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Joint Test for Seasonal Cointegrating Ranks†

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Abstract

In this paper we consider a joint test for seasonal cointegrating(CI) ranks that enables us to simultaneously model cointegrated structures across seasonal unit roots in seasonal cointegration. A CI rank test for a single seasonal unit root is constructed and extended to a joint test for multiple seasonal unit roots. Their asymptotic distributions and selected critical values for the joint test are obtained. Through a small Monte Carlo simulation study, we evaluate performances of the tests.

Keywords: Seasonal cointegration; seasonal unit roots; Gaussian reduced rank estimation; reduced rank regression.

1. Introduction

In empirical studies on nonstationary time series, the identification of the cointegrating(CI) rank is often a subject of major interest because it is performed at an earlier stage of the analysis and affects the model set-up and inference procedures at other stages. The commonly-used CI rank test in seasonal cointegration is the likelihood ratio(LR) type of tests proposed by Lee (1992), Johansen and Schaumburg (1999) (henceforth, JS), Cubadda (2001), Cubadda and Omtzigt (2005) and Seong *et al.* (2006), among others.

In all the previous literature, seasonal CI rank tests are performed independently by focusing on one unit root at a time, as in non-seasonal cointegration, even though more than one unit root(seasonal unit root) is involved in seasonal cointegration.

In this paper, we consider a joint test to simultaneously impose reduced-rank restrictions across several seasonal unit roots. For the purpose, we use Gaussian reduced rank(GRR) estimation of Ahn and Reinsel (1994) that simultaneously considers all seasonal unit roots in the estimation. If we use a partial regression approach, often used by the previous literature such as JS, it is impossible to construct the joint test because it ignores the constraints of reduced ranks(*i.e.*, cointegrated structures) of the other unit roots except one unit root focused by the partial regression.

The paper is organized as follows. In Section 2, the model is presented and the GRR estimation is described. In Section 3, we first construct a CI rank test for a single seasonal unit root and extend it to a joint test for multiple seasonal unit roots by using

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the GRR estimation. Also, their asymptotic distributions and selected critical values for the joint test are obtained. In Section 4, Monte Carlo simulations are conducted to evaluate performances of the tests. Conclusions are drawn in Section 5.

2. Model and Gaussian Reduced Rank Estimation

We consider a vector autoregressive(VAR) model for an m-dimensional process $\{X_t\}$ generated by

$$\Pi(L)X_t = \left(I_m - \sum_{j=1}^{p} \Pi_j L^j\right) X_t = \Phi D_t + \varepsilon_t, \tag{2.1}$$

where ε_t are *i.i.d.* $N_m(0, \Omega)$, D_t is a deterministic term that may contain a constant, a linear term, or seasonal dummies and I_m denotes an $m \times m$ identity matrix. We assume that the initial values $X_0, \ldots, X_{-p+1}$ are fixed and the roots of the determinant $|\Pi(z)| = 0$ are on or outside the unit circle.

For brevity, it is assumed that X_t are observed on a quarterly basis with no deterministic terms, $\Phi D_t = 0$. Models with the other seasonal periods, *e.g.*, monthly, can be easily implemented as in Ahn *et al.* (2004) and models with deterministic terms, $\Phi D_t \neq 0$, as in Cubadda (2001) and JS. Then, as in Ahn and Reinsel (1994), if the series are cointegrated of order $(1, 1)$ at seasonal unit roots $z = 1$, -1, i and $-i$ (*i.e.*, frequencies 0, π, $\pi/2$ and $3\pi/2$, respectively), model (2.1) can be rewritten in the following error correction model (ECM):

$$\begin{aligned}\Pi^*(L)Z_t = {} & A_1B_1U_{t-1} + A_2B_2V_{t-1} + (A_3B_4 + A_4B_3)W_{t-1} \\ & + (A_4B_4 - A_3B_3)W_{t-2} + \varepsilon_t,\end{aligned}$$

