Distributor's reliable inventory model for deteriorating product when the supplier offers an uncharged addition in a supply chain

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묶음판매가 허용되는 공급사슬에서 퇴화성 제품을 취급하는 중간분배자의 신뢰성있는 재고모형

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Abstract

본 연구는 공급자, 중간분배자 그리고 고객으로 구성된 2 단계 공급사슬에서 시간에 따라 일정률로 퇴화하는 퇴화성 제품을 취급하는 중간분배자의 경제적 재고모형을 분석하였다. 문제 분석을 위하여 공급자는 고객으로 부터의 수요를 증대시키기 위한 수단으로 일정한 묶음 단위별로 일정한 양의 덤을 제공한다는 가정 하에 재고모형을 설계하였고, 모형 분석을 통하여 이익을 최대화하는 경제적 주문량 결정 방법을 제시하였다. 또한 예제를 통하여 제시된 해법을 적용하고, 그 타당성을 보였다.
Keywords : Reliable inventory, EOQ, Uncharged addition, Deteriorating products

1. Introduction

An effective supply chain network requires a cooperative relationship between the vendor/supplier and the buyer/distributor. The cooperation include the sharing of information, resources and profit or cost saving. One of the realistic strategies is based on the assumption that the supplier gives some incentives to the buyer in order to stimulate the demand for the product he produces. In this regard, a considerable number of research papers have been studied on the subjects of inventory control involving lot sizing with quantity discount. The traditional quantity discount models have analyzed solely the unit purchasing price discount and considered two types of price discount, "all-units" and "incremental" discount. Abad[1, 2], and, Kim and Hwang[12] analyzed the effect of quantity discount on the optimal inventory policy. Recognizing another type of discount structure, Burwell et al.[3] and Lee[14] formulated the classical EOQ model with set up cost including a fixed cost and freight cost, where the freight cost has a quantity discount. The common assumption of the above extended models is that the customer must pay for the items as soon as he receives them from a supplier.

However, in some distribution systems, the supplier will allow a certain credit period for settling the amount the buyer owes to him for the items supplied. Trade credit would play an important role in the conduct of business for many reasons.

† 이 연구는 2007년도 한라대학교 교내연구비 지원에 의한 것 임.
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2008년 1월 접수; 2008년 2월 수정본 접수; 2008년 2월 게재확정

For a supplier who offers trade credit, it is an effective means of price discrimination which circumvents antitrust measures and is also an efficient method to stimulate the demand of the product. For a buyer, it is an efficient method of bonding a supplier when the retailer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stocks. Based upon the above observations, some research papers dealt with the inventory model under trade credit. Chapman et al.[4], Chung [6], Goyal[9], and Kingsman[13] examined the effects of trade credit on the optimal inventory policy. Also, Shinn et al.[16] introduced the joint price and lot size determination problem under conditions of trade credit and quantity discounts for freight cost.

In another distribution system, it is not uncommon that the suppliers offer an uncharged addition related to the bundle size for reasons of marketing policy. Therefore, the buyers can get some extra with no additive cost depending on the amount of the quantity purchased. The availability of opportunity to get some extra with no cost effectively reduces the buyer's total purchasing cost and it enables the buyer to choose an optimal ordering quantity from another options. In this regard, Shinn and Song[17] formulated the optimal inventory model when the supplier permits an uncharged addition for an order of a product depending on the amount of the quantity purchased by the buyer. And they showed that the optimal order quantity quite sensitive to the uncharged addition rate.

All the research works mentioned above implicitly assume that inventory is depleted by buyer's demand alone. This assumption is quite valid for nonperishable or nondeteriorating inventory items. However, there are numerous types of inventory whose utility does not remain constant over time. In this case, inventory is depleted not only by demand but also by deterioration. Ghare and Schrader[8], assuming exponential deterioration of the inventory in the face of constant demand, derived a revised form of the economic order quantity. Cohen[7] examined the joint price and lot size determination problem for an exponentially deteriorating product. Recently, Chu et al.[5], Hwang and Shinn[10], Jaggi and Aggarwal[11], and Shinn[15] analyzed the inventory model for deteriorating products under the condition of permissible delay in payments.

In this paper, we evaluate the distributor's economic lot sizing problem for an exponentially deteriorating product when the supplier permits a fixed uncharged addition depending on the amount of the quantity purchased by the buyer. We formulate a relevant mathematical model in Section 2 and the characteristics of the model are analyzed in Section 3. In Section 4, a solution algorithm is developed and a numerical example is provided, which is followed by concluding remarks.

