A BERRY-ESSEEN TYPE BOUND OF REGRESSION ESTIMATOR BASED ON LINEAR PROCESS ERRORS

HAN-YING LIANG AND YU-YU LI

ABSTRACT. Consider the nonparametric regression model $Y_{ni} = g(x_{ni}) + \epsilon_{ni}$ $(1 \leq i \leq n)$, where $g(\cdot)$ is an unknown regression function, x_{ni} are known fixed design points, and the correlated errors $\{\epsilon_{ni}, 1 \leq i \leq n\}$ have the same distribution as $\{V_i, 1 \leq i \leq n\}$, here $V_t = \sum_{j=-\infty}^{\infty} \psi_j e_{t-j}$ with $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ and $\{e_t\}$ are negatively associated random variables. Under appropriate conditions, we derive a Berry-Esseen type bound for the estimator of $g(\cdot)$. As corollary, by choice of the weights, the Berry-Esseen type bound can attain $O(n^{-1/4}(\log n)^{3/4})$.

1. Introduction

Consider the nonparametric regression model

$$(1.1) \qquad Y_{ni} = g(x_{ni}) + \epsilon_{ni}, \qquad i = 1, \ldots, n,$$

where g is an unknown regression function defined on A, x_{ni} are known fixed design points, and ϵ_{ni} are random errors. As an estimate of g, we consider the following weighted regression estimator:

$$(1.2) \qquad g_n(x) = \sum_{i=1}^{n} w_{ni} Y_{ni}, \ x \in A,$$

where $w_{ni} = w_{ni}(x)$ are weight functions.

The above estimator was first proposed by Georgiev [8] and subsequently have been studied by many authors. For instance, when ϵ_{ni} are assumed to be independent, consistency and asymptotic normality have been studied by Georgiev and Greblicki [10], Georgiev [9] and Müller [17] among others. Results for the case when ϵ_{ni} are dependent have also been studied by various authors. Fan [7] extended the work of Georgiev [9] and Müller [17] in the estimation of the regression model to the case where $\{\epsilon_{ni}\}$ form an L^q-mixingale sequence for some $1 \leq q \leq 2$. Roussas [19] discussed strong consistency and quadratic mean consistency for $g_n(x)$ under mixing conditions. Roussas et al. [22] established

asymptotic normality of $g_n(x)$ assuming that the errors are from a strictly stationary stochastic process and satisfy the strong mixing condition. Tran *et al* [26] discussed again asymptotic normality of $g_n(x)$ assuming that the errors form a linear time series, more precisely, a weakly stationary linear process based on a martingale difference sequence.

In this paper, we consider the model (1.1) and assume the following form for $\{\epsilon_{ni}\}$:

(A1) For each n, $\{\epsilon_{ni}, 1 \le i \le n\}$ have the same distribution as $V_1, \ldots, V_n$, where $V_t = \sum_{j=-\infty}^{\infty} \psi_j e_{t-j}$ and $\{e_t\}$ are identically distributed, negatively associated random variables with $Ee_i = 0$. Here $\{\psi_j\}$ is a sequence of real numbers with $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$.

Here, a finite family of random variables $\{X_i, 1 \le i \le n\}$ is said to be *negatively associated* (NA) if, for every pair of disjoint subsets A and B of $\{1, 2, \ldots, n\}$, we have

$$\text{Cov}(f_1(X_i, i \in A), f_2(X_j, j \in B)) \le 0,$$

whenever f_1 and f_2 are coordinatewise increasing provided the covariance exists. An infinite family of random variables is said to be NA, if every finite subfamily is NA.

The notion of negative association was first introduced by Alam and Saxena [1]. Joag-Dev and Proschan [11] showed that many well known multivariate distributions possess the NA property. Examples include (a) multinomial, (b) convolution of different multinomials, (c) multivariate hypergeometric, (d) Dirichlet, (e) Dirichlet compound multinomial, (f) negatively correlated normal distribution, (g) permutation distribution, (h) random sampling without replacement, and (i) joint distribution of ranks. The significance of NA, however, seems to come from the perception that it is an appropriate model when several species compete for the same limited resources. Because of its wide applications in multivariate statistical analysis and systems reliability, the notion of NA has recently received considerable attention. We refer to Joag-Dev and Proschan [11] for fundamental properties, Matula [16] for the three series theorem, Shao [23] for the Rosenthal-type inequality and the Kolmogorov exponential inequality, and Su et al. [25] for a moment inequality and weak convergence, Shao and Su [24] for the law of the iterated logarithm, Liang and Su [15] and Liang [12] as well as Baek et al. [2] for complete convergence, Liang and Baek [13] for some strong law, Roussas [20] studied the central limit theorems for weak stationary NA random fields. Asymptotic properties of estimates related to NA samples have also been studied by some authors. Cai and Roussas [3] gave uniform strong consistency, convergence rates and the asymptotic distribution of the Kaplan-Meier estimator for observations under randomly censored failure times. In Cai and Roussas [4], they established Berry-Esseen bounds for a smooth estimate of the distribution function; Roussas [21] derived the asymptotic normality of the kernel estimate of the probability density function; Chen et al. [6] studied strong consistency of estimator in heteroscedastic

regression model under NA samples; Yang [27] studied uniformly asymptotic normality of the regression weighted estimator for NA samples; Liang and Jing [14] discussed the asymptotic properties of $g_n(x)$ under NA setting. In particular, Liang and Jing [14] studied the asymptotic normality of $g_n(x)$ under assumption (A1) for $V_t = \sum_{j=0}^{\infty} \psi_j e_{t-j}$ setting.