where $Z_t = (1 - L^4)X_t$, $U_t = (1 + L)(1 + L^2)X_t$, $V_t = (1 - L)(1 + L^2)X_t$, $W_t = (1 - L^2)X_t$, $\Pi^*(L)$ is a matrix polynomial of order $p - 4$ and A_j and B_j are $m \times r_j$ and $r_j \times m$ matrices, respectively, with rank equal to r_j for $j = 1, \ldots, 4$ and $r_3 = r_4$. For a unique parameterization, we need to normalize the B_j's such that $B_1 = [I_{r_1}, B_{10}]$, $B_2 = [I_{r_2}, B_{20}]$, $B_3 = [I_{r_3}, B_{30}]$ and $B_4 = [O_{r_3}, B_{40}]$, where O_{r_j} is an $r_j \times r_j$ matrix of zeros and B_{j0}'s are $r_j \times (m - r_j)$ matrices of unknown parameters. Note that r_1, r_2 and $r_3(r_4)$ denote the CI ranks at the unit roots 1, -1 and $i(-i)$, respectively and B_1U_t, B_2V_t, $(B_3 + B_4L)W_t$ and $(B_4 - B_3L)W_t$ are stationary processes, *i.e.*, CI relationships.

For parameterization, we define $\alpha = \text{vec}\{(A_1, \ldots, A_4, \Pi_1^*, \ldots, \Pi_{p-4}^*)'\}$, $\beta_j = \text{vec}(B_{j0}')$ for $j = 1, \ldots, 4$ and $\gamma = (\beta_1', \beta_2', \beta_3', \beta_4', \alpha')'$, where $\text{vec}(\cdot)$ vectorizes a matrix columnwise from left to right. Then, the GRR estimate of γ, based on T observations $X_1, \ldots, X_T$ that maximizes the log likelihood function, $l(\gamma, \Omega) = -(T/2)\log|\Omega| - (1/2)\sum_{t=1}^{T} \varepsilon_t'\Omega^{-1}\varepsilon_t$, is obtained by the iterative Newton-Raphson method,

$$\hat{\gamma}^{(k+1)} = \hat{\gamma}^{(k)} + \left(\sum_{t=1}^{T} P_{t-1}\hat{\Omega}^{-1}P_{t-1}'\right)^{-1} \left(\sum_{t=1}^{T} P_{t-1}\hat{\Omega}^{-1}\varepsilon_t\right)\Bigg|_{\hat{\gamma}^{(k)}},$$

where $\hat{\gamma}^{(k)}$ is an estimate at the previous iteration,

$$P_{t-1} = \big[A_1 \otimes U'_{2t-1},\ A_2 \otimes V'_{2t-1},\ A_4 \otimes W'_{2t-1} - A_3 \otimes W'_{2t-2},$$
$$A_3 \otimes W'_{2t-1} + A_4 \otimes W'_{2t-2},\ I_m \otimes \tilde{P}'_{t-1}\big]',$$
$$\tilde{P}_{t-1} = \big[U'_{t-1}B'_1,\ V'_{t-1}B'_2,\ (B_4W_{t-1} - B_3W_{t-2})',\ (B_3W_{t-1} + B_4W_{t-2})',$$
$$Z'_{t-1}, \ldots, Z'_{t-p+4}\big]',$$

U_{1t}, V_{1t} and W_{1t} are the first r_1, r_2 and r_3 components of U_t, V_t and W_t, respectively, U_{2t}, V_{2t} and W_{2t} are the last $m - r_1$, $m - r_2$ and $m - r_3$ components of U_t, V_t and W_t, respectively, $\hat{\Omega}$ is the residual covariance matrix and $\otimes$ denotes the Kronecker product.

The GRR estimation has a special characteristic that it simultaneously imposes rank conditions at all existing seasonal unit roots. Then, it enables us to jointly model cointegrating restrictions across all the unit roots. As an example to utilize the characteristic, Seong *et al.* (2007) considered tests for hypotheses with generalized linear restrictions on CI vectors at possibly different unit roots. In the next section, their work for the CI vectors is extended to that for the CI ranks.

