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Multi-vehicle Route Selection Based on an Ant System

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Abstract

This paper introduces the multi-vehicle routing problem(MRP) which is different from the traveling sales problem(TSP), and presents the ant system(AS) applied to the MRP. The proposed MRP is a distributive model of TSP since many vehicles are used, not just one salesman in TSP and even some constraints exist. In the AS, a set of cooperating agents called vehicles cooperate to find good solutions to the MRP. To make the proposed MRP extended more, Tokyo city model(TCM) is proposed. The goal in TCM is to find a set of routes that minimizes the total traveling time such that each vehicle can reach its destination as soon as possible. The results show that the AS can effectively find a set of routes minimizing the total traveling time even though the TCM has some constraints.

Key words : ant system, traveling sales problem, multi-vehicle management problem, swarm intelligence

1. Introduction

Combinational optimization problems are intriguing because they often easy to state but very difficult to solve. Many of the problems arising in applications are $\mathcal{NP}$-hard, that is, it is strongly believed that they cannot be solved to optimality within polynomially bounded computation time. Hence, to practically solve large instances one often has to use approximate methods which return near-optimal solution in a relatively short time. Algorithms of this type are loosely called heuristics. Recently, many researchers have focused their attention on a new class of algorithms, called meta-heuristics. A meta-heuristic is a set of algorithmic concepts that can be used to define heuristic methods applicable to a wide set of different problems. The use of meta-heuristics has significantly increased ability of finding very high-quality solutions to hard, practically relevant combinatorial optimization problems in a reasonable time.

A particularly successful meta-heuristic is inspired by the behavior of real ants. Ant system(AS) is a meta-heuristic for the approximate solution of combinatorial optimization problems that has been inspired by the foraging behavior of ant colonies [1]. In AS algorithms the computational resources are allocated to a set of relatively simple agents that exploit stigmergic communication, that is, a form of indirect communication medicated by the environment, [2] to construct solution to the considered problem. The construction of good solution is a result of the agents' cooperative interaction.

The ant system has been applied to combinatorial optimization problems such as the traveling salesman problem(TSP) [3]-[10]. TSP is the problem of finding a shortest closed tour which visits all the cities in a given set. The AS, represented in [11], builds on the previous ant system in the direction of improving efficiency when applied to symmetric and asymmetric TSP's. There several reasons for the choice of the TSP as the problem to explain the working of AS algorithms: it is an important $\mathcal{NP}$-hard optimization problem that arises in several applications; it is a problem to which AS algorithms are easily applied; it is easily understandable, so that the algorithm behavior is not obscured by too many technicalities; and it is a standard test bed for new algorithmic ideas - a good performance on the TSP is often taken as a proof of their usefulness. Additionally, the history of AS shows that very often the most efficient AS algorithms for the TSP were also found to be among the most efficient ones for a wide variety of other problems.

In the paper, MRP, which is a distributive and more restrictive model of TSP, is proposed. In addition, to make the MRP extended more, Tokyo city model(TCM) is considered. Since multi-vehicles are used in MRP, not just one salesman in TSP, MRP where each vehicle has its own destination is a distributive model of TSP. MRP and TCP needs much more memory space than TSP. In the case of TSP, general approximation method such as 0.878 approximation can not succeed to solve TSP using a computer.

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In addition, MRP is more complex version of TSP since the road is not shared among other vehicles. The purpose is to find an optimal route in MRP and TCM for multi-vehicle routing using AS that is recently contrived for parallel stochastic optimization problems, which may deserve to be considered. The main novel idea introduced by AS algorithm will be discussed in the remainder of the paper.

This paper is organized as follows. Section 2 contains the description of the AS as it is currently implemented. MRP and TCM are introduced in Section 3 and 4, respectively, along with an overview of their results. Section 5 describes difference between TSP and TCM. Finally conclusion is drawn in Section 6.