2. Development of the Model

Let's consider the situation in which the supplier allows a certain uncharged addition depending on the amount of the quantity purchased by the buyer for reasons of marketing policy. For the buyer, the availability of opportunity to get some extra with no cost effectively reduces the buyer's total purchasing cost and it enables the buyer to choose an optimal inventory policy from another options. The objective of this model is to determine the optimal ordering quantity which maximizes the annual net profit for the distributor.

The following assumptions will be used for our mathematical model.

1) Replenishments are instantaneous with a known and constant lead time.
2) No shortages are allowed.
3) The inventory system deals with only one type of item.
4) The demand rate is known and constant.
5) Inventory is depleted not only by demand but also by deterioration. Deterioration follows an exponential distribution with parameter λ.
6) The supplier allows a certain uncharged addition depending on the amount of the quantity purchased by the buyer(distributor).

And the following notations will be used.

D : annual demand rate

P : distributor's selling price
C : unit purchasing cost
Q : order size
T : replenishment cycle time
A : ordering cost
H : inventory carrying cost
U : bundle size for a uncharged addition
α : uncharged addition rate(as a percentage of U)
λ : a positive number representing the inventory deteriorating rate
$q(t)$: inventory level at time t

In this model, we consider the situation that the supplier allows a certain uncharged addition to stimulate the demand of the product. Let U be the constant bundle size and α be a certain uncharged addition rate. For the first U, as products are ordered to $(1-\alpha)U$, the unit purchasing cost C is charged for each unit of product. Therefore, the total purchasing cost becomes CQ for $0 < Q < (1-\alpha)U$.

When the order size Q becomes $(1-\alpha)U$, the products are sold in a bundle of size U as the total purchasing cost is $C(1-\alpha)U$. Namely, there is no additive purchasing cost for αU units. <Figure 1> illustrates the total purchasing cost to the order size.

Note that the feasible quantities of Q are $Q \in [(j-1)U, (j-1)U + (1-\alpha)U)$, $j = 1, 2, \cdots, n$ and therefore, the total purchasing cost, $C(Q)$ is

$$C(Q) = C(1-\alpha)(j-1)U + C(Q-(j-1)U) = C(Q-\alpha(j-1)U), \quad Q \in [(j-1)U, (j-1)U + (1-\alpha)U), j = 1, 2, \cdots. n. \quad (1)$$

Note that when the uncharged addition rate, $\alpha = 0$, the total purchasing cost $C(Q)$ reduce to CQ.

For the case of exponential deterioration, as stated by Ghare and Schrader[8], the rate at which inventory deteriorates will be proportional to on hand inventory, $q(t)$. Thus, the depletion rate of inventory at any time t is

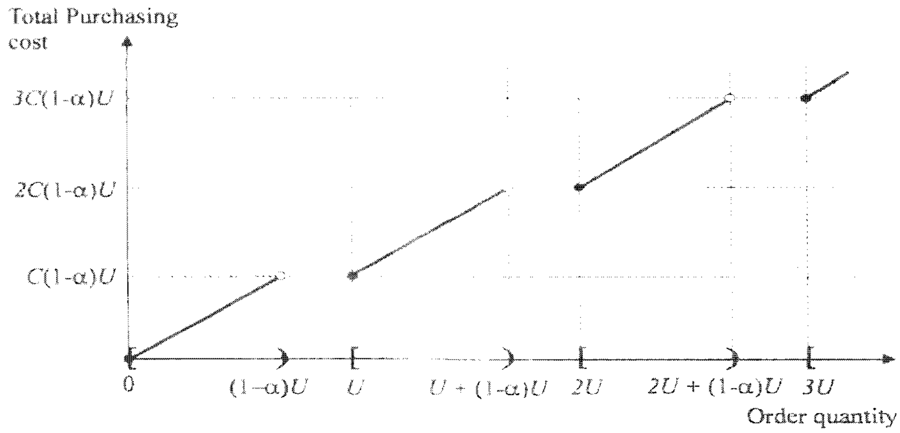
$$\frac{dq(t)}{dt} = -\lambda q(t) - D \ . \quad (2)$$

Observing that equation(2) is a first order linear differential equation, its solution is

$$q(t) = q(0)e^{-\lambda t} - \frac{D}{\lambda}(1 - e^{-\lambda t}) \ . \quad (3)$$

Equation (3) gives the inventory level at time t representing the combined effects of demand usage and exponential deterioration.