In this paper, we further investigate model (1.1) and derive a Berry-Esseen type bound for the estimator $g_n(x)$ of $g(\cdot)$ under the errors $\{\epsilon_{ni}\}$ satisfy assumption (A1). By choice of the weights, the Berry-Esseen type bound can attain $O(n^{-1/4}(\log n)^{3/4})$.

The layout of the paper is as follows. The main result is presented in Section 2. In Section 3, some preliminary lemmas are given. The proofs of the main result and preliminary lemmas are provided in Sections 4 and 5, respectively.

In the sequel, let $C, c, c_1, \ldots$ denote generic finite positive constants, whose values are unimportant and may change from line to line. For random variables X and Y, $X \overset{\mathcal{D}}{=} Y$ means that the distribution of X is the same as that of Y. All limits are taken as the sample size n tends to ∞, unless specified otherwise.

2. The main result

In order to formulate the main result, we now give the following assumptions.

(A2) The weights satisfy $\sum_{i=1}^{n} |w_{ni}(x)| \leq C$, $w_n := \max_{1 \leq i \leq n} |w_{ni}(x)| = O(\sigma_n^2(x))$, where $\sigma_n^2(x) := \mathrm{Var}(g_n(x)) > 0$.

(A3) There exist positive integers $p := p(n)$ and $q := q(n)$ such that for sufficiently large n, $p + q \leq 3n$, $qp^{-1} \leq c < \infty$.

Let $\gamma_{in} \to 0$ $(i = 1, 2, 3)$ and $u(q) \to 0$, where $\gamma_{1n} = nqp^{-1}w_n$, $\gamma_{2n} = pw_n$, $\gamma_{3n} = n(\sum_{|j|>n} |\psi_j|)^2$ and $u(q) = \sup_{j \geq 1} \sum_{|i-j| \geq q} |\mathrm{Cov}(e_i, e_j)|$.

Our main result is as follows.

Theorem 2.1. *Suppose that (A1)-(A3) are satisfied. If $E|e_0|^{2+\delta} < \infty$ for some $\delta > 0$, then, for each $x \in A$, we have*

$$\sup_u |P(\sigma_n^{-1}(x)\{g_n(x) - Eg_n(x)\} \leq u) - \Phi(u)| = O(a_n),$$

where $a_n = (\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2})(\log n)^{1/2} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/3} + u^{1/3}(q) + n^{-1} + ((nqp^{-1})^{-\delta/2} + p^{-\delta/2})(\log n)^{(\delta-2)/2} \to 0$, $\Phi(u)$ represents the standard normal distribution function.

Corollary 2.1. *Suppose that (A1)-(A2) are satisfied. Let*

$$\sup_{n \geq 1} n^{7/8}(\log n)^{-9/8} \sum_{|j|>n} |\psi_j| < \infty$$

and $u(n) = O(n^{9/4}e^{-3n/4})$. If $E|e_0|^3 < \infty$ and $w_n = O(n^{-1})$, then for each $x \in A$, we have

$$\sup_u |P(\sigma_n^{-1}(x)\{g_n(x) - Eg_n(x)\} \leq u) - \Phi(u)| = O(n^{-1/4}(\log n)^{3/4}).$$

Corollary 2.2. *Suppose that* (A1) *is satisfied, and that spectral density function* $\tau(w)$ *of* V_i *is bounded away from zero and infinity, i.e.,* $0 < c_1 \leq \tau(w) < \infty$*, for* $w \in (\pi, \pi]$*. Let* $\sup_{n \geq 1} n^{7/8}(\log n)^{-9/8} \sum_{|j| > n} |\psi_j| < \infty$ *and* $u(n) = O(n^{9/4} e^{-3n/4})$*. If* $E|e_0|^3 < \infty$*,* $w_n = O(n^{-1})$ *and* $n \sum_{i=1}^{n} w_{ni}^2(x) \geq c_0 > 0$*, then, for each* $x \in A$*, we have*

$$\sup_u |P(\sigma_n^{-1}(x)\{g_n(x) - Eg_n(x)\} \leq u) - \Phi(u)| = O(n^{-1/4}(\log n)^{3/4}).$$

Remark 2.1. (a) In Theorem 2.1, we use $u(q) \to 0$, which is easily satisfied. For example:
(i) If $u(1) < \infty$ (cf. Roussas [21]), then $u(q) \to 0$ as $q \to \infty$.
(ii) For stationary NA sequence, Cai and Roussas [4] use the covariance coefficient: $u'(n) := \sum_{j=n}^{\infty} |\mathrm{cov}(e_1, e_{j+1})|^{1/3}$ and $u'(1) < \infty$. In this case, we have $|\mathrm{cov}(e_1, e_{j+1})| = o(j^{-3})$. Hence

$$u(q) := \sum_{j=q}^{\infty} |\mathrm{cov}(e_1, e_{j+1})| = O(\sum_{j=q}^{\infty} |\mathrm{cov}(e_1, e_{j+1})|^{1/3} j^{-2}) = O(q^{-2}).$$

(b) In Roussas et al. [22], the weights are required to satisfy the conditions: $\sum_{i=1}^{n} |w_{ni}(x)| \leq C$, $\max_{1 \leq i \leq n} |w_{ni}(x)| = O(\sum_{i=1}^{n} w_{ni}^2(x))$, $\sum_{i=1}^{n} w_{ni}^2(x) = O(\sigma_n^2(x))$. Clearly, these conditions imply Assumption (A2).

