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Determination of Equivalent Roughness for Estimating Flow Resistance in Stabled Gravel-Bed River:

Ⅰ. Theory and Development of the Model

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Abstract

Flow resistance in a natural stream is caused by complex factors, such as the grains on the bed, vegetation, and bed-form, reach profile. Flow resistance in a generally stable gravel bed stream is due to protrudent grains from bed. Therefore, the flow resistance can be calculated by equivalent roughness in gravel bed stream, but estimation of equivalent roughness is difficult because nonuniform size and irregular arrangement of distributed grain on natural stream bed. In previous study, equivalent roughness is empirically estimated using characteristic grain size. However, application of empirical equation have uncertainty in stream that stream bed characteristic differs. In this study, we developed a model using an analytical method considering grain diameter distribution characteristics of grains on the bed and also taking into account flow resistance acting on each grain. Also, the model consider the protrusion height of grain.

Key Words : Grain protrusion, Grain shear stress, Drag force, Equivalent roughness

1. Introduction

The flow resistance occurring in the boundary between the water flow and the surface of materials on the bed in a natural stream is called frictional resistance. Frictional resistance is related to the boundary roughness, or the size of grains distributed on the bed. Frictional resistance is produced in a stream that has a bed of clay or sand. For instance, a gravel bed stream has great boundary roughness, thereby serving as a major factor on the frictional resistance regarding flow resistance.

Frictional resistance is expressed as a term of the size of bed materials and can be represented as the equivalent roughness or roughness height. Equivalent roughness means the uniform roughness on the boundary producing the same loss of head as the loss of head produced by the roughness of the channel distributed irregularly and non-uniformly on the boundary in the completely rough turbulent flow. The equivalent roughness is used to estimate vertical velocity flow distribution, the mean velocity and the friction coefficient, among other things, and through this it is used for various flow analyses like the calculation of water depth and discharge, and the water level-discharge relationship analysis. Also, the flow resistance parameters such as Chezy's coefficient C, the Darcy Weisbach friction coefficient f, and the Manning's roughness coefficient n are related to one another, it is possible to estimate the values of these coefficients.

Generally, researchers experimentally applied the relationship between a characteristic grain diameter distributed on the bed and a constant to estimate equiv-

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alent roughness or to analyze the flow resistance of the gravel bed stream. However, the analyses of these studies regarded the flow resistance of grains distributed in the entire bed as an impact by a characteristic grain diameter or representative grain diameter. If the grains diameter distribution of bed materials, their arrangement, and the direction and shapes of the space and grain are the same, this assumption would be reasonable. However, a natural stream has different grain diameters distributions and characteristics, so it is difficult to apply uniformly. Moreover, as experimental equivalent roughness estimation methods that have been proposed use each other different kinds of grain diameters and constants corresponding to the characteristics of their study subject streams, such methods are confusing to use. Therefore, estimating the equivalent roughness in streams that have different stream bed characteristics should be performed according to the grain characteristics on the bed.

Therefore, in this study, we aim to develop an equivalent roughness estimation model that can consider the distribution characteristics of grain diameter through an analytical method. Also, the model consider the protrusion height of grain.

2. The flow resistance in the gravel bed stream

In a stabilized a gravel bed stream, sand ripples or dunes are not particularly developed, unlike in a sand stream, so that it is not necessary to consider the resistance by the bed form. However, as grains diameter in the gravel bed stream are big, there is significant flow resistance caused by the shape of the grain, and as it is difficult to distinguish such flow resistance from that caused by boundary grains, they are treated as a whole[1,2]. Also, although roughness as determined by the reach profile has to be considered when the flow analysis is performed for long stream channel, if flow resistance in a certain section or a random section where the reach profile is not largely changed, it is not necessary to consider roughness by the reach profile[3].

