

Gradient-based Fast Connectivity Weighted Hough Transform

金 廷 泰[†] · 申 智 令^{*}
(Jeongtae Kim · Jiyoung Shin)

Abstract - The connectivity weighted Hough transform is a useful method for detecting well-connected short lines without generating false lines yet requires extensive computation. This letter describes a method that reduces the computation of the connectivity weighted Hough transform by removing unnecessary weight calculations using the gradient angles of feature points. In simulations with synthetic images and experiments with liquid crystal display panel images, the proposed method showed significantly improved speed without compromising detectability.

Key Words : Hough transform, Connectivity, Gradient, LCD panel inspection, Line defects.

1. Introduction

$$\rho = x \cos \theta + y \sin \theta, \quad (1)$$

For detecting lines composed of feature points, the Hough transform (HT) and its variants have been applied widely due to their robustness to noise and gaps in the lines. However, many HT methods that do not consider the connectivity of a line suffer from missing well-connected short lines as well as detecting false lines composed of unrelated feature points [1]. To alleviate the problems, methods considering the connectivity of a line such as connectivity Hough transform, connectivity oriented fast Hough transform have been investigated [1]-[3]. Among those, the connectivity weighted Hough transform (CWHT) is a promising approach since it is robust to noise and effective in detecting short line segments. The disadvantage of the CWHT is extensive computation. In experiments using some test images [2], although the CWHT showed better performance than the standard Hough transform (SHT) in terms of detectability, it was approximately three times slower than the SHT. The lack of fast connectivity weighted Hough transform motivated us to investigate a new method for detecting line defects on a liquid crystal display (LCD) panel image. Fast and reliable detection of such defects is important for quality control, and has been investigated continuously.

Many HT methods are based on the following polar equation of a line:

where (x, y) denotes the location of each feature point on the line, ρ is the distance of the line from the origin and θ is the angle between the normal of the line and the x axis. For each feature point, the SHT generates N uniformly sampled θ_n , and for each θ_n , it computes the corresponding ρ . After quantizing the computed ρ to ρ_m , $m = 1, \dots, M$, the SHT increases the value of the Hough cell at (ρ_m, θ_n) by one. The $M \times N$ Hough transform $H(\rho, \theta)$ is acquired by repeating the preceding procedure for every feature point. Note that the connectivity of a line is not considered since each feature point on the line accumulates the same amount to the corresponding Hough cell regardless of how many neighboring feature points exist on the line. Unlike the SHT, the CWHT determines the distances and angles of possible lines using pairs of feature points [4]. For each pair, the CWHT computes and quantizes the distance and angle of the line that connects two feature points in the pair, and accumulates the following weight w to the corresponding Hough cell [2]:

$$w = \begin{cases} m \log\left(\frac{m}{lp_o}\right) + (l-m) \log\left(\frac{l-m}{l(1-p_o)}\right), & \frac{m}{l} > p_o \\ 0, & \frac{m}{l} \leq p_o \end{cases} \quad (2)$$

where l and m denote the number of pixels and the number of feature points on the line segment that connects the two points in the pair, respectively, and p_o is the ratio of the number of total feature points to the number of total pixels in the image. $H(\rho, \theta)$ is determined

[†] 교신저자, 正會員 : 梨花女子大學 電子情報通信學科
助教授 · 工博

E-mail : jtkim@ewha.ac.kr

^{*} 正會員 : (주)포스레이타 研究員

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by repeating the preceding weight accumulation for every pair. The weight w is designed to be larger if there are more feature points on the line segment, thereby improving the ability of detecting well-connected short lines. However, the method requires extensive computation since it examines every pixel on the line segment to decide whether it is a feature point or not for calculating the weight.

