Power System Nonlinearity Modal Interaction by the Normal Forms of Vector Fields

Jing Zhang†, J. Y. Wen* and S. J. Cheng**

Abstract – Because of the robust nonlinear characteristics appearing in today's modern power system, a strong interaction exists between the angle stability and the voltage stability, which were conventionally studied insularly. However, as the power system is a complex unified system, angle instability always happens in conjunction with voltage instability. The authors propose a novel method to analyze this type of stability problem. In the proposed method, the theory of normal forms of vector fields is utilized to treat the auxiliary dynamic system. By use of this method, the interaction between response modes caused by the nonlinearity of the power system can be analyzed. Consequently, the eigenvalue analysis method is extended to cope with performance analysis of the power system with heavy nonlinearity. The effectiveness of the proposed methodology is verified on a 3-bus power system.

Keywords: angular stability, modal interaction, nonlinearity, normal forms of vector fields, voltage stability.

1. Introduction

For the purpose of simplifying the analysis, studies of the angle stability and the voltage stability for power systems are usually performed independently. Reference [1] studied the voltage instability decoupled from angle instability. Reference [2] decomposed the power system into separate subsystems and studied the relationship between angle stability and voltage stability. Reference [3] studied the instability modes, angle instability, and voltage instability, at different operation conditions, and highlighted the factors that distinguished the angle and voltage stability. Reference [4] constituted a general energy function, and examined the relationship between angle instability and voltage instability from the viewpoint of this energy function. Reference [5] studied the characterization of the power system small disturbance stability with models incorporating voltage variation.

However, as the power system is a unified system, all kinds of instabilities are inter-connected. Therefore, they are indivisible problems. Reference [6] proposed such a viewpoint and investigated this problem. According to this standpoint, power system angle stability and voltage stability are two aspects of power system stability that cannot be separated from each other clearly in the real power system. Insular investigation of them may result in an inaccuracy or even incorrect result. In fact, interaction between the angle and the voltage instability do exist in the power system, especially when the system operating condition is close to the stability limit resulting from the restriction of the environment and the interconnected networks. Investigation results indicate that in these situations, because of the existence of the nonlinearity, the angle stability and the voltage stability cannot be distinguished exactly [7]. Therefore, new methodologies should be developed to study such problems.

In recent years, a novel method called normal forms of vector fields, which can be used to analyze the dynamic behaviors of nonlinear systems, have been introduced to the power system analysis [8-14]. This paper applies the normal form method (NFM) to investigate an auxiliary dynamic system corresponding to the power flow equations and studies the interaction between the angle stability and the voltage stability in different system operating conditions. The result shows that interaction between different stability modes arise if strong nonlinearity exists in the power system. The investigation results also indicate that both the angle stability mode and the voltage stability mode are sub-modes of the system dynamic behavior, which is contributed by all sub-modes and their interaction.

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2. Auxiliary Dynamic System

In this paper, the power system is modeled by the power flow equations shown below:

$$P_i - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0$$
$$Q_i - V_i \sum_{j=1}^{n} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 \quad (1)$$

where V_i is the voltage of bus i, θ_{ij} is the angle difference between buses i and j, G_{ij} and B_{ij} are the conductance and susceptance of the line connecting the buses i and j respectively, P_i and Q_i are the active and the reactive power injection of bus i respectively, and m and n are the number of PQ buses and total buses of the system respectively.

In order to calculate the stability margin, reference [15] proposed an auxiliary dynamic system associated with the power flow equations (1) to form an energy function. In this paper, the same auxiliary dynamic system (2) is adopted.

$$\varepsilon\dot{\theta}_i = P_i - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij})$$
$$\varepsilon\dot{V}_i = Q_i - V_i \sum_{j=1}^{n} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \quad (2)$$

where $\varepsilon>0$ is a constant.