As the relationship between grain resistance and roughness height when grains of various sizes are mixed in a gravel bed has not been clearly revealed, roughness is generally assumed to be the same as certain sizes(D_{50}, D_{84} etc. : the grain diameter D when the cumulative pass weight of the bed materials are 50% and 80%) of the grain diameter distribution[4,5]. However, a gravel bed has greatly varying grain diameter distribution so much that it imposes a high level of uncertainty in expressing bed roughness.

Several previous studies on flow resistance applied the most generalized flow resistance equation of Equation (1) below proposed by Keulegan[6].

$$\frac{1}{\sqrt{f}} = 2.03\log\left(\frac{aR}{C_e D_n}\right) \tag{1}$$

Here, a is constant, R is hydraulic radius, C_e is a coefficient to estimate the equivalent roughness and D_n is the grain diameter when the cumulative pass weight of bed materials is n%. In Equation. (1), $C_e D_n$ is the term of equivalent roughness that the results of existing studies are arranged as shown in the following Table 1.

In Table 1, the range of coefficient C_e applied against D_{50}, D_{84} and D_{90} is 1.5~8.2, 2.65~3.78 and

Table 1. The equivalent roughness used the flow resistance equations in previous studies

D_{50}		D_{84}		D_{90}		
Investigator	C_e	Investigator	C_e	Investigator		C_e
Thompson et al.[7]	4.5	Leopold et al.[12]	3.78	Kamphuis[14]		2.0
Bray[8]	6.8	Limerinos[13]	3.16	Van Rijn[15]		3.0
Griffiths[9]	5.0	Hey[1]	3.50	Gomez[16]	round gravel	2.4~14.5
Thorne et al.[10]	1.5~8.2	Azmon[11]	2.65		flat gravel	2.6~6.6
Azmon[11]	4.3	-	-		angular gravel	5.3~16.2

2.0~16.2, respectively, so that it is found that the range of C_e applied against each grain diameter is quite wide. The reason for this is judged as the value of C_eD_n and is an experimentally proposed value and that bed materials have different distribution characteristics.

If flow resistance is caused only by each grain distributed on the bed, then the flow resistance is influenced by each grain on the bed[4]. In other words, to analyze the equivalent roughness and flow resistance in streams with each other difference bed characteristics, it is necessary to apply equivalent roughness and flow resistance analysis through an analytical method that considers the distribution characteristics of the grain diameter on the bed in order to reflect the flow resistance of each grain.

3. Theoretical background

3.1. Average bed shear stress

The average bed shear stress τ_0 in the steady uniform flow is led by considering the balance of power acting on the water pillar located to be parallel with the free surface, and the balance of power is the balance between the component force in the flowing direction of gravity acting on the water pillar and the average bed shear stress, the boundary friction, as shown in Equation (2) below. In other words, if flow resistance is produced only by surface friction of the bed, the average bed shear stress can be said to be the same as the shear stress acting on the grains in a unit area producing friction force on the boundary, as shown in Equation (3), and the grain shear stress τ_g can be estimated by the drag force F_D[17].

$$\tau_0 = \rho g R s_0 \tag{2}$$

$$\tau_0 = \tau_g = F_D = C_D A_g \frac{\rho u_m^2}{2} \tag{3}$$

Here, ρ is the density of water, g is the gravitational acceleration, R is the hydraulic radius, s_0 is the bed slope, and A_g is the projected grain area to the flow. Also, C_D is the drag coefficient, and u_m is the mean velocity hit to the grain.

3.2. Flow velocity distribution on the rough boundary

The gravel bed has a rough boundary. The vertical velocity distribution for completely rough turbulent flow follows the logarithmic law of the wall[18], a vertical velocity distribution equation using k_s as in Equation (4).

$$\frac{u_y}{u^*} = \frac{1}{\kappa} ln\left(\frac{y}{k_s}\right) + B \tag{4}$$

Here, the von Karman constant κ has the value of 0.4 in the turbulent flow. And B is the integration constant, the function of the Reynolds number, and 8.5 is the value used in the completely rough turbulent flow. If we compute Equation (4) by substituting the κ is 0.4 and the B is 8.5, it will be as shown in the following Equation (5).

$$\frac{u_y}{u^*} = 5.75\log\left(\frac{30y}{k_s}\right) \tag{5}$$

Theoretically, Equation (5) finds that $u_y = 0$ for $y = k_s/30$ or, that the velocity becomes zero at a distance $k_s/30$ above the bed[19].