In addition, if we choose

$$w_{ni}(x) = \frac{x_{n,i} - x_{n,i-1}}{h_n} K(\frac{x - x_{n,i}}{h_n}),$$

where $\{h_n\}$ is a sequence of positive constants tending to 0 and $nh_n \to \infty$, and the design points satisfy $0 = x_{n,0} \leq x_{n,1} \leq \cdots \leq x_{n,n} = 1$, this weight also was used by Tran et al. [26]. Assume that
(iii) there exist positive constants c_1 and c_2 such that $c_1 n^{-1} \leq x_{n,i} - x_{n,i-1} \leq c_2 n^{-1}$ for $i = 1, 2, \ldots, n$,
(iv) $K(x)$ is nonnegative, bounded and continuous almost everywhere on R and has a majorant; that is, $K(x) \leq H(x)$ all $x \in R$, where H is symmetric, bounded, nonincreasing on $[0, \infty)$ with $\int H(y)dy < \infty$, where the integral is over R.
(v) $\{\epsilon_{ni}\}$ are stationary with $E\epsilon_{ni} = 0$ and its spectral density function $f(w)$ is bounded away from zero and infinity, i.e., $0 < c_3 \leq f(w) < \infty$ for $w \in (\pi, \pi]$.
Note that (v) implies that

$$\sigma_n^2(x) = E[\sum_{i=1}^{n} w_{ni}(x)\epsilon_{ni}]^2 = \int_{-\pi}^{\pi} f(w)|\sum_{k=1}^{n} w_{nk}(x)e^{-ikw}|^2 dw \geq c_4 \sum_{i=1}^{n} w_{ni}^2(x).$$

While, by using Lemma 3.1 in Tran et al. [26], (iii) and (iv) imply $\max_{1\leq i \leq n} |w_{ni}(x)| \leq C/nh_n$, $\sum_{i=1}^{n} w_{ni}(x) \leq C$ and

$$\frac{1}{h_n} \sum_{i=1}^{n} (x_{n,i} - x_{n,i-1}) K^2 (\frac{x - x_{n,i}}{h_n}) \to \int_{-\infty}^{\infty} K^2(u) du > 0.$$

Thus $\sigma_n^2(x) \geq c_4 \sum_{i=1}^{n} w_{ni}^2(x) \geq \frac{c_5}{nh_n}$. Hence $\max_{1\leq i \leq n} |w_{ni}(x)| = o(\sigma_n^2(x))$, which shows that (A2) is mild.

(c) In Roussas et al. [22] (cf. (2.21) there), they assume that

$$nqp^{-1} \sum_{i=1}^{n} w_{ni}^2(x) \to 0, \ p^2 \sum_{i=1}^{n} w_{ni}^2(x) \to 0.$$

Obviously, the limits above are stronger than $\gamma_{1n} \to 0$ and $\gamma_{2n} \to 0$ here, which were also used by Yang [27]. Under some regularity conditions, $\gamma_{3n} \to 0$ holds with the usual AR, MA and ARMA processes which are extensively used to model serially correlated data.

3. Some preliminary lemmas

We write $\sigma_n^2 = \sigma_n^2(x)$ and

$$S_n \ := \ \sigma_n^{-1}\{g_n(x) - Eg_n(x)\} \overset{\mathcal{D}}{=} \sigma_n^{-1} \sum_{i=1}^{n} w_{ni} V_i$$

$$= \ \sigma_n^{-1} \sum_{i=1}^{n} w_{ni} \sum_{j=-n}^{n} \psi_j e_{i-j} + \sigma_n^{-1} \sum_{i=1}^{n} w_{ni} \sum_{|j|>n} \psi_j e_{i-j}$$

$$(3.3) \qquad := \ S_{1n} + S_{2n}.$$

Note that

$$S_{1n} \ = \ \sigma_n^{-1} \sum_{i=1}^{n} w_{ni} \sum_{j=-n}^{n} \psi_j e_{i-j} = \sum_{l=1-n}^{2n} \sigma_n^{-1} \left(\sum_{i=\max\{1,l-n\}}^{\min\{n,l+n\}} w_{ni} \psi_{i-l} \right) e_l$$

$$:= \ \sum_{l=1-n}^{2n} \sigma_n^{-1} t_{nl} e_l.$$

Set $Z_{nl} = \sigma_n^{-1} t_{nl} e_l$, $l = 1-n, 2-n, \ldots, 2n$. Then $S_{1n} = \sum_{l=1-n}^{2n} Z_{nl}$. Let $k = [\frac{3n}{p+q}]$ and

$$(3.4) \qquad\qquad\qquad S_{1n} = S_{1n}' + S_{1n}'' + S_{1n}''',$$

where

$$\begin{cases} S_{1n}' = \sum_{m=1}^{k} y_{nm}, & y_{nm} = \sum_{i=k_m}^{k_m+p-1} Z_{ni}, \\ S_{1n}'' = \sum_{m=1}^{k} y_{nm}', & y_{nm}' = \sum_{i=l_m}^{l_m+q-1} Z_{ni}, \\ S_{1n}''' = y_{nk+1}', & y_{nk+1}' = \sum_{i=k(p+q)-n+1}^{2n} Z_{ni}, \end{cases}$$

$k_m = (m-1)(p+q) + 1 - n,\ l_m = (m-1)(p+q) + p + 1 - n,\ m = 1, \ldots, k.$
From (3.3) and (3.4) we have

$$(3.5) \qquad\qquad S_n = S'_{1n} + S''_{1n} + S'''_{1n} + S_{2n}.$$

Lemma 3.1. *Suppose that* (A1)-(A3) *are satisfied. If* $Ee_0^2 < \infty$, *then*

$$E(S''_{1n})^2 \leq C\gamma_{1n},\ E(S'''_{1n})^2 \leq C\gamma_{2n},\ E(S_{2n})^2 \leq C\gamma_{3n}.$$