2. Problem Statement and Ant System

Ant system used in the paper is based on [12]. The main phase of the AS algorithm constitutes the ants' solution construction and the pheromone update. In AS, m artificial ants concurrently build a path of the MRP. Initially, ants are put on randomly chosen points. At each construction step, ant k applies a probabilistic action choice rule, called *random proportional* rule, to decide which point to come by next. In particular, the probabilistic with which ant k, currently at point i, chooses to go to point j is

$$p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in \mathcal{N}_i{}^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, \quad if\ j \in \mathcal{N}_i{}^k \tag{1}$$

where τ_{ij} is a desirability measure called pheromone and t_{ij} is the time that takes from points i to j. $\eta_{ij} = \frac{1}{t_{ij}}$ is a heuristic value that is available a priori, α and β are two parameters which determine are the relative influence of the pheromone trail and the heuristic information. t_{ij} is the cost measure. In TSP, d_{ij} was used to represent the distance from points i to j instead of t_{ij}. $\mathcal{N}_i{}^k$ is the feasible neighborhood of ant k when being at point i, that is, the set of points that ant k has not moved yet (the probability of choosing a point outside $\mathcal{N}_i{}^k$ is 0).

By this probabilistic rule, the probability of choosing a particular arc (i, j) increases with the value of the associated pheromone trail τ_{ij} and of the heuristic information value η_{ij}. The role of the parameters α and β is the following. If $\alpha = 0$, the closest points are more likely to be selected. This corresponds to a stochastic greedy algorithm (with multiple starting points since ants are initially randomly distributed over the points). If $\beta = 0$, only pheromone amplification is at work, that is, only pheromone is used, without any heuristic bias. This generally leads to rather poor results and, in particular, for values of $\alpha > 1$ it leads to the rapid emergence of a stagnation situation, that is, a situation in which all the ants follow the same road and construct the same path, which, in general, is strongly suboptimal [1], [14].

Each ant k maintains a memory M^k which contains the points already passed, in the order they were passed. This memory is used to define the feasible neighborhood $\mathcal{N}_i{}^k$ in the construction rule given by (1). In addition, the memory M^k allows ant k both to compute the time of the whole path T^k it generated for each vehicles and to retrace the path to deposit pheromone.

After all the ants have constructed their paths to destination, the pheromone trails are updated. This is done by first lowering the pheromone value on all path between two points by a constant factor, and then adding pheromone on the roads the ants have crossed in their paths. Pheromone evaporation is implemented by

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij}, \quad \forall (i,j) \in L \tag{2}$$

where $0 < \rho \leq 1$ is the pheromone evaporation rate. The parameter ρ is used to avoid unlimited accumulation of the pheromone trails and it enables the algorithm to "forget" bad decisions previously taken. In fact, if a road between two points is not chosen by the ants, its associated pheromone value decreases exponentially in the number of iterations. After evaporation, all ants deposit pheromone on the road they have crossed in their road

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^k, \quad \forall (i,j) \in L \tag{3}$$

where $\Delta\tau_{ij}^k$ is the amount of pheromone and k deposits on the roads it has visited. It is defined as follows

$$\Delta\tau_{ij}^k = \begin{cases} \frac{1}{C^k}, & if\ road(i,j)\ belongs\ to\ T^k \\ 0, & otherwise \end{cases} \tag{4}$$

where C^k is the summation of the time T^k taken by the k-th ant that is taken by each car. In other word, C^k is computed as the sum of the time of the roads belonging to T^k. By means of (4), the faster an ant's road is taken, the more pheromone the roads belonging to this path receive. In general, roads that are used by many ants and which are part of fast roads, receive more pheromone and are therefore more likely to be chosen by ants in future iterations of the algorithm.

3. Multi-vehicle Routing Problem(MRP)

The authors proposes the multi-vehicle routing problem(MRP) which is different from the traveling sales problem(TSP), and presents the ant system(AS) applied to the MRP. The proposed MRP is a distributive model of TSP since many vehicles are used, not just one salesman in TSP and even some constraints exist. In the AS, a set of cooperating agents called vehicles cooperate to find good solutions to the MRP. MRP is the problem of finding a total