3.3. Drag coefficient

Drag coefficient C_D is the function of the Reynolds number R_e and it changes according to grain shape. If the grain is spherical, C_D rapidly decreases as R_e increases having 0.4 ~ 0.5 value of 10^3 ~ 10^6, and it is rapidly reduced again when it is over 10^6. Coleman[20] proposed that if the grain is spherical in a completely turbulent flow, C_D has a value of 0.5, Flammer et al.[21] estimated the drag coefficient by relative roughness (k_s/R) and Froude number, and analyzed that when the relative roughness is smaller than 0.25, it has the constant value of 0.6 ~ 0.5 regardless of the Froude numbers. Further, Wiberg and Smith[5] studied the flow velocity distribution using the average value of 0.45, as C_D has the value of 0.4 ~ 0.5 when R_e is between 10^4 ~ 10^6. Therefore, in this study, we used the average value of 0.5 of the results of previous studies as the drag coefficient C_D under the assumption that the grains distributed on the bed are spherical.

The actual grains distributed on the bed are not protruding into the flow of water on the whole, but as grains are in contact with one another, the actual protrusion height becomes shorter than grain diameter D, and the actual bed could be regarded as the above contact points of the grains. Therefore, the drag coefficient should have different values according to the protrusion height from the contact point of the grain instead of the value of 0.5. For this reason, Martin[19] proposed the drag coefficient C_D^* for the relative protrusion of grains, H/D, as an exponent equation like Equation (6) using the data of Nikuradse[18] and the Coleman[20] under the assumption that grains on the bed are spherical.

$$C_D^* = 0.54(1-\exp(-2.6\frac{H}{D})) \tag{6}$$

Here, H is the grain protrusion height from the bed surface(contact point of grains) and the height that subtracted the height $k_s/30$, which the actual flow velocity becomes zero. It is shown in Equation (7) and Figure 1.

$$\frac{H}{D} = \frac{H_0 - \frac{k_s}{30}}{D} \tag{7}$$

Here, H_0 is the distance from the joint point with a neighboring grain to the top of the grain.

3.4. Mean velocity acting on the grain

If grains on the bed are spherical, their shapes shift from spherical to hemispherical etc. according to the grain protrusion height, so that the mean velocity height y_m. Therefore, in this study, we estimated y_m, the height that the mean velocity acts upon, according to the protrusion height through the numerical integration against the drag force acting on the grain with the diameter D. Fig. 2. shows the drag force acting on the slice section of the grain with a diameter D.

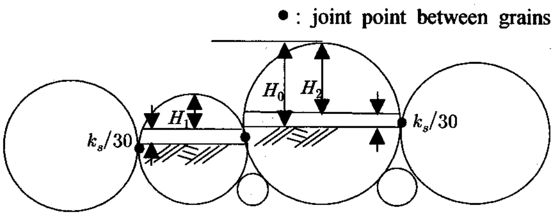

Fig. 1. The grains protrusion height distributed on the bed.

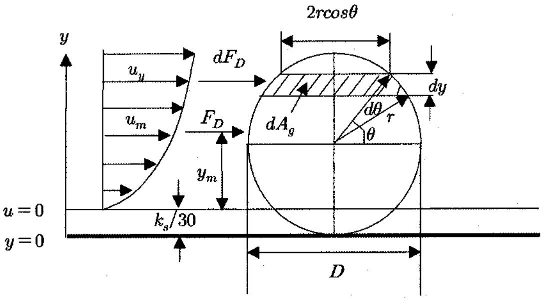

Fig. 2. Drag force acting on the slice section of the grain.