Now, we determine the inventory loss caused by deterioration. Let $q^0(t)$ be the inventory level at time t where there were no deterioration. Then, the inventory loss caused by deterioration becomes



<Figure 1> Total purchasing cost vs. Order quantity

$$q^0(t)-q(t)=(q(0)-Dt)-(q(0)e^{-\lambda t}-\frac{D}{\lambda}(1-e^{-\lambda t})) \quad (4)$$

$$=q(t)(e^{\lambda t}-1)-Dt+\frac{D}{\lambda}(e^{\lambda t}-1)\ . \quad (5)$$

Therefore, the quantity ordered per cycle becomes

$$Q=(q^0(T)-q(T))+DT\ . \quad (6)$$

Note that because of the inventory carrying costs, it is clearly better off to have the inventory level reach zero just before reordering, i.e., $q(T)=0$. <Figure 2> illustrates the time behavior of the inventory level. Demand rate, D is indicated by the slope of the dashed line. With $q(T)=0$, we have

$$Q=\frac{D}{\lambda}(e^{\lambda T}-1)\ . \quad (7)$$

Also, from the condition of $q(0)=Q$, the inventory level at time t is

$$q(t)=\frac{D}{\lambda}(e^{\lambda(T-t)}-1), 0\le t\le T\ . \quad (8)$$

Now, we formulate the annual net profit $\Pi(T)$. The annual net profit consists of the following four elements.

1) Annual sales revenue = DP
2) Annual ordering cost = $\frac{A}{T}$.
3) Annual inventory carrying cost = $\frac{H}{T}\int_0^T q(t)dt$

$$=\frac{HD}{\lambda^2 T}(e^{\lambda T}-\lambda T-1).$$

4) Annual purchasing cost = $\frac{C(Q)}{T}$

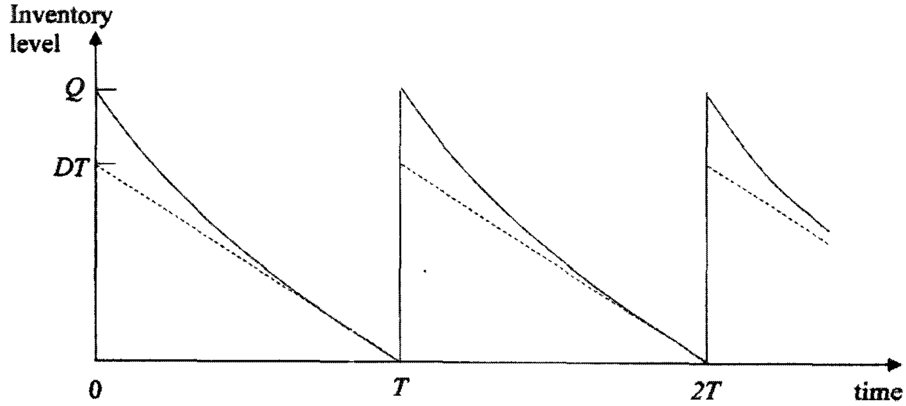
$$=\frac{C}{T}(Q-\alpha(j-1)U),$$
$$Q\in[(j-1)U,(j-1)U+(1-\alpha)U).$$

Also, by equation (7), the annual purchasing cost, $\frac{C}{T}(Q-\alpha(j-1)U)$ can be rewritten as $\frac{C}{T}((e^{\lambda T}-1)\frac{D}{\lambda}-\alpha(j-1)U)$. And if let $I_j^L=\frac{1}{\lambda}ln\left(\frac{\lambda}{D}(j-1)U+1\right)$ and $I_j^U=\frac{1}{\lambda}ln\left(\frac{\lambda}{D}(j-1)U+1+\frac{\lambda}{D}(1-\alpha)U\right)$, then the annual purchasing cost becomes $\frac{C}{T}((e^{\lambda T}-1)\frac{D}{\lambda}-\alpha(j-1)U)$, $T\in[I_j^L, I_j^U)$.

Then, the annual net profit $\Pi(T)$ can be expressed as

$\Pi(T)$ = Sales revenue - Ordering cost - Inventory carrying cost - Purchasing cost.