Lemma 3.2. *Suppose that* (A1)-(A3) *are satisfied. Set* $s_n^2 = \sum_{m=1}^k \mathrm{Var}(y_{nm})$.
If $Ee_0^2 < \infty$, *then*

$$|s_n^2 - 1| \leq C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2} + u(q)).$$

Let $\eta_{nm}, m = 1, 2, \ldots, k$ be independent random variables and $\eta_{nm} \overset{\mathcal{D}}{=} y_{nm}$ for
$m = 1, 2, \ldots, k.$ Put $T_n = \sum_{m=1}^k \eta_{nm}.$

Lemma 3.3. *Under the assumptions of Theorem* 2.1, *we have*

$$\sup_u |P(T_n/s_n \leq u) - \Phi(u)| \leq C\gamma_{2n}^{\delta/2}.$$

Lemma 3.4. *Under the assumptions of Theorem* 2.1, *we have*

$$\sup_u |P(S'_{1n} \leq u) - P(T_n \leq u)| \leq C\{\gamma_{2n}^{\delta/2} + u^{1/3}(q)\}.$$

Lemma 3.5. *Under the assumptions of Theorem* 2.1, *we have*

$$(3.6) \qquad P(|S''_{1n}| > c\mu_n) \leq C\{n^{-1} + (nqp^{-1})^{-\delta/2}(\log n)^{(\delta-2)/2}\},$$

$$(3.7) \qquad P(|S'''_{1n}| > c\nu_n) \leq C\{n^{-1} + p^{-\delta/2}(\log n)^{(\delta-2)/2}\},$$

$$(3.8) \qquad P(|S_{2n}| > \tau_n) \leq C\gamma_{3n}^{1/3},$$

where $\mu_n = \gamma_{1n}^{1/2}(\log n)^{1/2},\ \nu_n = \gamma_{2n}^{1/2}(\log n)^{1/2},\ \tau_n = \gamma_{3n}^{1/3}.$

Lemma 3.6. *Let* X *and* Y *be random variables. Then for any* $a > 0$

$$\sup_u |P(X + Y \leq u) - \Phi(u)| \leq \sup_u |P(X \leq u) - \Phi(u)| + \frac{a}{\sqrt{2\pi}} + P(|Y| > a).$$

The proof of Lemma 3.6 can be found in Chang and Rao [5].

4. Proof of Theorem 2.1

We observe that

$$\sup_u |P(S'_{1n} \leq u) - \Phi(u)|$$

$$\leq \sup_u |P(S'_{1n} \leq u) - P(T_n \leq u)| + \sup_u |P(T_n \leq u) - \Phi(\frac{u}{s_n})|$$

$$+ \sup_u |\Phi(\frac{u}{s_n}) - \Phi(u))|$$

$$:= J_{1n} + J_{2n} + J_{3n}.$$

From Lemma 3.4 we have $J_{1n} \le C(\gamma_{2n}^{\delta/2} + u^{1/3}(q))$, Lemma 3.3 yields that $J_{2n} \le C\gamma_{2n}^{\delta/2}$, according to Lemma 3.2 we obtain that

$$
\begin{aligned}
J_{3n} &= |\Phi(u/s_n) - \Phi(u))| \\
&\le C|s_n^2 - 1|/s_n^2 \\
&\le C|s_n^2 - 1| \le C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2} + u(q)).
\end{aligned}
$$

Therefore

$$
\sup_u |P(S_{1n}' \le u) - \Phi(u)| \le C\{\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/2} + u^{1/3}(q)\}
$$

and from Lemmas 3.5 and 3.6 we have

$$
\begin{aligned}
&\sup_u |P(S_n \le u) - \Phi(u)| \\
&\le \sup_u |P(S_{1n}' + S_{1n}'' + S_{1n}''' + S_{2n} \le u) - \Phi(u)| \\
&\le \sup_u |P(S_{1n}' \le u) - \Phi(u)| + (\mu_n + \nu_n + \tau_n)/\sqrt{2\pi} \\
&\quad + P(|S_{1n}'' + S_{1n}''' + S_{2n}| > \mu_n + \nu_n + \tau_n) \\
&\le C\{\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/2} + u^{1/3}(q)\} + \mu_n + \nu_n + \tau_n \\
&\quad + P(|S_{1n}''| > \mu_n) + P(|S_{1n}'''| > \nu_n) + P(|S_{2n}| > \tau_n) \\
&= O\bigg((\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2})(\log n)^{1/2} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/3} + u^{1/3}(q) + n^{-1} \\
&\quad + ((nqp^{-1})^{-\delta/2} + p^{-\delta/2})(\log n)^{(\delta-2)/2}\bigg) \\
&= O(a_n).
\end{aligned}
$$

Proof of Corollary 2.1. In Theorem 2.1, taking $\delta = 1$, $p = [n^{1/2}(\log n)^{1/2}]$, $q = [\log n]$, then

$$
\begin{aligned}
u(q) &= O(n^{-3/4}(\log n)^{9/4}) \text{ by } u(n) = O(n^{9/4}e^{-3n/4}); \\
\gamma_{3n}^{1/3} &= n^{-1/4}(\log n)^{3/4}\bigg(n^{7/8}(\log n)^{-9/8} \sum_{|j|>n} |\psi_j|\bigg)^{2/3} \\
&= O(n^{-1/4}(\log n)^{3/4})
\end{aligned}
$$

by $\sup_{n \ge 1} n^{7/8}(\log n)^{-9/8} \sum_{|j|>n} |\psi_j| < \infty$; $\gamma_{1n}^{1/2} = \gamma_{2n}^{1/2} = n^{-1/4}(\log n)^{1/4}$ by $w_n = O(n^{-1})$.