From equation (2), the following general form of the auxiliary dynamic system (2) can be obtained:

$$\dot{x} = g(x) \quad (3)$$

where $\boldsymbol{x}=(\theta_1, \theta_2, \cdots\cdots, \theta_{n-1}, V_1, V_2, \cdots\cdots, V_m)^T$

Since the right hand sides of equations (1) and (2) are the same, they have the same Jacobian matrix. Consequently, all equilibrium points of (2) are the solutions of the power flow equations (1). Moreover, the solutions of the power flow equations cannot be classified as stable or unstable, but they can be distinguished by the type of equilibrium points, therefore, it is possible to prove that type 0 equilibrium points of (1) will correspond to stable equilibrium points (*SEP*) of (2) while type n equilibrium points of (1) with $n>0$ will correspond to unstable equilibrium points (*UEP*) of (2) [16]. Thus, it is possible to study the characteristics of the load flow solutions through analyzing the equilibrium points of the auxiliary dynamic system. Furthermore, it is recognized that the power flow equations represent the equilibrium condition of the auxiliary dynamic system.

3. Characteristic Of Equilibrium Points

As it is well known that the power flow equation (1) is a nonlinear equation, under the stable operating condition, the number of the solutions for this equation is 2^{n-1}, for example, for a 3-bus power system, 4 solutions can be obtained for the power flow equations. One of the four solutions corresponds to the *SEP* and the other three solutions correspond to the *UEP*s. Gradually changing the operating condition of the power system towards the stability limits, the equilibrium points of the system will coalesce in pairs by saddle-node bifurcation (*SNB*). Finally, the coalescent of the survived *SEP* and *UEP* will result in an instability of the system.

For the *SEP*, the eigenvalues of relative Jacobian represent the dynamic modes of the power system according to the linear analysis theory. The participation factor can be used as a measure to weigh the system state variable participating system dynamic mode. Reference [11] distinguished different modes by the participation factor. A similar idea used in reference [11] is adopted in this paper in order to class the type of mode. Modes with large participation factor of the generator angle in a power system will be identified as the angular stability modes, while modes with large participation factor of the power system voltage will be identified as the voltage stability modes. The power system is characterized by all modes. Reasonably, they are named the sub-mode of stability modes, angular stability sub-mode, and voltage stability sub-mode, in this paper.

Moreover, according to reference [4], the equilibrium point with big angle and high voltage corresponds to the angular instability, the equilibrium point with small angle and low voltage corresponds to the voltage instability, and the equilibrium point with big angle and low voltage corresponds to both the angular and the voltage instability.

4. The Normal Form Method

4.1 Taylor's Series Expansion

The Taylor's series expansion of equation (3) about an equilibrium point (*EP*) is given by

$$\dot{x}_i = J_i x + x^T H^i x / 2 + H.O.T \quad (4)$$

where $\boldsymbol{J}_i$ is the ith row of the Jacobian matrix $\boldsymbol{J}$, $\boldsymbol{H}^i$ is the ith Hessian matrix, and $H.O.T$ represents the high order terms.

Using the similarity transformation $\boldsymbol{x}=\boldsymbol{Uy}$, where $\boldsymbol{U}$ is the matrix of right eigenvectors for $\boldsymbol{J}$, the original system can be represented by the following Taylor expansion with up to second order in the y-coordinates:

$$\dot{y}_i = \lambda_i y_i + y^T C^i y = \lambda_i y_i + \sum_{j=1}^{n} \sum_{k=1}^{n} C^i_{jk} y_j y_k \quad (5)$$

where λ_i is the ith eigenvalue of $\boldsymbol{J}$, and

$$C^i = \frac{1}{2}\sum_{j=1}^{n+m-1} v_{ij} U^T H^i U = [C^i_{jk}]$$

v_{ij} is the element of the matrix $\boldsymbol{V}=\boldsymbol{U}^{-1}$.