3. Joint Test for Seasonal Cointegrating Ranks

We first construct a CI rank test for a single seasonal unit root, which is based on the GRR estimation. Consider the hypothesis

$$H_0 : r_j = r_j^0 \quad vs. \quad H_1 : r_j > r_j^0, \qquad \text{for some } r_j^0 < m \text{ and } j = 1,\ 2,\ 3, \tag{3.1}$$

where the subscript j means the frequency corresponding to a given single seasonal unit root. Note that the CI rank at one complex unit root equals the rank at its conjugate. For testing the hypothesis, the LR test statistic can be obtained by

$$\mathrm{LR}(r_j^0) = -T \log \left(\frac{\max_{H_1} |\tilde{\Omega}|}{\max_{H_0} |\tilde{\Omega}|} \right), \tag{3.2}$$

where $\tilde{\Omega}$ denotes the residual covariance matrix from the GRR estimation. We note that $\max_{H_1} |\tilde{\Omega}|$ refers to the value of $|\tilde{\Omega}|$ in a model with full ranks at all existing unit roots, while $\max_{H_0} |\tilde{\Omega}|$ implies the value of $|\tilde{\Omega}|$ in a model with reduced rank only at a given unit root z_j, *i.e.*, $r_j = r_j^0$. Therefore, the asymptotic distribution of the LR test by the GRR estimation can be easily obtained by Lee (1992) and JS.

Theorem 3.1 In model (2.2), the LR test statistic for hypothesis (3.1) is asymptotically distributed as

$$\mathrm{LR}(r_j^0) \xrightarrow{D} \mathrm{tr} \left\{ \int_0^1 (dB_C^j)(B_C^j)^* \left(\int_0^1 B_C^j (B_C^j)^* du \right)^{-1} \int_0^1 B_C^j (dB_C^j)^* \right\} \equiv \mathrm{TR}(r_j^0), \tag{3.3}$$

where $\xrightarrow{D}$ denotes the convergence in distribution, $\mathrm{tr}\{\cdot\}$ and $(\cdot)^*$ denote the trace and transpose of the argument matrix, respectively and $B_C^j(u) = B_R^j(u) + iB_I^j(u)$, $B_R^j(u)$

and $B_I^j(u)$ are independent standard Brownian motions of dimension $m - r_j^0$. When the given unit root corresponds to 0 or 1, $B_C^j(u)$ reduces to $B_R^j(u)$, *i.e.*, $B_I^j(u) = 0$.

As another approach for constructing the LR test, JS considered a switching algorithm based on the partial regression to avoid the complexity generated with the simultaneous estimation of the GRR estimation. See also the complex ECM by Cubadda (2001).

Now, we are interested in the following joint hypothesis

$$H_0 : r_1 \le r_1^0,\ r_2 \le r_2^0,\ r_3 \le r_3^0 \quad \textit{vs.} \quad H_1 : \text{ not } H_0, \qquad \text{for some } r_j^0 < m,\ (j = 1, 2, 3). \tag{3.4}$$

For testing the joint hypothesis, the LR test statistic can be obtained by

$$\mathrm{LR}(r_1^0, r_2^0, r_3^0) = -T \log\left(\frac{\max_{H_1} |\tilde{\Omega}|}{\max_{H_0} |\tilde{\Omega}|}\right). \tag{3.5}$$

Then, the test statistic can not be calculated by the partial regression approach but the GRR estimation because the hypothesis is related to reduced rank structure at all existing unit roots.

For finding the asymptotic distribution, we decompose the test statistic as

$$\begin{aligned}
\mathrm{LR}(r_1^0,\, r_2^0,\, r_3^0) &= -\,T \log\left(\frac{|\tilde{\Omega}(m,\, m,\, m)|}{|\tilde{\Omega}(r_1^0,\, r_2^0,\, r_3^0)|}\right) \\
&= -\,T \log\left(\frac{|\tilde{\Omega}(m,\, m,\, m)|}{|\tilde{\Omega}(r_1^0,\, m,\, m)|} \times \frac{|\tilde{\Omega}(r_1^0,\, m,\, m)|}{|\tilde{\Omega}(r_1^0,\, r_2^0,\, m)|} \times \frac{|\tilde{\Omega}(r_1^0,\, r_2^0,\, m)|}{|\tilde{\Omega}(r_1^0,\, r_2^0,\, r_3^0)|}\right) \\
&= -\,T \log\left(\frac{|\tilde{\Omega}(m,\, m,\, m)|}{|\tilde{\Omega}(r_1^0,\, m,\, m)|}\right) - T \log\left(\frac{|\tilde{\Omega}(r_1^0,\, m,\, m)|}{|\tilde{\Omega}(r_1^0,\, r_2^0,\, m)|}\right) \\
&\quad - T \log\left(\frac{|\tilde{\Omega}(r_1^0,\, r_2^0,\, m)|}{|\tilde{\Omega}(r_1^0,\, r_2^0,\, r_3^0)|}\right) \\
&\equiv \mathrm{LR}_1 + \mathrm{LR}_2 + \mathrm{LR}_3,
\end{aligned}$$

