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Reconstruction of Dispersive Lamb Waves in Thin Plates Using a Time Reversal Method

Hyunjo Jeong

Abstract Time reversal (TR) of nondispersive body waves has been used in many applications including ultrasonic NDE. However, the study of the TR method for Lamb waves on thin structures is not well established. In this paper, the full reconstruction of the input signal is investigated for dispersive Lamb waves by introducing a time reversal operator based on the Mindlin plate theory. A broadband and a narrowband input waveform are employed to reconstruct the A_0 mode of Lamb wave propagations. Due to the frequency dependence of the TR process of Lamb waves, different frequency components of the broadband excitation are scaled differently during the time reversal process and the original input signal cannot be fully restored. This is the primary reason for using a narrowband excitation to enhance the flaw detectability.

Keywords: Time Reversal, Dispersive Waves, Reconstruction of Source Waveforms

1. Introduction

The origin of the time reversal (TR) concept traces back to time reversal acoustics (Fink, 1999; Fink and Prada, 2001; Draeger et al, 1997). In time reversal acoustics, an input body wave can be exactly reconstructed at the source location if a response signal measured at a distinct location is time-reversed and reemitted to the original excitation location. This phenomenon is referred to as TR of body waves and has been used in many applications including ultrasonic nondestructive evaluation and underwater communications.

While the TR method for nondispersive body waves in fluids has been well established, the study of the TR method for Lamb waves on plates is relatively new (Ing and Fink, 1998a, 1998b; Park et al, 2007). The effect of dispersion on the time reversal analysis of Lamb waves in a homogeneous plate was first studied by Wang et al. (Wang et al, 2003) by introducing the time reversal operator into the Lamb wave equation based on the Mindlin plate theory (Rose and Wang, 2004).

Due to the frequency dependence of the time reversal process of Lamb waves, different frequency components of the broadband excitation are scaled differently during the time reversal process and the original input signal cannot be fully restored.

In this paper, the full reconstruction of the input signal is attempted for Lamb waves. The TR of the A_0 mode Lamb wave is investigated by introducing a time reversal operator in a frequency domain using the Mindlin plate theory. To achieve this goal, a narrowband input waveform is employed to enhance the TR of Lamb waves. The complete reconstruction of the input signal can be achieved when a narrowband excitation is employed for Lamb wave propagations.

Received: December 23, 2007, Accepted: February 13, 2008. Division of Mechanical and Automobile Eng, Wonkwang University, 344-2 Sinyong-dong, Iksan, Jeonbuk 570-749, Korea, (E-mail: hjjeong@wonkwang.ac.kr)

2. TR of A_0 Mode Lamb Wave in a Thin Plate

The time reversibility of waves is based on the spatial reciprocity and time reversal invariance of linear wave equations. In the time reversal method, an input signal can be reconstructed at an excitation point (point A) if an output signal recorded at another point (point B) is reemitted to the original source point (point A) after being reversed in a time domain as illustrated in Fig. 1.

When a sensor A is used as an actuator and another sensor B is used as a receiver (Fig. 1), the received voltage at sensor B can be written as

$$V_B(r,\omega) = I_A(\omega)K_a(\omega)G(r,\omega)K_s(\omega) \qquad (1)$$

where r is the wave propagation distance from the sensors, $I_A(\omega)$ the input at patch A, K_a and K_s, the mechanical-electro efficiency factor of sensor A and B, G the frequency response function of sensor B as a result of the input at sensor A. The frequency response function G for the A_0 mode Lamb wave is obtained by applying appropriate transformation techniques in the spatial and time domain to the wave equation based on the Mindlin plate theory (Wang et al, 2003).

$$G(r,\omega) = -\frac{i\pi h^2}{2D}\frac{\gamma_1 k_1^3 a J_1(k_1 a) H_0^{(1)}(k_1 r)}{k_1^2 - k_2^2} \qquad (2)$$

where h, D, γ_1, a, $J_1(\cdot)$ and $H_0^1(\cdot)$ are the plate thickness, the flexural stiffness of the plate, the amplitude ratio of dilatational to shear wave potentials at the wave number k_1, radius of sensor A, the first order Bessel function and the zeroth order Hankel function of the first kind, respectively. The wave numbers k_1 and k_2 are determined at the A_0 and A_1 modes of the Lamb waves, respectively.