Depending on the relative size of T to I_j^L and I_j^U,



<Figure 2> Inventory level($q(t)$) vs. time(t)

$$\Pi_j(T) = DP - \frac{A}{T} - \frac{HD}{\lambda^2 T}(e^{\lambda T} - \lambda T - 1) - \frac{CD}{\lambda T}(e^{\lambda T} - 1) + \frac{C}{T}\alpha(j-1)U,\ T \in [I_j^L,\ I_j^U),\ j = 1,2,\cdots,n. \quad (9)$$

Note that if $\alpha = 0$, then equation (9) reduce to the total cost function of the classical EOQ model for deteriorating product.

3. Determination of Optimal Policy

The problem is to find an optimal replenishment cycle time T^* which maximizes $\Pi_j(T)$. Once T^* is found an optimal ordering quantity Q^* can be obtained by equation (7). Although the objective function can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find an optimal solution in explicit form. Thus the model will be solved approximately by using a truncated Taylor series expansion for the exponential term, i.e.,

$$e^{\lambda T} \fallingdotseq 1 + \lambda T + \frac{1}{2}\lambda^2 T^2\ , \quad (10)$$

which is a valid approximation for smaller values of λT. With the above approximation, the annual total cost function can be rewritten as

$$\Pi_j(T) = DP - \left(\frac{A}{T} + \frac{1}{2}DT(H + C\lambda) + CD - \frac{C}{T}\alpha(j-1)U\right),\ T \in [I_j^L,\ I_j^U), j = 1,2,\cdots,\ n. \quad (11)$$

Note that equation (10) is exact when $\lambda = 0$ so that equation (9) reduces to the exact formula equation (11) for non-deteriorating product. Also, it is self evident that for any fixed T, $\Pi_j(T) < \Pi_{j+1}(T)$, $j = 1,2,\cdots,\ n$. So, $\Pi_j(T)$ is strictly increasing for any fixed T as j increases.

For $j < 1 + A/(C\alpha U)$, $\Pi_j(T)$ is a concave function for every j and thus, there exists a unique value T_j, which maximizes and they are:

$$T_j = \sqrt{\frac{2(A - C\alpha(j-1)U)}{D(H + C\lambda)}}\ . \quad (12)$$

And if T_j exists, then the value of T_j is strictly decreasing as j increases, i.e.,

$$T_{j-1} > T_j \text{ holds for } j < 1 + A/(C\alpha U). \quad (13)$$

Also, for $j \geqq 1 + A/(C\alpha U)$, $\Pi_j(T)' < 0$ and so, $\Pi_j(T)$ is an decreasing function of T.
From the above results, we have the following property.

Property 1.
There exists at least one $T_j \geqq I_j^L$.

Proof.
Because $A/(C\alpha U) > 0$, $\Pi_1(T)$ must be a concave function and thus, there exists at least one T_j. Also, from property 1, if all $T_j < I_j^L$ for every j, then $T_1 \langle 0$ holds, which contradicts the feasibility of T, i.e., $0 \langle T \langle \infty$. Q.E.D.

Now, we can make Property 2 and 3 about the characteristics of $\Pi_j(T)$ for $T \in I_j^{LU} = \{T | I_j^L \leqq T < I_j^U\}$, $j = 1,\ 2,\ \cdots,\ n$. These properties simplifies our search process such that only a finite number of candidate values of T need to be considered to find an optimal value T^*. Let m be the largest index such that $T_m \geqq I_m^L$ and $T^0 = I_m^U - \epsilon$.

Property 2.
For j > m, we only need to consider $T = I_j^L$ as candidate of an optimal value T^* for $T \in I_j^{LU}$.

Proof.
By definition of T_j and inequality (13), $\Pi_j(T)$ is decreasing in $T \in I_j^{LU}$ for $j > m$. So, we only need to consider $T = I_j^L$, $j > m$, in finding an optimal value T^* for $T \geqq I_{m+1}^L$. Q.E.D.

Property 3.
(i) If $T_m > T^0$, then we only have to consider $T = T^0$ for $T \in I_m^{LU}$ as candidate for T^* and

$T^* \geqq T^0$.

(ii) Else if $T_m \geqq I_m^L$, then we only have to consider $T = T_m$ for $T \in I_m^{LU}$ as candidate for T^* and $T^* \geqq T_m$.

Proof.