The drag force F_D acting on the entire projection section area of the grain is estimated like in Equation (8) as integrating dF_D, the drag force acting on each slice section like in Equation (6), and it is the same as the drag force F_D when the mean velocity u_m acting on the entire projection cross-section area A_g of grain.

$$F_D = C_D\frac{1}{2}\rho u_m^2 A_g = \int dF_D = \int C_D\frac{1}{2}\rho u_y^2 dA_g$$
$$= \int C_D\frac{1}{2}\rho u_y^2 2r\cos\theta dy \tag{8}$$

Also, y_m, the height that the mean velocity u_m acting on the entire projection cross-section area A_g of grain was calculated by drag force F_D according to the protrusion height based on Equation (8), and the acting height of the mean velocity u_m, corresponding to this, was estimated by substituting it in Equation (5). As a result, y_m according to the relative protrusion H/D of the grain is as shown in the following Table 2, and the average value of y_m for each H/D is $0.4H$.

Regarding the value on Table 2, if it is made as the function of the relative protrusion H/D through the regression analysis and added with $k_s/30$, the height that the flow velocity becomes zero from the bed, it becomes as in Equation (10) below.

$$y_m = \left[0.490 - 0.538\frac{H}{D} + 0.704(\frac{H}{D})^2 - 0.213(\frac{H}{D})^3\right] \times H + \frac{k_s}{30} \tag{10}$$

Table 2. Mean velocity height for the relative protrusion of the grain

H/D	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
y_m/D	0.448	0.414	0.396	0.385	0.379	0.375	0.377	0.382	0.398	0.450

Therefore, the mean velocity acting on the grain is like in Equation (11), and Fig. 3. shows the flow velocity distribution acting on the grain.

$$u_m = u^* 5.75 \log\left(\frac{30 y_m}{k_s}\right) \tag{11}$$

3.5. Comparison of mean velocity

The mean velocity acting on protrusive grain from the bed becomes different according to the estimation method of the mean velocity height, and as the mean velocity is proportional to the square of the drag force acting on the protruding grain, the result of the estimation of the mean velocity height directly affects the result of the equivalent roughness estimation. Therefore, in this study, we compared and examined the mean velocity height acting on the protruded grain v_1 with the mean velocity v_2 and v_3 against $0.35H$ proposed by Einstein and El-Samni[22], and $0.5H$ proposed by Coleman[20]. For this purpose, the considered height $k_s/30$ of the flow velocity is zero, to substitute it to Equation (5), and the results are as shown in Table 3. To estimate the shear velocity u^* in the Equation (3),

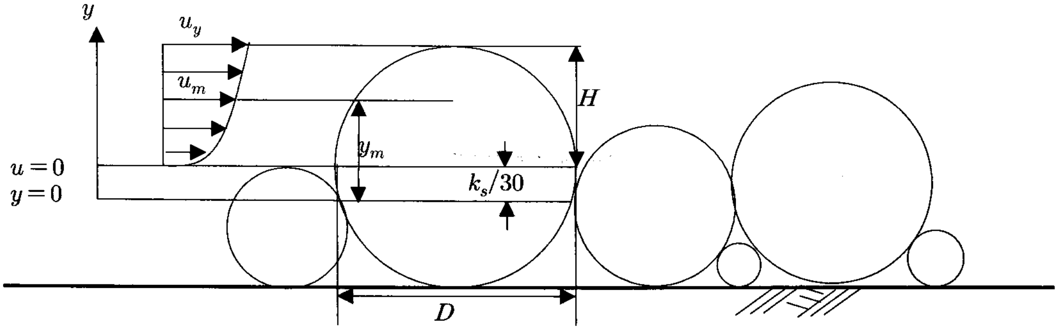

Fig. 3. The flow velocity distribution acting on the grain.