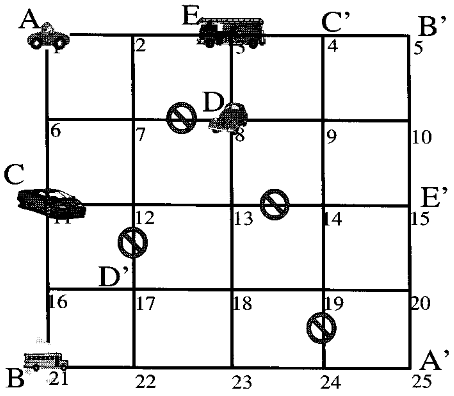
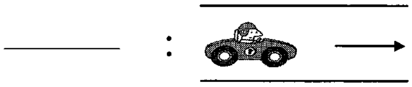


Figure 1: Multi-vehicle routing problem

Target : 5 vehicles, 25 points,

Assumption :

1. Same time is taken to drive from point to point.
2. Destination of vehicle located in A Route: from A to A′



one-way road

Constraint : Same road can be taken only by one car.

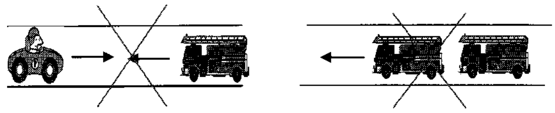


Figure 2: Description of multi-vehicle routing problem

fastest path that each of given vehicles drives from its starting point to its destination. In this paper we will restrict attention to MRP in which points are are on a plane and a road exists between each pair of points. The last point of each vehicle is destination.

Informally, AS works as follows. Each ant generates a path by choosing the roads according to a probabilistic state transition rule, that is, ant prefer to move to roads which are connected by short time with high amount of pheromone. Once all ants have arrived their destinations, a pheromone updating rule is applied. In other words, a fraction of the pheromone evaporates on all roads that are not refreshed become less desirable. And then each ant deposits an amount of pheromone on roads which belongs to its path to destination in proportion to how fast its path was. In other words, roads which belong to many fast paths are the roads which receive the greater amount of pheromone. The process is then iterated.

The idea was that a set of agents, called ants, search in parallel for good solutions to the MRP and cooperate through pheromone-mediated indirect and global communication. Informally, each ant constructs a MRP solution in an iterative way: it adds new points to a partial solution by exploiting both information gained from past experience and a greedy heuristic. Memory takes the form of pheromone deposited by ants on each road between points, while heuristic information is simply given by the time that takes from point to point. The main idea is the synergistic use of cooperation among many relatively simple agents which communicate by distributed memory implemented as pheromone deposited on roads between points.

Fig. 1 shows the MRP where it is composed of 5 vehicles and 25 points. Its proposition and constraints are described in Fig. 2.

Success condition is organized as follows

A-A' : 9 points, B-B' : 9 points, C-C' : 6 points, D-D' : 4 points, E-E' : 5 points .

The number of nodes used in simulation is 36 where 10 vehicles have their own destination. The number of iterations is 10. Design parameters are set to $\alpha = 1$, $\beta = 0$, and $\rho = 0.9$. In the case of a conventional study (0.878 approximation method), only one solution is obtained taking around 42 [s] using a Pentium computer as follows.

A-A' = [1 2 3 8 13 18 19 20 25]
B-B' = [21 22 23 18 19 14 9 10 5]
C-C' = [11 6 7 2 3 4]
D-D' = [8 13 18 17]
E-E' = [3 4 9 14 15]

But the AS (Swarm Intelligence) provides many different solutions with the same success condition whenever it runs. In addition, it takes just less than 1 [s] using a Pentium computer as follows.

A-A' = [1 6 11 12 13 18 23 24 25]
B-B' = [21 22 23 18 19 14 9 4 5]
C-C' = [11 6 1 2 3 4]
D-D' = [8 13 18 17]
E-E' = [3 4 5 10 15]

A-A' = [1 2 7 12 13 18 23 24 25]
B-B' = [21 22 23 18 19 20 15 10 5]
C-C' = [11 6 7 2 3 4]
D-D' = [8 13 18 17]
E-E' = [3 8 9 14 15]

more

4. Tokyo City Model(TCM)

To make the proposed MRP extended more, Tokyo city model(TCM) is proposed by the authors. The goal in TCM is to find a set of routes that minimizes the total traveling