where $|\tilde{\Omega}(r_1,\, r_2,\, r_3)|$ denotes the value of $|\tilde{\Omega}|$ in a model with ranks r_1, r_2 and r_3 at unit roots 1, -1 and i, respectively. Then, based on the asymptotic uncorrelatedness property by corollary 7 in JS, we can show that

$$\mathrm{LR}_j - \mathrm{LR}(r_j^0) \xrightarrow{P} 0,$$

where $\xrightarrow{P}$ denotes the convergence in probability and $\mathrm{LR}(r_j^0)$ is defined in (3.2). Since the asymptotic distribution of LR_j is the same with that of $\mathrm{LR}(r_j^0)$, we obtain the asymptotic distribution of $\mathrm{LR}(r_1^0,\, r_2^0,\, r_3^0)$ by using the asymptotic uncorrelatedness among LR_j for $j = 1,\ 2,\ 3$.

Theorem 3.2 In model (2.2), the LR test statistic for hypothesis (3.4) is asymptotically distributed as

$$\mathrm{LR}(r_1^0,\, r_2^0,\, r_3^0) \xrightarrow{D} \mathrm{TR}(r_1^0) + \mathrm{TR}(r_2^0) + \mathrm{TR}(r_3^0). \tag{3.6}$$

Table 3.1: Selected asymptotic critical values for the joint LR test in identifying seasonal CI ranks when nonstationary quarterly time series is considered

$m-r_1$	$m-r_2$	$m-r_3$	5%	1%	$m-r_1$	$m-r_2$	$m-r_3$	5%	1%
1	1	1	10.04	13.82	3	1	1	28.36	34.16
1	1	2	16.98	21.45	3	1	2	34.44	40.64
1	1	3	28.34	33.65	3	1	3	45.10	51.91
1	1	4	43.93	50.71	3	1	4	59.89	67.73
1	2	1	23.41	28.59	3	2	1	40.45	46.67
1	2	2	29.68	35.41	3	2	2	46.37	53.36
1	2	3	40.47	46.77	3	2	3	56.57	64.04
1	2	4	55.39	62.70	3	2	4	71.36	79.67
1	3	1	45.07	51.82	3	3	1	61.28	69.00
1	3	2	51.01	58.10	3	3	2	67.09	75.01
1	3	3	61.18	68.81	3	3	3	77.19	85.95
1	3	4	75.81	84.15	3	3	4	91.67	100.92
1	4	1	74.72	83.15	3	4	1	90.24	99.25
1	4	2	80.39	89.07	3	4	2	95.94	105.22
1	4	3	90.31	99.51	3	4	3	105.80	115.53
1	4	4	104.70	114.39	3	4	4	120.04	130.68
2	1	1	17.02	21.59	4	1	1	43.77	50.46
2	1	2	23.43	28.63	4	1	2	49.66	56.48
2	1	3	34.49	40.48	4	1	3	59.93	67.37
2	1	4	49.54	56.65	4	1	4	74.66	83.13
2	2	1	29.73	35.42	4	2	1	55.36	62.77
2	2	2	35.78	42.11	4	2	2	61.18	69.02
2	2	3	46.46	53.10	4	2	3	71.47	79.76
2	2	4	61.22	69.15	4	2	4	85.60	94.76
2	3	1	50.97	58.17	4	3	1	75.92	84.26
2	3	2	56.78	64.19	4	3	2	81.58	90.43
2	3	3	67.03	74.86	4	3	3	91.53	100.75
2	3	4	81.66	90.54	4	3	4	105.80	115.91
2	4	1	80.35	88.67	4	4	1	104.78	114.36
2	4	2	85.95	94.98	4	4	2	110.14	120.24
2	4	3	95.96	105.13	4	4	3	120.05	130.70
2	4	4	110.20	120.31	4	4	4	134.14	144.94

The asymptotic critical values of (3.3) can be obtained in JS but those of (3.6) need to be calculated. This can be done by the formulae in Seong *et al.* (2006) where explicitly describes the method to approximate $\mathrm{TR}(r_j^0)$ by using a finite-step random walk. Table 3.1 reports the selected values for a quarterly cointegrated process.