Table 3. Comparison of velocity by the mean velocity height estimation method

H (m)	H/D	v_1 (m/s)	v_2 (m/s)	v_3 (m/s)	$\Delta v_{1,2}$ (%)	$\Delta v_{1,3}$ (%)	v_1^2 $(m/s)^2$	v_2^2 $(m/s)^2$	v_3^2 $(m/s)^2$	$\Delta v_{1,2}^2$ (%)	$\Delta v_{1,3}^2$ (%)
0.020	1.0	0.1104	0.1029	0.1144	6.74	-3.61	0.0122	0.0106	0.0131	13.03	-7.34
0.018	0.9	0.1054	0.0996	0.1110	5.46	-5.33	0.0111	0.0099	0.0123	10.63	-10.95
0.016	0.8	0.1001	0.0959	0.1072	4.20	-7.11	0.0100	0.0092	0.0115	8.22	-14.72
0.014	0.7	0.0946	0.0917	0.1029	3.07	-8.81	0.0090	0.0084	0.0106	6.04	-18.39
0.012	0.6	0.0889	0.0869	0.0981	2.25	-10.27	0.0079	0.0076	0.0096	4.45	-21.60
0.010	0.5	0.0830	0.0813	0.0923	1.96	-11.30	0.0069	0.0066	0.0085	3.88	-23.87
0.008	0.4	0.0765	0.0746	0.0854	2.45	-11.67	0.0059	0.0056	0.0073	4.83	-24.70
0.006	0.3	0.0690	0.0662	0.0767	4.01	-11.19	0.0048	0.0044	0.0059	7.85	-23.64
0.004	0.2	0.0591	0.0549	0.0648	7.06	-9.70	0.0035	0.0030	0.0042	13.61	-20.35
0.002	0.1	0.0431	0.0377	0.0462	12.65	-7.03	0.0019	0.0014	0.0021	23.70	-14.55

we randomly applied 1m as the hydraulic radius R, 0.0006 as the bed slope s_0, and 10 mm as the equivalent roughness k_s to estimate the mean velocity u_m acting on the grain with a diameter of 20 mm.

The fast flow velocity is estimated as the vertical height is higher that, as a result of the comparison of those three methods, the values were estimated as v_3, v_2, v_1 in the order of size similar to the size order of the mean velocity action height. Comparing flow velocity v_1 by numerical integration and the flow velocities for v_2 ($y_m = 0.35H + k_s/30$) and $v_3(y_m = 0.50H + k_s/30)$, deviation $\Delta v_{1,2}$ of v_2 against v_1 according to the protrusion height H, showed 1.96~12.65% of flow velocity, and the deviation $\Delta v_{1,3}$ of v_3 against v_1 showed -3.61~-11.67% of flow velocity. However, as drag force is proportional to the square of the flow velocity, the deviation becomes even larger. The deviation $\Delta v_{1,2}^2$ against the square of the flow velocity showed in the range of 3.88~23.70% of deviation, and $\Delta v_{1,3}^2$ showed -7.34~-24.70% of deviation. Thus, it is judged that the mean velocity height acting on each grain will give an impact on the estimation of the drag force and equivalent roughness as much as the Δv^2.

4. The model development for the equivalent roughness estimation

Equivalent roughness is estimated according to Equation (3), which is the equation between the average bed shear stress and the grain shear stress per unit area. If the average bed shear stress and the grain shear stress by unit area are put as the same, it is shown in the following Equation (12).

$$\tau_0 = \tau_g = \rho g R s_0 = C_D^* A_g \frac{\rho u_m^2}{2} \tag{12}$$

If Equation (11) of the mean velocity is substituted for u_m in Equation (12), it is as follows.

$$\rho g R s_0 = C_D^* \frac{\rho}{2}\left[u^* 5.75 \log\left(\frac{30 y_m}{k_s}\right)\right]^2 \tag{13}$$

As the shear velocity u^* on the right side of Equation (13) is $\sqrt{gRs_0}$, if $\rho g R s_0$ is eliminated by squaring that, Equation (13) becomes the same as Equation (14).

$$C_D^* \frac{1}{2}\left[5.75 \log\left(\frac{30 y_m}{k_s}\right)\right]^2 - 1 = 0 \tag{14}$$