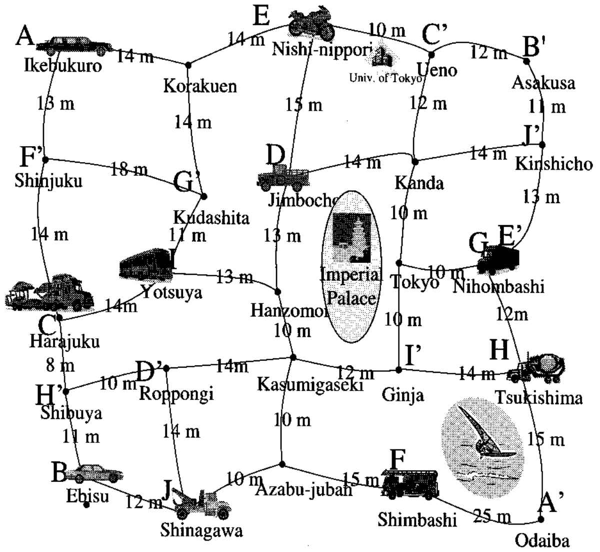


Figure 3: Tokyo city model

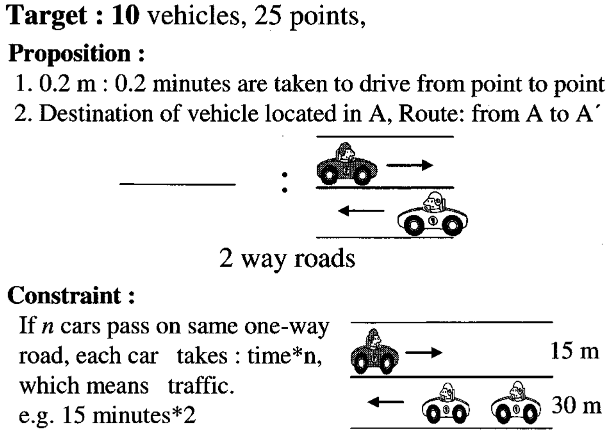


Figure 4: Description of Tokyo city model

time such that each vehicle can reach its destination as soon as possible. Fig. 3 shows the MRP where it is composed of 10 vehicles, 25 points and different time from point to point.

TCM is the model of extended MRP. It has different time from point to point and includes a constraint called traffic. $\eta_{ij} = \frac{1}{t_{ij} * n_{ij}}$ where n_{ij} is the number of vehicles between points i and j.

Fig. 4 shows the summary of difference between MRP and TCM. In TCM, there is a traffic that each car takes time (being taken to drive from point to point) $\times$ number of cars (driving on same road if exist). Such a traffic condition may be modified according to circumstances.

Considering TCM, the version of the extended MRP, the reason that shows the difference between TSP and TCM follows as:

1. TCM is a distributive model of TSP. $\rightarrow$ Many vehicles are used, not just one salesman in TSP.
2. The constraint exists. $\rightarrow$ Different time is required even the same point to point depending on how many vehicles are which corresponds traffic.
3. In TSP, a salesman returns to starting point(city) while each vehicle has its own destination in TCM. $\rightarrow$ The destination is not a starting point.
4. Time is used from point to point, unlike a distance in TSP. $\rightarrow$ It is used to calculate traffic. Hence, TCM is a complicated distributive model of TSP. Here, 'complicated' means TCM has constraint(different time). 'distributive' means TCM has more vehicles and different their destination.

Thus, TCM is deserve to be considered importantly.

The number of nodes used in simulation is 36 where 10 vehicles have their own destination with traffic in different time-consuming roads. The number of iterations is 10. Design parameters are set to $\alpha = 1$, $\beta = 0$, and $\rho = 0.9$. As a simulation result, computation time swarm intelligence (Ant System) takes less than 2 seconds using a Pentium computer as follows. Fig. 5 shows each point number in TCM.

times visit city =
9 9 6 6 5 7 7 5 6 7 longitudinal cars do not exist.
sum = 67
Path matrix =
A 1 6 7 12 13 18 19 20 25
B 21 16 11 6 7 2 3 4 5
C 11 12 7 2 3 4 0 0 0
D 8 13 12 11 16 17 0 0 0 turn round to avoid the same route 18 $\rightarrow$ 17 with car H
E 3 4 5 10 15 0 0 0 0
F 24 23 22 21 16 11 6 0 0
G 15 14 9 4 3 2 7 0 0
H 20 19 18 17 16 0 0 0 0
I 12 11 16 17 18 19 0 0 0 turn round to avoid the same route 18 $\rightarrow$ 19 with car J
J 22 17 18 19 14 9 10 0 0

The above result is depicted Fig. 6. Each vehicles takes from its starting point to its destination as follows:

A: 106 m, B: 101 m, C: 63 m, D: 58 m, E: 46 m, F: 70 m, G: 70 m, H: 50 m, I: 58 m, J: 74m .