Therefore, the conclusion follows from Theorem 2.1. $\qquad\square$

Proof of Corollary 2.2. Note that

$$
\sigma_n^2(x) = E\bigg(\sum_{k=1}^n w_{nk} V_k\bigg)^2 = \int_{-\pi}^{\pi} \tau(w)\bigg|\sum_{k=1}^n w_{nk} e^{-ikw}\bigg|^2 dw \ge C \sum_{k=1}^n w_{nk}^2.
$$

Hence, from $n \sum_{i=1}^{n} w_{ni}^2(x) \geq c_0 > 0$ and $w_n = O(n^{-1})$, it follows that

$$w_n \leq C \sum_{i=1}^{n} w_{ni}^2(x) = O(\sigma_n^2(x)) \text{ and } \sum_{i=1}^{n} |w_{ni}(x)| \leq C,$$

i.e., (A2) is satisfied. The rest proofs are the same as the proof of Corollary 2.1.   □

5. Proofs of lemmas

Lemma 5.1. *Let $\{X_j, 1 \leq j \leq n\}$ be NA random variables with $EX_j = 0$ and $E|X_j|^p < \infty$ for some $p > 1$, and let $\{a_j, j \geq 1\}$ be a sequence of real numbers. Then, there exist $A_p > 0$ and $B_p > 0$ such that $E \max_{1 \leq k \leq n} |\sum_{j=1}^{k} a_j X_j|^p \leq A_p \sum_{j=1}^{n} E|a_j X_j|^p$ for $1 < p \leq 2$ and*

$$E \max_{1 \leq k \leq n} \left| \sum_{j=1}^{k} a_j X_j \right|^p \leq B_p \left\{ \sum_{j=1}^{n} E|a_j X_j|^p + \left(\sum_{j=1}^{n} E(a_j X_j)^2 \right)^{p/2} \right\} \quad \text{for} \quad p > 2.$$

From Theorem 2 of Shao [23], it is easy to prove Lemma 5.1.

Lemma 5.2 (Yang [27]). *Suppose that $\{X_j, j \geq 1\}$ is a sequence of NA random variables.*

(a) *Let $\{a_j, j \geq 1\}$ be a sequence of real numbers, $1 = m_0 < m_1 < \cdots < m_k = n$. Denote by $Y_l := \sum_{j=m_{l-1}+1}^{m_l} a_j X_j$ for $1 \leq l \leq k$. Then*

$$\left| E \exp \left\{ it \sum_{l=1}^{k} Y_l \right\} - \prod_{l=1}^{k} \exp\{itY_l\} \right| \leq 4t^2 \sum_{1 \leq s < j \leq n} |a_s a_j| |\mathrm{Cov}(X_s, X_j)|.$$

(b) *Let $EX_j = 0$ and $|X_j| \leq b$ a.s. Set $\Delta_n = \sum_{j=1}^{n} EX_j^2$. Then, for any $\epsilon > 0$,*

$$P(|\sum_{j=1}^{n} X_j| \geq \epsilon) \leq 2 \exp \left\{ -\frac{\epsilon^2}{2(2\Delta_n + 2b\epsilon)} \right\}.$$

Proof of Lemma 3.1. According to the definition of S_{1n}'', from Lemma 5.1 and (A1)-(A2) we have

$$E(S_{1n}'')^2 = E\left(\sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} Z_{ni} \right)^2$$

$$\leq C \sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} E(Z_{ni}^2)$$

$$= C \sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} \sigma_n^{-2} t_{ni}^2 E e_i^2$$

$$\leq C \sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} \sigma_n^{-2} \left(\sum_{j=\max\{1,\ i-n\}}^{\min\{n,\ i+n\}} w_{nj}\psi_{j-i} \right)^2$$

$$\leq C \sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} w_n \left(\sum_{j=\max\{1,\ i-n\}}^{\min\{n,\ i+n\}} |\psi_{j-i}| \right)^2$$

$$\leq Ckqw_n \left(\sum_{j=-\infty}^{\infty} |\psi_j| \right)^2$$

$$\leq Cnqp^{-1}w_n = C\gamma_{1n}.$$

Similarly,

$$
\begin{aligned}
E(S_{1n}''')^2 &= E\left(\sum_{i=k(p+q)-n+1}^{2n} Z_{ni} \right)^2 \\
&\leq C \sum_{i=k(p+q)-n+1}^{2n} \sigma_n^{-2} t_{ni}^2 E e_i^2 \\
&\leq C \sum_{i=k(p+q)-n+1}^{2n} w_n \left(\sum_{j=\max\{1,\ i-n\}}^{\min\{n,\ i+n\}} |\psi_{j-i}| \right)^2 \\
&\leq C[3n - k(p+q)]w_n \left(\sum_{j=-\infty}^{\infty} |\psi_j| \right)^2 \\
&\leq Cpw_n = C\gamma_{2n}.
\end{aligned}
$$