4.2 Normal Form analysis

The condition given by equation (6) for system (5) is called second order resonance condition.

$$\lambda_i = \lambda_j + \lambda_k \tag{6}$$

When the condition of (6) is unsatisfied, the second order terms in equation (5) can be annihilated by the following normal form transformation to the z-coordinates [8-9].

$$y = z + h(z) \tag{7}$$

where,

$$h^i(z) = \sum_{k=1}^{n+m-1}\sum_{l=1}^{n+m-1} h^i_{kl} z_k z_l \tag{8}$$

$$h^i_{kl} = \frac{C^i_{kl}}{\lambda_k + \lambda_l - \lambda_i} \tag{9}$$

which is named nonlinear interaction coefficient.

In z-coordinates, system (5) takes the following form:

$$\dot{z}_i = \lambda_i z_i \tag{10}$$

In this way, an explicit second order solution for the original system (3) in z-coordinates can be obtained, which is represented by (11) and (12):

$$z_i(t) = z_{i0} e^{\lambda_i t} \tag{11}$$

$$x_i(t) = \sum_{j=1}^{n+m-1} u_{ij} z_{j0} e^{\lambda_j t} + \sum_{j=1}^{n+m-1} u_{ij} \sum_{k=1}^{n+m-1}\sum_{l=1}^{n+m-1} h^j_{kl} z_{k0} z_{l0} e^{(\lambda_k+\lambda_l)t} \tag{12}$$

where u_{ij} is an element of the matrix $\boldsymbol{U}$, and $\boldsymbol{z_0}$ is the value in z-coordinates corresponding to the initial value $\boldsymbol{x_0}$ in x-coordinates.

It can be seen from equation (12) that for this system, there are n+m-1 modes corresponding to the n+m-1 eigenvalues, if only the linear portion of the original system is considered. Moreover, these modes are independent with each other. However, if taking the 2nd order term into account, the interactions between modes can be represented by h^j_{kl}.

Nonlinear interaction coefficient h^j_{kl} indicates the strength of the nonlinear interaction between modes, i.e. it shows the interaction of mode k and mode l to mode j. The larger the magnitude of h^j_{kl} is, the stronger the nonlinear interaction will be [11]. This paper is based on the following considerations. One dynamic mode of the power system corresponds to one eigenvalue, and the dynamic mode of the power system can be distinguished into the angular stability mode and the voltage stability mode. Therefore, the magnitude of h^j_{kl} can be used to specify the interaction between different dynamic modes, including the angular stability mode and the voltage stability mode.

It should be pointed out that linear participation factor is not a new concept to analyze the behavior of the dynamic system. Linear participation factor has been a well-known concept in the small signal analysis (*SSA*) and is given below

$$p_{ki} = u_{ki} v_{ik} \tag{13}$$

where p_{ki} is considered as a measure to weigh the participation of the kth state in the ith mode.

5. Test Results And Analysis

A 3-bus power system shown in Figure 1 is used for the present investigation. The parameters of this system given in the figure are all in per unit. In this system, bus 2 is the generation bus, at which the magnitude of voltage is 1.02, bus 3 is the slack bus, at which the voltage is 1.05 $\angle 0$, and bus 1 is the PQ bus with Q=P/2.

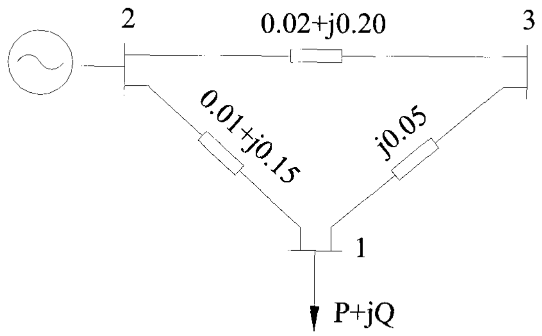

Fig. 1. Simple 3-bus system

5.1 Case 1

Fixing the injecting active power of generator, P_m, at 4, Figure 2 presents the *PV* curve when the active power *P* at bus 1 increases from 1 p.u.. It can be seen from Figure 2 that, as the active power increases continually, $x3$ and $x4$ coalesce and vanish at first by the *SNB*, and then $x1$ and $x2$ survive. Further increase of the active power will result in $x1$ and $x2$ coalescing and the power system loses the voltage stability eventually.