Once a response signal is measured at sensor B, the reconstructed input signal at sensor A can be obtained by reemitting the time-reversed response signal at sensor B. The time reversal operation of a signal in the time domain is equivalent to taking the complex conjugate of the Fourier Transform of the signal in the frequency domain. Therefore, the reconstructed signal at sensor A from the reemitted signal at sensor B can be written in a similar fashion to eqn. (1) as

$$V_A(r,\omega) = V_B^*(r,\omega)K_a(\omega)G(r,\omega)K_s(\omega) \qquad (3)$$

where a superscript * denotes a complex conjugate. By using eqn. (1), the signal received at sensor A is rewritten as

$$V_A(r,\omega) = I_A^*(\omega)K_a^*(\omega)K_s^*(\omega)K_a(\omega)K_s(\omega)G(r,\omega)G^*(r,\omega) \qquad (4)$$

Performing an inverse Fourier transform, the reconstructed input signal V_A at sensor A is

$$V_A(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I_A^*(\omega)K_{as}^*(\omega)K_{as}(\omega)G^*(r,\omega)G(r,\omega)e^{-iwt}d\omega \qquad (5)$$

where K_{as} denotes the product between K_a and K_s. If the TR of waves is satisfied, the

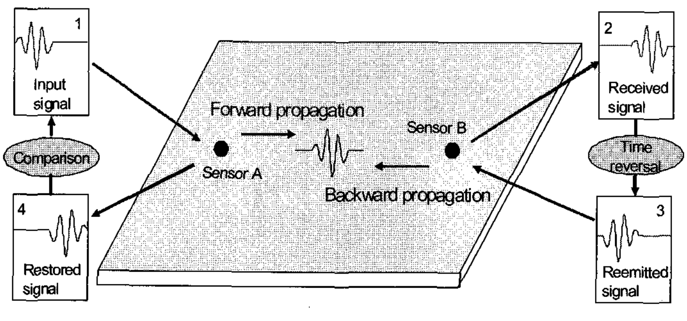

Fig. 1 Time reversal process of A_0 mode Lamb wave in a thin plate

reconstructed signal $V_A(t)$ in eqn. (5) would be identical to the time-reversed original signal $I_A(T-t)$ where T represents the total time duration of the signal. To directly compare with the original input signal $I_A(t)$ at sensor A, eqn. (5) should be time reversed, thus

$$V_A(T-t) = \frac{1}{2\pi}\int I_A(\omega)K_{TR}(\omega)G_{TR}(r,\omega)e^{-i\omega(T-t)}d\omega \quad (6)$$

where

$$K_{TR}(\omega) = K_{as}(\omega)K^*_{as}(\omega),\ G_{TR}(\omega) = G(r,\omega)G^*(r,\omega) \quad (7)$$

Here, K_{TR} is a factor determined by the electro-mechanical efficiency of the sensor, and G_{TR} is referred to as a time reversal operator of A_0 mode Lamb wave determined by the Mindlin plate theory. In enq. (6), assuming K_{TR} is constant, the TR is achieved only if G_{TR} is independent of the angular frequency ω. However, because the impulse response function $G(r,\omega)$ of a plate structure is frequency dependent, the time reversal operator G_{TR} varies with respect to the frequency. This indicates that the wave components are non-uniformly scaled depending on the frequency. Therefore, the original input signal cannot be reconstructed if a broadband input signal is used.

3. Numerical Examples

To justify the use of a narrowband excitation for the time reversal process, a numerical example of the time reversal process is provided here. In particular, a broadband Gaussian pulse as shown in eqn. (8) and Figs. 2(a), (b) and a narrowband 100 kHz toneburst as shown in eqn. (9) and Figs. 3(a), (b) are used as input signals in a numerical simulation of the time reversal process.