As to S_{2n}, (A2) and $Ee_0^2 < \infty$ yield that

$$
\begin{aligned}
ES_{2n}^2 &= E\left| \sigma_n^{-1} \sum_{i_1=1}^{n} w_{ni_1} \sum_{|j_1|>n} \psi_{j_1} e_{i_1-j_1} \right| \left| \sigma_n^{-1} \sum_{i_2=1}^{n} w_{ni_2} \sum_{|j_2|>n} \psi_{j_2} e_{i_2-j_2} \right| \\
&\leq CE\left\{ \sum_{i_1=1}^{n} |w_{ni_1}| \sum_{i_2=1}^{n} \left| \sum_{|j_1|>n} \psi_{j_1} e_{i_1-j_1} \right| \left| \sum_{|j_2|>n} \psi_{j_2} e_{i_2-j_2} \right| \right\} \\
&\leq Cn\left(\sum_{|j|>n} |\psi_j| \right)^2 = C\gamma_{3n}.
\end{aligned}
$$

$\square$

Proof of Lemma 3.2. Let $\Gamma_n = \sum_{1 \leq i < j \leq k} \text{Cov}(y_{ni}, y_{nj})$, then $s_n^2 = E(S_{1n}')^2 - 2\Gamma_n$. Note that $ES_n^2 = 1$, so

$$
\begin{aligned}
E(S_{1n}')^2 &= E[S_n - (S_{1n}'' + S_{1n}''' + S_{2n})]^2 \\
&= 1 + E(S_{1n}'' + S_{1n}''' + S_{2n})^2 - 2ES_n(S_{1n}'' + S_{1n}''' + S_{2n}).
\end{aligned}
$$

Hence, from Lemma 3.1 we have

$$
|E(S'_{1n})^2 - 1|
$$

$$
\leq E(S''_{1n} + S'''_{1n} + S_{2n})^2 + 2|ES_n(S''_{1n} + S'''_{1n} + S_{2n})|
$$

$$
\leq E(S''_{1n} + S'''_{1n} + S_{2n})^2 + 2(ES_n^2)^{1/2}[E(S''_{1n} + S'''_{1n} + S_{2n})^2]^{1/2}
$$

(5.9)
$$
\leq \{[E(S''_{1n})^2]^{1/2} + [E(S'''_{1n})^2]^{1/2} + (ES_{2n}^2)^{1/2}\}^2
$$

$$
+ 2\{[E(S''_{1n})^2]^{1/2} + [E(S'''_{1n})^2]^{1/2} + (ES_{2n}^2)^{1/2}\}
$$

$$
\leq (\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2})^2 + 2(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2})
$$

$$
\leq C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2}).
$$

On the other hand,
(5.10)
$$
|\Gamma_n|
$$

$$
= |\sum_{1 \leq i < j \leq k} \mathrm{Cov}(y_{ni}, y_{nj})|
$$

$$
\leq \sum_{1 \leq i < j \leq k} \sum_{s=k_i}^{k_i+p-1} \sum_{t=k_j}^{k_j+p-1} |\mathrm{Cov}(Z_{ns}, Z_{nt})|
$$

$$
\leq \sum_{i=1}^{k-1} \sum_{s=k_i}^{k_i+p-1} \sum_{j=i+1}^{k} \sum_{t=k_j}^{k_j+p-1} \sum_{u=\max\{1,s-n\}}^{\min\{n,s+n\}} \sum_{v=\max\{1,t-n\}}^{\min\{n,\,t+n\}} \sigma_n^{-2} |w_{nu} w_{nv}| |\psi_{u-s} \psi_{v-t}| |\mathrm{Cov}(e_s, e_t)|
$$

$$
\leq C \sum_{i=1}^{k-1} \sum_{s=k_i}^{k_i+p-1} \sum_{u=\max\{1,s-n\}}^{\min\{n,s+n\}} |\psi_{u-s}||w_{nu}| \sum_{j=i+1}^{k} \sum_{t=k_j}^{k_j+p-1} |\mathrm{Cov}(e_s, e_t)| \sum_{v=\max\{1,t-n\}}^{\min\{n,t+n\}} |\psi_{v-t}|
$$

$$
\leq C \sum_{i=1}^{k-1} \sum_{s=k_i}^{k_i+p-1} \sum_{u=\max\{1,s-n\}}^{\min\{n,s+n\}} |\psi_{u-s}||w_{nu}| \sum_{t:\,|t-s| \geq q} |\mathrm{Cov}(e_s, e_t)|
$$

$$
\leq C u(q) \sum_{i=1}^{k-1} \sum_{s=k_i}^{k_i+p-1} \sum_{u=1}^{n} |w_{nu}||\psi_{u-s}|
$$

$$
\leq C u(q).
$$

Therefore, (5.9) and (5.10) follow that

$$
|s_n^2 - 1| \leq C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2} + u(q)).
$$

$\square$

Proof of Lemma 3.3. By using Berry-Esseen inequality (cf. Petrov [18], p. 154, Theorem 5.7) we have

$$
\sup_u |P(T_n/s_n \leq u) - \Phi(u)| \leq C \sum_{m=1}^{k} E|\eta_{nm}|^{2+\delta}/s_n^{2+\delta}.
$$