2. Proposed Method

We propose gradient-based connectivity weighted Hough transform (GCWHT) that incorporates the gradient angle information of each feature point to reduce the computation of the CWHT. The information is usually available for detecting lines composed of edge pixels since those are generally determined by a gradient-based edge detection method. The gradient angle information had been used previously in other HT methods [5]. The GCWHT is based on the intuition that time consuming weight calculations are not required for edge pairs composed of two edge pixels that are not likely to exist on the same line. For example, if the gradient angles of two edge pixels are considerably different, those are not likely to exist on the same line since the gradient angles of edge pixels on the same line are identical to the normal direction of the line in an ideal case. Even if the gradient angles are similar, the two pixels may exist on two different parallel lines, respectively. In such a case, the distances of the two parallel lines from the origin would be different. Based on the observation, we design the procedure of the GCWHT as follows. Within the same procedure as the CWHT, before computing the weight for an edge pair composed of two edge points p_i , p_j whose gradient angles are g_i , g_j , the GCWHT computes the absolute difference of g_i and g_j . If the difference is larger than some threshold T_g , the method proceeds to another pair without accumulating weight. Otherwise, the method computes the distances of the lines from the origin whose normal directions are g_i , g_j and on which the points p_i , p_j lie, respectively. If the absolute difference of the distances is larger than some threshold T_ρ , the method proceeds to another pair without accumulating weight. Only if the absolute differences of the gradient angles and the distances are less than the respective thresholds, the method computes weight using (2) and accumulates it to the corresponding Hough cell. The GCWHT algorithm is summarized as follows.

Input: Edge locations $p_k = (x_k, y_k)$ and corresponding gradient angles $g_k, k = 1, \dots, K$.

Step1: Initialize $H(\rho_m, \theta_n) = 0, m = 1, \dots, M, n = 1, \dots, N$.

Step2: Select an edge pair (p_i, p_j) that was not previously selected. If $|g_i - g_j| > T_g$, go to Step 4. Otherwise, compute the distances $\hat{\rho}_i = x_i \cos g_i + y_i \sin g_i$ and $\hat{\rho}_j = x_j \cos g_j + y_j \sin g_j$.

Step 3: If $|\hat{\rho}_i - \hat{\rho}_j| > T_\rho$, go to Step 4. Otherwise, solve for (ρ, θ) using p_i , p_j , and quantize it to (ρ_m, θ_n) . Compute w using (2) for the line segment connecting p_i , p_j , and accumulates it to $H(\rho_m, \theta_n)$ cell.

Step 4: Repeat Step 2 to Step 3 until every edge pair is done.

3. Experimental Results

We conducted experiments using a 144×144 pixel rectangular image with additive Gaussian noise shown in Fig. 1(a). Fig. 1(b) shows the edge image of Fig. 1(a) acquired using the Laplacian of Gaussian edge detector.

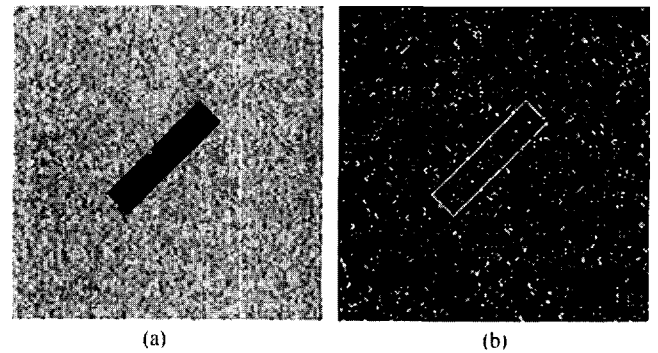


Fig. 1 Rectangular images: (a) grayscale (b) edge.

We applied the GCWHT, the CWHT and the SHT to the edge image, and detected four lines by searching the four highest peaks in the Hough space. The quantization step sizes for ρ and θ were 1 (pixel) and 0.5 (degree), respectively, and the thresholds for the GCWHT were T_g (degree) and T_ρ (pixel). We repeated the preceding detection for 100 times with different noise realizations and measured the average computation time (for a 3GHz Pentium PC). We also evaluated the average detectability by the percentage of successes in detecting the two short lines. We repeated the preceding experiment while increasing the lengths of the short lines from 15 pixels to 40 pixels gradually. Table 1 summarizes the results. As shown in the table, not only the GCWHT was significantly faster than the CWHT but also it was faster than the SHT. Both the GCWHT and the CWHT showed better detectability than the SHT, which is consistent with the results of the previous investigation [2]. An interesting observation is that the detectability of the

GCWHT was better than the CWHT. We think that this difference is due to the fact that the GCWHT accumulates non-zero weight only for edge pairs composed of two edge pixels that are likely to exist on the same line while the CWHT accumulates non-zero weight for every edge pair as long as there are some edge pixels on the corresponding line segment.