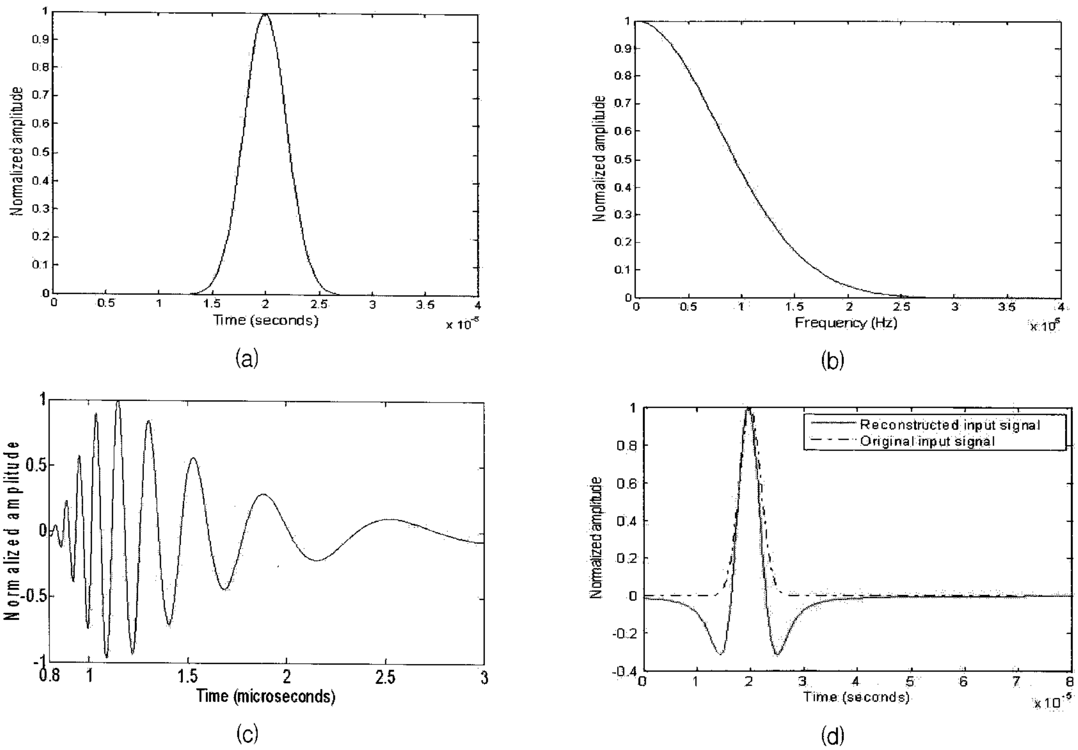

Fig. 2 Time reversal process of a broadband Gaussian pulse. (a) time domain input signal (b) its magnitude spectrum, (c) response signal of sensor B at the propagation distance r=152.4 mm, and (d) reconstructed TR signal compared with the original input signal

For the broadband simulation, the following Gaussian pulse is employed to derive the sensor A:

$$y(t) = Ae^{-(t-\mu)^2/2\sigma^2} \tag{8}$$

where μ=20 μs and σ=2 μs. For the narrowband toneburst signal, the following input function is used.

$$y(t) = \begin{cases} A[1-\cos(2\pi ft/N)]\cos(2\pi ft) & (0<t<N/f) \\ 0 & otherwise \end{cases} \tag{9}$$

where the frequency f=100 kHz and the number of cycles N=5. An aluminum plate (E=73 GPa, ν=0.3, ρ=2770 kg/m^3, h=1.02 mm) is used with sensor diameter a= 3.175 mm and propagation distance r=152.4 mm.

The time domain input signals are shown in Figs. 2(a) and 3(a) for broadband and narrowband cases, respectively. Their frequency spectra are also shown in Figs. 2(b) and 3(b). The response signal received at sensor B is shown in Figs. 2(c) and 3(c). The broadband input signal causes a velocity dispersion on the A_0 mode propagation even at a short distance of 152.4 mm. On the contrary, the narrowband input signal remains its shape throughout the propagation.