In Equation (14), the drag coefficient C_D^* is the function of the relative protrusion H/D in Equation (6), and the mean velocity height y_m in Equation (11) is also the function of the relative protrusion of the grain H/D by Equation (10). In addition, the relative protrusion H/D is the function by the equivalent roughness k_s. Therefore, Equation (14) can be expressed as the function of equivalent roughness k_s like in Equation (15).

$$f(k_s) = C_D^* \frac{1}{2}\left[5.75 \log\left(\frac{30 y_m}{k_s}\right)\right]^2 - 1 = 0 \tag{15}$$

Equations (12)~(15) are values calculated by unit area of the grain. As grains of varying sized are distributed on the bed, it is required to apply the weight value consisting of the unit area of each grain. Therefore, we calculated the composition ratio of weight for each grain diameter through the cumulative grain diameter analysis and used it as the weight value. We also estimated the grain shear stress per unit area for each grain diameter, applied the weight composition ratio and added the grain shear stress for each grain diameter to make a final estimate of the grain shear stress per unit area. Therefore, Equation (15) is transformed as the equation for each grain as Equation (16).

$$f(k_s) = \sum\left\{C_{Di}^* \frac{1}{2}\left[5.75 \log\left(\frac{30 y_{mi}}{k_s}\right)\right]^2\right\} F_i - 1 = 0 \tag{16}$$

Here, i is each grain diameter, F_i is the weight composition ratio of each grain diameter. The equivalent roughness k_s is assumed as an unknown value in the calculation initial stage, and then found using the trial and error method or simple numerical analysis to have the value that satisfies Equation (16) or within the allowable error e. The general flow chart of this model is shown in Fig. 4.

In this model made by fortran language, there are required some simple input data like as the diameter,

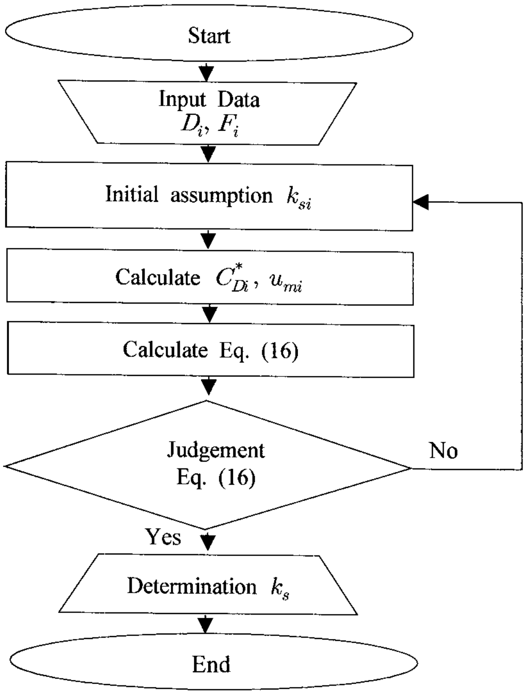

Fig. 4. Flowchart of the model.

weight composition ratio of the grain distributions and calculation conditions like as the initial equivalent roughness k_{si}, allowable error e and maximum return number.

5. Conclusions

In this study, we proposed a model for the estimation of equivalent roughness considering protrusion height of grains in the gravel stream bed, and the details are the following.

1) The distribution characteristics of the grain diameter is considered in the model. Therefore, it is a model that can be applied in the analysis of various streams with different grain diameter distributions.

2) Also, previous studies did not consider the grain protrusion height but applied a constant, whereas in this study, we applied the grain protrusion height condition to ensured the physical validity.

3) For the mean velocity hit on the protruding spherical grains from the bed, we made the estimation using the mean velocity height. The mean velocity height was proposed as a regression for the grain protrusion height through numerical integration.

In this study, we developed the equivalent roughness estimation model considering only the grains on the bed that is the applicable model to the gravel bed stream, where the effect of vegetation is minimal. In the future, further studies should be performed considering other factors like vegetation, the movement of grains on the bed and the grain shape, among other factors, to broaden the model's application range.

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