Total travel time that all vehicles take is 696 m , so to speak, 3 hour 36 minute.

In the proposed scheme, a set of vehicles, called ants, search in parallel for fast convergence to the MRP and TCM. Each vehicle prefers to move to roads which are connected by fast distance with high amount of pheromone. 10 ants are used in AS, since the number of vehicles in TCM is 10. Once all vehicles have arrived their destinations, a pheromone updating rule is applied. And then the vehicle deposits an amount of pheromone on roads which be-

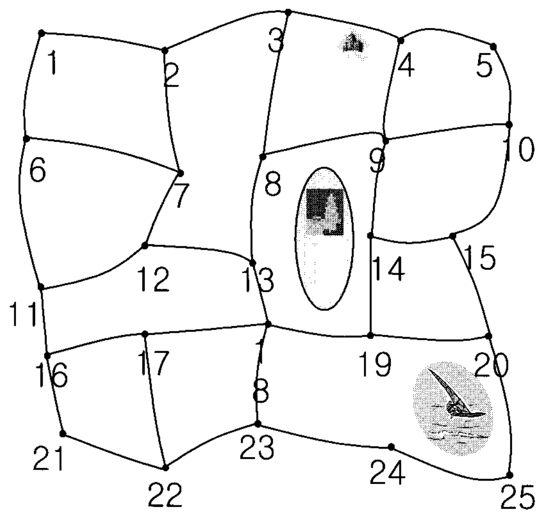


Figure 5: Point number in Tokyo city model

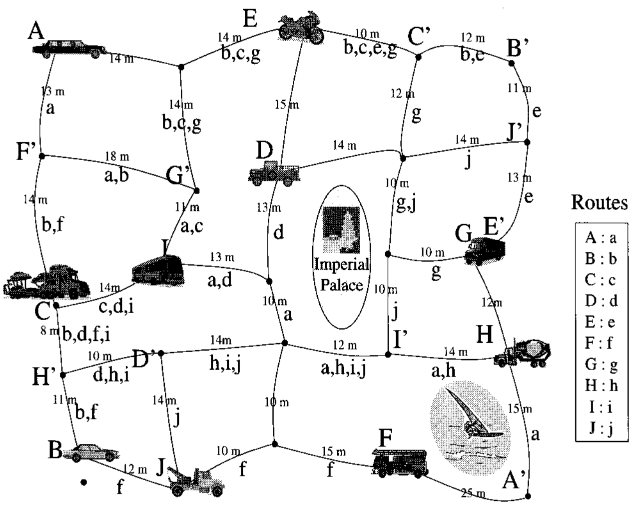


Figure 6: Solution of Tokyo city model

longs to its path to destination in proportion to how fast its path was. In other words, roads which belong to many fast paths in total travel are the roads which receive the greater amount of pheromone. In TCM, traffic is considered as a penalty value, and the road in a traffic condition receives less pheromone. The process is then iterated. After all, by the help of AS algorithm, each vehicle chooses the path taking the shortest time in the total travel. The result shows that the AS can effectively find a set of routes minimizing the total travel time even though the TCM has some constraints.

5. A Study of Some Characteristics of the TCM

In the Section, the reason why swarm intelligence such as AS is appropriate to TCM and the convergence to the fastest path in AS are analyzed.

• The reason that TCM should be considered for MRP follows as:

- TCM can be extended and applicable to actual situation since many vehicles and roads can be added. As for the problems, optimal routing selection should meet the requirements depending on the number of vehicles and road conditions.