When the response signal is reversed in time and reemitted to the input sensor, the velocity dispersion of Lamb waves is compensated as shown in Figs. 2(d) and 3(d). However, the shape of the original pulse is not fully recovered when the Gaussian input is used as illustrated in Fig. 2(d). This is because the various frequency components of the Gaussian input are differently scaled and superimposed during the time reversal process. On the other hand, as shown in Fig. 3(d), the shape of reconstructed toneburst waveform is practically identical to that of the original input because the amplification of the

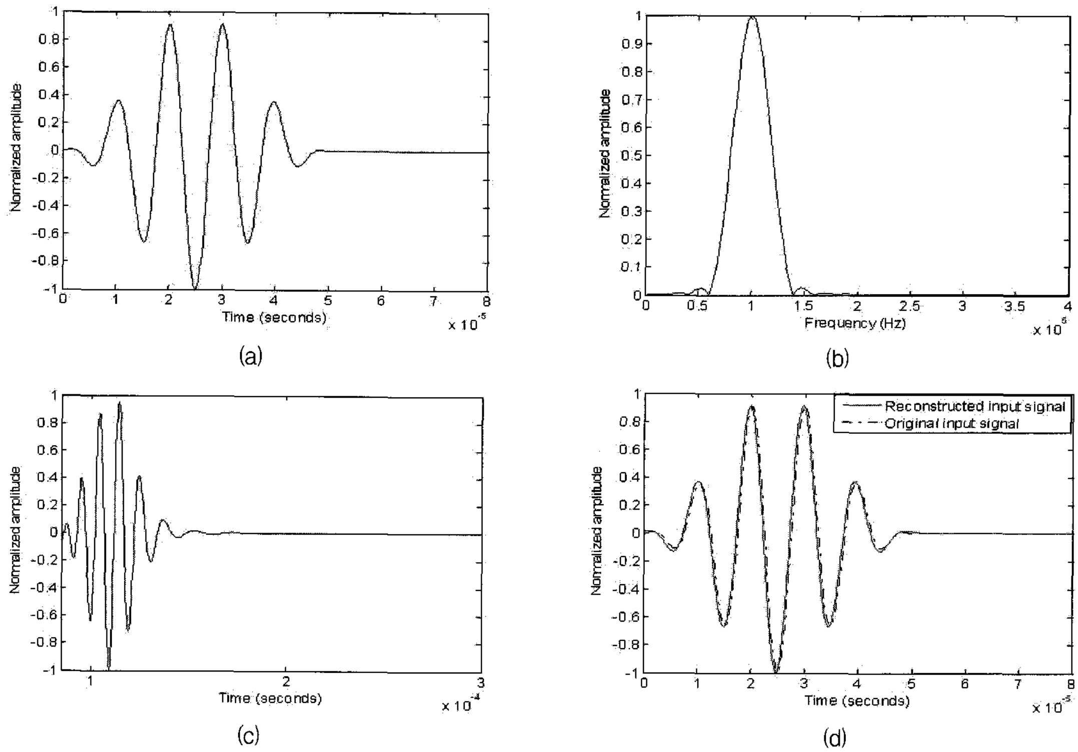

Fig. 3 Time reversal process of a narrowband toneburst waveform. (a) time domain input signal (b) its magnitude spectrum, (c) response signal of sensor B at the propagation distance r=152.4 mm, and (d) reconstructed TR signal compared with the original input signal

time reversal operator is almost uniform in this limited frequency band.

4. Conclusions

The full reconstruction of the known excitation signal at the original input location is attempted through the time reversal process of dispersive Lamb waves. The TR of the A_0 mode Lamb wave is investigated by introducing a time reversal operator in a frequency domain using the Mindlin plate theory. The complete reconstruction of the input signal cannot be achieved when a broadband excitation is employed for Lamb wave propagations. Due to the frequency dependence of the time reversal process of Lamb waves, different frequency components of the broadband excitation are scaled differently during the time reversal process and the original input signal cannot be fully restored. The use of narrowband input waveform fully restores the original input signal in the time reversal process of dispersive Lamb wave propagations. This is the primary reason for using a narrowband toneburst excitation in most experiments. The ultimate goal of future work is to enhance the flaw detectability in thin-walled structures through spatial ad temporal focusing due to the TR of Lamb waves.

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