While, according to Lemma 5.1, from (A1) and (A2) it follows that

$$\sum_{m=1}^{k} E|\eta_{nm}|^{2+\delta}$$

$$\leq C \sum_{m=1}^{k} \left\{ \sum_{j=k_m}^{k_m+p-1} \left| \sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} \sigma_n^{-1} w_{ni}\psi_{i-j} \right|^{2+\delta} E|e_j|^{2+\delta} \right.$$

$$\left. + \left[\sum_{j=k_m}^{k_m+p-1} \left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} \sigma_n^{-1} w_{ni}\psi_{i-j} \right)^2 Ee_j^2 \right]^{1+\delta/2} \right\}$$

$$\leq C\sigma_n^{-(2+\delta)} \sum_{m=1}^{k} \sum_{j=k_m}^{k_m+p-1} \sum_{i=\max\{1,\ j-n\}}^{\min\{n,\ j+n\}} |w_{ni}||\psi_{i-j}| \cdot \left| \sum_{i=\max\{1,\ j-n\}}^{\min\{n,\ j+n\}} w_{ni}\psi_{i-j} \right|^{1+\delta}$$

$$+ C\sum_{m=1}^{k} \left[\sum_{j=k_m}^{k_m+p-1} \left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} \sigma_n^{-1} w_{ni}\psi_{i-j} \right)^2 \cdot \left(\sum_{j=k_m}^{k_m+p-1} \left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} \sigma_n^{-1} w_{ni}\psi_{i-j} \right)^2 \right)^{\delta/2} \right]$$

$$\leq C \left\{ w_n^{\delta/2} \sum_{i=1}^{n} |w_{ni}| \left(\sum_{m=1}^{k} \sum_{j=k_m}^{k_m+p-1} |\psi_{i-j}| \right) \right.$$

$$\left. + (pw_n)^{\delta/2} \sum_{m=1}^{k} \left[\sum_{j=k_m}^{k_m+p-1} \left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} |w_{ni}\psi_{i-j}| \right) \cdot \left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}} \sigma_n^{-2} |w_{ni}\psi_{i-j}| \right) \right] \right\}$$

$$\leq C\{ w_n^{\delta/2} + (pw_n)^{\delta/2} \}$$

$$\leq C(pw_n)^{\delta/2} = C\gamma_{2n}^{\delta/2}.$$

Note that Lemma 3.2 implies $s_n^2 \to 1$ and therefore

$$\sup_u |P(T_n/s_n \leq u) - \Phi(u)| \leq C\gamma_{2n}^{\delta/2}.$$

$$\square$$

Proof of Lemma 3.4. Assume that $\varphi(t)$ and $\psi(t)$ are the characteristic function of S_n' and T_n, respectively. By Esseen inequality (cf. Petrov [18], p. 146, Theorem 5.3), for any $T > 0$

$$\sup_u |P(S_n' \leq u) - P(T_n \leq u)|$$

(5.11)
$$\leq \int_{-T}^{T} \left| \frac{\varphi(t) - \psi(t)}{t} \right| dt$$

$$+ T \sup_u \int_{|u| \leq \frac{C}{T}} |P(T_n \leq u + y) - P(T_n \leq u)| dy$$

$$:= I_{1n} + I_{2n}.$$

According to Lemma 5.2(a), following the line as in (5.10) we have

$$\left| \frac{\varphi(t) - \psi(t)}{t} \right|$$

$$= |E \exp\{itS'_{1n}\} - \Pi_{m=1}^{k} E \exp\{ity_{nm}\}|$$

$$\leq 4t^2 \sum_{i=1}^{k-1} \sum_{s=k_i}^{k_i+p-1} \sum_{j=i+1}^{k} \sum_{t=k_j}^{k_j+p-1} \sum_{u=\max\{1,\ s-n\}}^{\min\{n,\ s+n\}} \sum_{v=\max\{1,\ t-n\}}^{\min\{n,\ t+n\}} \sigma_n^{-2} |w_{nu}w_{nv}|\, |\psi_{u-s}\psi_{v-t}|\, |\mathrm{Cov}(e_s, e_t)|$$

$$\leq Ct^2 u(q).$$

Therefore $I_{1n} \leq Cu(q)T^2$. On the other hand, from Lemmas 3.2 and 3.3 it follows that

$$\sup_u |P(T_n \leq u + y) - P(T_n \leq u)|$$

$$\leq \sup_y |P(T_n/s_n \leq (u+y)/s_n) - \Phi((u+y)/s_n)|$$

$$+ \sup_u |P(T_n/s_n \leq u/s_n) - \Phi(u/s_n)| + \sup_u |\Phi((u+y)/s_n) - \Phi(u/s_n)|$$

$$\leq C\{\gamma_{2n}^{\delta/2} + |y|/s_n\} \leq C\{\gamma_{2n}^{\delta/2} + |y|\}.$$

Therefore $I_{2n} \leq C(\gamma_{2n}^{\delta/2} + 1/T)$ and choosing $T = u^{-1/3}(q)$ from (5.11) we obtain that

$$\sup_u |P(S'_n \leq u) - P(T_n \leq u)| \leq C(\gamma_{2n}^{\delta/2} + u^{1/3}(q)).$$

$\square$

Proof of Lemma 3.5. Note that

$$S''_{1n} = \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} \sigma_n^{-1} t_{nj} e_j = \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} \sigma_n^{-1}(t_{nj}^+ - t_{nj}^-) e_j,$$

where $a^+ = \max\{0, a\}$, $a^- = \min\{0, -a\}$, so, without loss of generality, we assume that $t_{nj} \geq 0$ for $j \geq 1$ and $n \geq 1$. Let

$$\alpha_{nj} = -b_n I(Z_{nj} < -b_n) + Z_{nj} I(|Z_{nj}| \leq b_n) + b_n I(Z_{nj} > b_n),$$

$$\alpha'_{nj} = (Z_{nj} + b_n) I(Z_{nj} < -b_n) + (Z_{nj} - b_n) I(Z_{nj} > b_n),$$

$$\beta_{nj} = \alpha_{nj} - E\alpha_{nj}, \quad \beta'_{nj} = \alpha'_{nj} - E\alpha'_{nj},$$

where $b_n > 0$, which will be specialized later. It is clear that $\{\beta_{nj}, \ j \geq 1\}$ and $\{\beta'_{nj}, \ j \geq 1\}$ are mean zero NA sequences from the definition of α_{nj} and α'_{nj}.