Table 1 Computation time and line detectability for the image shown in Fig. 1(b) (DA stands for detectability)

Line length (pixel)	SHT		CWHT		GCWHT		Edge number (pixel)
	DA (%)	time (msec)	DA (%)	time (msec)	DA (%)	time (msec)	
15	0	120	5.5	1675	98.5	20	894
20	0	122	49	1713	100	22	906
25	0	125	85	1739	100	23	916
28	3.5	125	98	1800	100	25	932
30	42	126	99	1824	100	25	938
35	87.5	127	100	1852	100	26	947
40	96.5	129	100	1954	100	27	972

We conducted another experiment using a 215×129 pixel LCD panel image (provided by LG Electronics, Korea). Fig. 2(a) and Fig. 2(b) show the gray scale and the edge image of the LCD panel, respectively. The number of edge pixels in Fig. 2(b) was 2274.

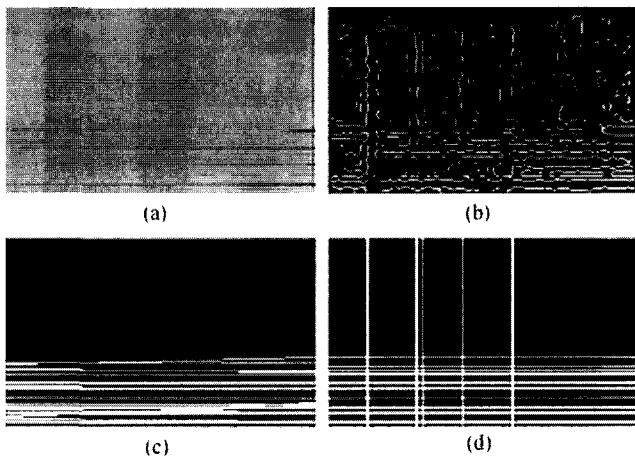


Fig. 2 LCD panel images and lines determined by HT methods: (a) gray scale image (b) edge image (c) lines determined by the SHT (d) lines determined by the GCWHT

As shown in the images, the LCD has horizontal and vertical line defects. To detect such defects, we applied the GCWHT, the CWHT and the SHT to the edge image, and determined the 23 largest local maxima in the Hough space. Fig. 2(c) and Fig. 2(d) show the detected lines using the SHT and the GCWHT, respectively. The SHT was not able to detect the vertical lines while detecting false lines that connect edge pixels from

neighboring horizontal lines. As shown in Fig. 2(d), the GCWHT was able to detect the horizontal and vertical lines without generating false lines. In addition, the GCWHT was more than three times faster than the SHT. Although the CWHT was also able to generate the similar result as the GCWHT, it was approximately 90 times slower than the GCWHT as summarized in Table 2.

Table 2 Computation time for the LCD image

SHT	CWHT	GCWHT
0.303 (sec)	9.456 (sec)	0.091 (sec)

4. Discussions and Conclusions

For an image that has k edge pixels, the SHT has the computational complexity of $O(k^2)$ with a heavy overhead since each edge point generates a curve in the Hough space. The CWHT has the computational complexity of $O(k^2)$ with a heavy overhead since even though each pair of two edge points generates weight for only one Hough cell, computationally expensive weight calculation is performed for every pair. On the other hand, although the GCWHT has the same computational complexity of $O(k^2)$ as the CWHT, its overhead can be much less since weight calculations are performed only for edge pairs composed of two edge points that are likely to exist on the same line. By virtue of its high speed and good detectability, we believe that the GCWHT should be useful for applications that require fast detection of short line segments without generating false lines, such as inspection of LCD panels.

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