control design problem can be reformulated as a general robust control problem and thus can be solved by the linear matrix inequality approach.

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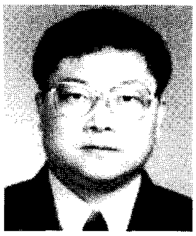

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