(i) Because $T_m > T^0$, $\Pi_m(T)$ is a increasing function for $T \in I_m^{LU}$ and so, we have

$$\Pi_m(T^0) > \Pi_m(T) \text{ for } T \in I_m^{LU}.$$

Therefore, if $T_m > T^0$, then $T = T^0$ yields the maximum annual net profit for $T \in I_m^{LU}$. Also, because $\Pi_j(T)$ is strictly increasing for any fixed T as j increases,

$$\Pi_m(T) > \Pi_j(T), \quad j < m.$$

Hence,

$$T^* \geqq T^0.$$

(ii) Since $I_m^L \leqq T_m \leqq T^0$, $\Pi_m(T)$ is concave function for $T \in I_m^{LU}$. So, by definition of T_m, we have

$$\Pi_m(T_m) \geqq \Pi_m(T) \text{ for } T \in I_m^{LU}.$$

Therefore, if $I_m^L \leqq T_m \leqq T^0$, then the annual net profit becomes the maximum at $T = T_m$. Also, because $\Pi_j(T)$ is strictly increasing for any fixed T as j increases,

$$\Pi_m(T) > \Pi_j(T), \quad j < m.$$

Hence,

$$T^* \geqq T_m$$

Q.E.D.

4. Solution Algorithm and Numerical Example

Based on the above properties, we develop the following solution algorithm to determine an optimal solution for the approximate model.

Step 1. Compute T_1 by equation (12) and find the index k such that $T_1 \in [I_k^L, I_{k+1}^L)$.

Step 2. Find the largest index l such that $l < 1 + A/(C\alpha U)$.

Step 3. Compute T_j, $j \leqq \min\{k, l\}$ by equation (12) and find the largest index m such that $T_m \geqq I_m^L$.

Step 4. If $T_m > I_m^U - \epsilon$, then compute the annual net profit for $T = I_m^U - \epsilon$.
Otherwise, compute the annual net profit for $T = T_m$.

Step 5. Compute the annual net profit for $T = I_j^L$, $j = m+1, m+2, \cdots, n+1$.

Step 6. Select the one that yields the maximum annual net profit as T^* and stop.

And to illustrate the proposed solution algorithm, let us consider the following problem.

D = 3,000 units, P = \$5, C = \$3, A = \$500, H = \$0.25, U = 300 units(j = 1, 2, …, 10), a = 0.1(=10 %) and λ = 0.2.

The solution procedure generates an optimal solution for the approximate model through the following steps.

Step 1. Since T_1 = 0.6262, k = 7.

Step 2. Since $1 + A/(C\alpha U) = 6.55$, l = 6.

Step 3. Since min{7, 6} = 6, compute T_j for $j \leqq 6$. And since $T_5 (= 0.3314) < I_5^L (= 0.3848)$ and $T_4 (= 0.4247) > I_4^L (= 0.2913)$, m = 4.

Step 4. Since $T_4 (= 0.4247) \geq I_4^U (= 0.3755)$, compute $\Pi_4(I_4^U - \epsilon)$.

Step 5. Since m = 4, compute $\Pi_j(I_j^L)$, $j = 5, 6, \cdots, 11$.

Step 6. Since $\Pi_8(I_8^L)$ = 5363.191 = min{ $\Pi_4(I_4^U - \epsilon)$ and $\Pi_j(I_j^L), j = 5, 6, \cdots, 11\}$, an optimal replenishment cycle time becomes 0.6551 with its maximum annual net profit of $5363.191.

5. Conclusions

In this paper, we evaluated the distributor's economic lot sizing problem for an exponentially deteriorating product when the supplier offers a fixed uncharged addition depending on the amount of the quantity purchased by the buyer. In many distribution systems in Korea, it is not uncommon that a supplier offers some uncharged addition to a certain degree expecting that he can make more profit by stimulating the buyer's demand.

After formulating the mathematical model, we proposed the solution procedure which leads to an economic lot size. To illustrate the validity of the procedure, an example problem was solved and the results are consistent with our expectation. The model developed in this paper may help the buyers find an economic replenishment policy for deteriorating products when the supplier offers a fixed uncharged addition depending on the amount of the quantity purchased by the buyer.

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인하대학교 산업공학과를 졸업했으며, 동 대학원에서 석사, 박사학위를 취득하였고, 한국산업개발연구원과 한국생산성본부에서기업지도 및 교육업무를 담당하였으며, 현재는 한라대학교 산업경영공학과에 재직중이다. 관심분야는 TQM, TPM, 생산관리, 경영혁신(공장합리화), 물류관리 등이다.

주소: 강원도 원주시 흥업면 흥업리 산 66번지 한라대학교 산업경영공학과