• The reason that swarm intelligence should be considered for TCM follows as:

- For TCM, general optimization techniques are not available due to requiring too many memory space and too long computation time. However, swarm intelligence gives good solutions to TCM with small memory space and fast time.

• Even TSP has more cities, TCM is not transformed to TSP by following reasons.

- TCM is a central problem in distribution management like vehicle routing problem (VRP). But TSP is a single management problem. VRP is a more difficult problem extended from TSP.

- The goal in TCM is to find a set of routes that minimizes the total travel time such that each vehicle can reach its destination as soon as possible.

- Each vehicle has different locations between start and destination, unlike VRP and TSP.

- The constraint in TCM depends on how many vehicles pass on the same road, unlike VRP and TSP.

- TCM is more similar as VRP rather than TSP in terms of a central problem in distribution management. But, TCM is a different model from VRP in terms of the constraint and having different destination from start.

- Needless to say, such problems (TSP, VRP, TCM) are $\mathcal{NP}$-hard, that is, it is strongly believed that they cannot be

	0.878 approximation method	Swarm Intelligence (Ant System)
Computation time	42 [s] (MRP) Not applicable (TCP)	Less than 1 [s] (MRP) Less than 2 [s] (TCP)
Complicated problem (e.g. TCM model)	Not applicable	Applicable
Support diverse solutions	No (Just one solution!)	Yes
Guarantee to convergence	Yes	No

Table 1: Comparison between 0.878 approximation and swarm intelligence (AS)

solved to optimality within polynomially bounded computation time.

Previous studies have shown the convergence to a value and convergence in solution of Ant Colony Optimization (ACO) [15], [16]. AS is one of particular ACO algorithms. It was shown through theorems that AS is guaranteed to find an optimal solution with a probability that can be made arbitrarily close to 1 if given enough time.

• Convergence to the fastest path in Ant System, that is, optimal route selection in for multiple vehicles follows as:

- By pheromone evaporation, AS does not make loops that a vehicle may turn around same path.
- During AS runs, the probability of choosing the fastest path becomes close to 1, while for all the others, it becomes arbitrarily close to 0.
- Even if AS does not select the fastest path (i.e. suboptimal path), the path is from a start point to corresponding destination, not a different destination. Since the path is identified and decided as a total path by AS in optimal route selection for multiple vehicles, which means that AS guarantees the vehicle reaches the destination for sure.
- It is believed that AS is guaranteed to find an optimal solution, the shortest total travel time, in TCM through previous study results about convergence in AS.

Tab. 1 shows the comparative results between 0.878 approximation method [17], [18] and swarm intelligence (AS) applied to MRP and TCM which are proposed by the author.

In the paper, the author proposed new two $\mathcal{NP}$-hard optimization problems, MRP and TCM that are more distributive and restrictive than TSP. TCM is a more complex model than MRP since it has different time from point to point and includes a constraint called traffic. The author applied AS algorithm to the two problems and found an optimal route for multiple vehicles for each. Such NP-hard optimization problems have not been proposed and dealt with in other previous studies.

AS used in the paper is a comparatively simple approach method rather than other ant colony optimization techniques [5]-[7],[10],[15],[16]. However, in the paper, our focus is to find an optimal route in the proposed MRP and TCM for multi-vehicle routing using AS that is recently contrived for parallel stochastic optimization problems. The use of optional local search phases in variant ant algorithms is not considered to the MRP and TCM in the paper since AS provides enough good result for the proposed MRP and TCP. However, it may be considered for more complex TCP model in future.

6. Conclusions

This paper proposes the MRP and TCM which are more complex and distributive model of TSP. In addition, a design framework based on AS for MRP and TCM is proposed. The main objectives are to find an optimal route in TCM problem for multiple vehicles using AS, swarm intelligence that is recently contrived for parallel stochastic optimization problems. As for the problems, the design method are proposed. As a result of simulation, AS shows much fast time rather than 0.878 approximation method. AS is applied to a complicated and distributed problem with constraints such as the MRP and TCM. As an applicable example, for the best transportation flow, a navigation base center can send a optimal route information to each vehicle, based on the proposed scheme. As well, the proposed method is easily applicable to NP-hard optimization problems that arises in other applications like TSP. Further research is on its application to TCP in the presence of more constraints.

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