Firstly, we prove (3.6). Taking $b_n = \gamma_{1n}^{1/2}(\log n)^{-1/2}$. Set

$$H_n = \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} \beta_{nj}, \quad H'_n = \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} \beta'_{nj}.$$

Then $S''_{1n} = H_n + H'_n$ and

(5.12) $$P(|S''_{1n}| > c\mu_n) \leq P(|H_n| > c\mu_n/2) + P(|H'_n| > c\mu_n/2).$$

To evaluate $P(|H_n| > c\mu_n/2)$, we shall use Lemma 5.2(b). Note that $|\beta_{nj}| \leq 2b_n$ and the proof of Lemma 3.1 shows that

$$\Delta_{1n} := \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} E(\beta_{nj}^2) \leq \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} E(Z_{nj}^2) \leq C\gamma_{1n}.$$

Then, by using Lemma 5.2(b), for large $c > 0$ we have
(5.13)
$$P\left(|H_n| > \frac{c\mu_n}{2}\right) \leq 2\exp\left\{-\frac{c^2\mu_n^2}{8(2\Delta_{1n} + b_n\mu_n)}\right\} \leq 2\exp\{-cc_1 \log n\} \leq Cn^{-1}.$$

We observe that

$$\begin{aligned} E(\beta_{nj}')^2 &\leq E(\alpha_{nj}')^2 \leq 2E(Z_{nj}^2 + b_n^2)I(|Z_{nj}| > b_n) \\ &\leq 4b_n^{-\delta}E|Z_{nj}|^{2+\delta} \leq C(\sigma_n^{-1}|t_{nj}|)^{2+\delta}b_n^{-\delta}. \end{aligned}$$

Hence, by using Lemma 5.1 we have

$$\begin{aligned} P\left(|H_n'| > \frac{c\mu_n}{2}\right) &\leq C\frac{E(H_n')^2}{\mu_n^2} \leq C\mu_n^{-2} \sum_{m=1}^{k} \sum_{j=l_m}^{l_m+q-1} (\sigma_n^{-1}|t_{nj}|)^{2+\delta}b_n^{-\delta} \end{aligned}$$

(5.14)
$$\qquad\qquad \leq C\frac{\gamma_{1n}w_n^{\delta/2}}{\mu_n^2 b_n^\delta} \leq C(nqp^{-1})^{-\delta/2}(\log n)^{(\delta-2)/2}.$$

Therefore, (5.13) and (5.14) follow (3.6).

Next we prove (3.7). Following the line as for (3.6). Choosing

$$b_n = \gamma_{2n}^{1/2}(\log n)^{-1/2}.$$

Set $Q_n = \sum_{j=k(p+q)-n+1}^{2n} \beta_{nj}$, $Q_n' = \sum_{j=k(p+q)-n+1}^{2n} \beta_{nj}'$. Hence $S_{1n}''' = Q_n + Q_n'$. Note that

$$\Delta_{2n} := \sum_{j=k(p+q)-n+1}^{2n} E\beta_{nj}^2 \leq \sum_{j=k(p+q)-n+1}^{2n} EZ_{nj}^2 \leq C\gamma_{2n}.$$

So, by using Lemma 5.2(b), for large $c > 0$, we have
(5.15)
$$P\left(|Q_n| > \frac{c\nu_n}{2}\right) \leq 2\exp\left\{-\frac{c^2\nu_n^2}{8(2\Delta_{2n} + b_n\nu_n)}\right\} \leq C\exp\{-cc_2 \log n\} \leq Cn^{-1}.$$

From $E(\beta_{nj}')^2 \leq C(\sigma_n^{-1}|t_{nj}|)^{2+\delta}b_n^{-\delta}$, and using Lemma 5.1 we have

$$\begin{aligned} P\left(|Q_n'| > \frac{c\nu_n}{2}\right) &\leq C\frac{E(Q_n')^2}{\nu_n^2} \leq C\nu_n^{-2} \sum_{j=k(p+q)-n+1}^{2n} (\sigma_n^{-1}|t_{nj}|)^{2+\delta}b_n^{-\delta} \end{aligned}$$

(5.16)
$$\qquad\qquad \leq C\frac{\gamma_{2n}w_n^{\delta/2}}{\nu_n^2 b_n^\delta} \leq Cp^{-\delta/2}(\log n)^{(\delta-2)/2}.$$

Therefore, (5.15) and (5.16) yield (3.7).

As to (3.8), choosing $\tau_n = \gamma_{3n}^{1/3} = n^{1/3}(\sum_{|j|>n} |\psi_j|)^{2/3}$. In view of Lemma 3.1 we have

$$P(|S_{2n}| > \tau_n) \leq E(S_{2n})^2/\tau_n^2 \leq C\gamma_{3n}^{1/3}.$$

$\square$

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HAN-YING LIANG
DEPARTMENT OF MATHEMATICS
TONGJI UNIVERSITY
SHANGHAI 200092, P. R. CHINA
E-mail address: hyliang83@yahoo.com

YU-YU LI
DEPARTMENT OF MATHEMATICS
TONGJI UNIVERSITY
SHANGHAI 200092, P. R. CHINA