Since we simultaneously consider all existing unit roots in the joint CI rank test, the identification of the CI ranks may be complicated. The suggested inferential procedure goes as follows. Determine a specific unit root and identify the corresponding CI rank by using the joint CI rank test with reduced ranks at the other unit roots fixed. For example, if the specific unit root is $z = 1$, use the test statistic $\mathrm{LR}(r_1^0, \tilde{r}_2^0, \tilde{r}_3^0)$ for some $r_1^0 < m$ and fixed values $\tilde{r}_2^0$ and $\tilde{r}_3^0$. Then, keeping the identified CI rank as fixed, identify another CI rank. For example, if we identified $r_1 = r_1^*$ at $z = 1$ and determined the next unit root as $z = -1$, use $\mathrm{LR}(r_1^*, r_2^0, \tilde{r}_3^0)$ for some $r_2^0 < m$ and fixed $\tilde{r}_3^0$. Through these sequential identifications, we can jointly identify the CI ranks at all the existing unit roots.

4. Monte Carlo Experiment

In this section, we conduct Monte Carlo experiments to compare the relative performances of the LR tests by the GRR estimation of Ahn and Reinsel (1994)(denoted by AR-LR) with those by the switching algorithm of JS(denoted by JS-LR) in identifying seasonal CI ranks.

The data generating process considered is identical to the one used in Ahn and Reinsel (1994):

$$(1 - L^4)X_t = A_1 B_1 U_{t-1} + A_2 B_2 V_{t-1} + A_4 B_2 W_{t-1} - A_3 B_2 W_{t-2} + \varepsilon_t, \tag{4.1}$$

where $A_1 = (0.6,\, 0.6)'$, $A_2 = (-0.4,\, 0.6)'$, $A_3 = (0.6,\, -0.6)'$, $A_4 = (0.4,\, -0.8)'$, $B_1 = (1,\, -0.7)$, $B_2 = (1,\, 0.4)$ and

$$\mathrm{cov}(\varepsilon_t) = \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix}, \quad \text{for } \rho = -0.5,\, 0,\, 0.5 \ \text{ and } \ \sigma^2 = 0.5,\, 1,\, 2.$$

We consider the various forms of the covariance matrix of ε_t to check the sensitivity to the size of the variances and to the degree and sign of correlation between the innovations. Note that the roots of the characteristic equation $|\Pi(z)| = 0$ are ± 1, $\pm i$, $0.9715 \pm 0.7328\, i$ and $-1.3508 \pm 0.3406\, i$ and that X_t is seasonally cointegrated with CI rank of one at unit roots 1, -1 and i each.

The series lengths considered are $T = 50$, 100 and 200 and 10,000 replications of the sample series are generated for each value of T. We use initial values that are set to zero, but discard the first 50 observations in order to eliminate dependence on the starting conditions. We estimate a VAR(4) with no deterministic terms.

The following hypotheses are considered:

$$H_0 : r_1 \le 1 \quad vs. \quad H_1 : r_1 > 1, \quad (\text{for } z = 1), \tag{4.2}$$

$$H_0 : r_2 \le 1 \quad vs. \quad H_1 : r_2 > 1, \quad (\text{for } z = -1), \tag{4.3}$$

$$H_0 : r_3 \le 1 \quad vs. \quad H_1 : r_3 > 1, \quad (\text{for } z = i), \tag{4.4}$$

$$H_0 : r_1 \le 1,\, r_2 \le 1,\, r_3 \le 1 \quad vs. \quad H_1 : \text{ not } H_0. \tag{4.5}$$

Hypotheses (4.2)–(4.4) are for the CI tests at a single seasonal unit root, whereas hypothesis (4.5) for the joint CI test at multiple seasonal unit roots. In JS-LR, hypothesis (4.5) is jointly tested in such a manner that if all the null hypotheses in (4.2)–(4.4) are accepted, the joint null of (4.5) is accepted, whereas if even one of the null hypotheses is rejected, the joint null is rejected. The performances are based on the 5% asymptotic critical values.

Table 4.1 presents sizes(rejection rates) of JS-LR and AR-LR with respect to hypothesis (4.4). We observe that the two tests have similar performances, irrespective of the sample sizes and the various forms of the covariance matrix. Both tests have the desirable sizes that converge to the asymptotic significance level, as series length T increases. It is also observed that the correlation and the ratio between the variances of ε_t do not play an important role in determining relative performances between the tests. In the cases of hypotheses (4.2) and (4.3), the table is omitted because we observed a similar result to the case of hypothesis (4.4).

Table 4.1: Comparison of the rejection rates at the asymptotic 5% level tests with respect to hypothesis (4.4)

σ^2	ρ	$T = 50$		$T = 100$		$T = 200$	
		JS-LR	AR-LR	JS-LR	AR-LR	JS-LR	AR-LR
0.5	−0.5	0.0623	0. 0619	0.0512	0. 0516	0.0558	0.0558
	0.0	0.0653	0. 0646	0.0549	0. 0549	0.0531	0.0531
	0.5	0.0684	0. 0683	0.0525	0. 0536	0.0502	0.0503
1.0	−0.5	0.0613	0. 0599	0.0562	0. 0563	0.0518	0.0524
	0.0	0.0609	0. 0607	0.0535	0. 0541	0.0557	0.0558
	0.5	0.0675	0. 0684	0.0552	0. 0547	0.0500	0.0505
2.0	−0.5	0.0641	0. 0631	0.0537	0. 0541	0.0527	0.0530
	0.0	0.0679	0. 0678	0.0566	0. 0584	0.0530	0.0532
	0.5	0.0663	0. 0663	0.0585	0. 0574	0.0573	0.0574

Table 4.2: Comparison of the rejection rates at the asymptotic 5% level tests with respect to hypothesis (4.5)

σ^2	ρ	$T = 50$		$T = 100$		$T = 200$	
		JS-LR	AR-LR	JS-LR	AR-LR	JS-LR	AR-LR
0.5	−0.5	0.1846	0. 0744	0.1614	0. 0637	0.1552	0.0574
	0.0	0.1882	0. 0720	0.1615	0. 0644	0.1594	0.0579
	0.5	0.1937	0. 0828	0.1593	0. 0614	0.1555	0.0561
1.0	−0.5	0.1845	0. 0759	0.1615	0. 0619	0.1545	0.0578
	0.0	0.1854	0. 0720	0.1667	0. 0631	0.1606	0.0604
	0.5	0.1880	0. 0798	0.1606	0. 0646	0.1554	0.0606
2.0	−0.5	0.1900	0. 0784	0.1661	0. 0615	0.1574	0.0597
	0.0	0.1891	0. 0777	0.1671	0. 0624	0.1501	0.0546
	0.5	0.1871	0. 0788	0.1698	0. 0629	0.1580	0.0580

Table 4.2 presents sizes of the two tests with respect to hypothesis (4.5). For the AR-LR, we employed asymptotic critical values from Table 3.1. The sizes of the JS-LR test for hypothesis (4.5) are highly distorted above 15%, irrespective of the series length. However, the sizes of the AR-LR test properly converge to the significance level of 5% although they are distorted between about 6% and 8%, as the series lengths are $T = 50$ and 100. Similarly to Table 4.1, the various forms of the correlation matrix do not make a significant change on the relative performances of the tests.

Through the results of these small Monte Carlo studies, we observe that without the AR-LR test(*i.e.*, the joint test for seasonal CI ranks by using the GRR estimation), the joint(or simultaneous) test for seasonal CI ranks is substantially inflated because the commonly-used CI rank test by JS focuses only a single seasonal unit root at one time.

5. Conclusion

In this paper, we construct a joint test to simultaneously identify seasonal CI ranks, which is often a subject of major interest in the analysis of seasonal cointegration. The Monte Carlo simulation reveals that without the joint test for seasonal CI ranks by using the GRR estimation, the joint(or simultaneous) test for seasonal CI ranks is substantially inflated.

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