In the light loading condition (P=2), there exist 4 *EP*s as shown in Table 1, in which $x1$ corresponds to the *SEP*, $x2$ is a *type*-1 *EP* of the voltage stability mode with small angle and low voltage, $x3$ is a *type*-1 *EP* of angular stability mode with big angle and high voltage, and $x4$ is a *type*-2 *EP* with big angle and low voltage.

In the procedure when the power system approaches its critical state, the nonlinearity increases especially when the power system operates close to the vicinity of the critical point.

Analyzing the power system by *NF* under the condition of P=8.74, the states vector is $[\theta_1\ \theta_2\ V_1]^T$, the eigenvalues are λ_1=-24.5623, λ_2=-0.2507, and λ_3=-7.3739, and the largest magnitude of h_{kl}^j is $|h_{22}^2| = -132.1755$, which shows a strong nonlinearity resulting from the mode corresponding to λ_2. Table 2 shows the linear participation factors of this mode for comparison.

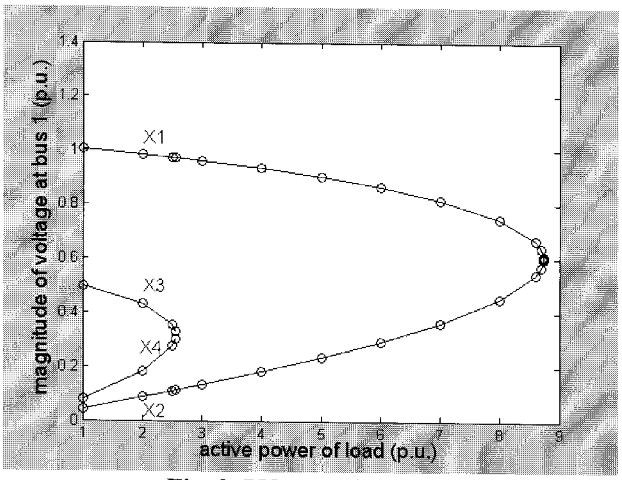

Fig. 2. PV curve for case 1

Table 1. 4 equilibriums under light load condition (P_m=4, P=2)

平衡点	$x1$	$x2$	$x3$	$x4$
θ_1	0.0080	-0.8773	-0.2008	-0.6254
θ_2	0.3403	0.5757	2.7810	2.6256
V_1	0.9850	0.0874	0.4320	0.1814

Table 2. Linear participation factors for λ_2 under the condition (P_m=4, P=8.74)

K	1	2	3
p_{k2}	0.3668	0.0069	0.6264

5.2 Case 2

Fixing the active power of load, *P*, at 2, Figure 3 shows the *PV* curve when the generation active power output increases from 4 p.u.. It can be see from Figure 3, that as the generation active power increases, equilibriums $x2$ and $x4$ coalesce and vanish at first by the *SNB*, and then $x1$ and $x3$ survive. Further increase of the generation active power output will result in $x1$ and $x3$ coalescing and the power system loses the angular stability.

Analyzing the dynamic behavior of the power system by NF under the condition of Pm=11.24, the states vector is $[\theta_1\ \theta_2\ V_1]^{\mathrm{T}}$, the eigenvalues are λ_1=-21.8756, λ_2=-15.3182, λ_3=-0.2505, and the largest magnitude of h_{kl}^j is $|h_{33}^3| = 39.9054$, representing a strong nonlinearity resulting from the mode corresponding toλ_3. Table 3 shows the linear participation factors of this mode for comparison.

Table 3. Linear participation factors for λ_3 under the condition (P_m=11.24, P=2)

K	1	2	3
p_{k3}	0.0002	0.9139	0.0859

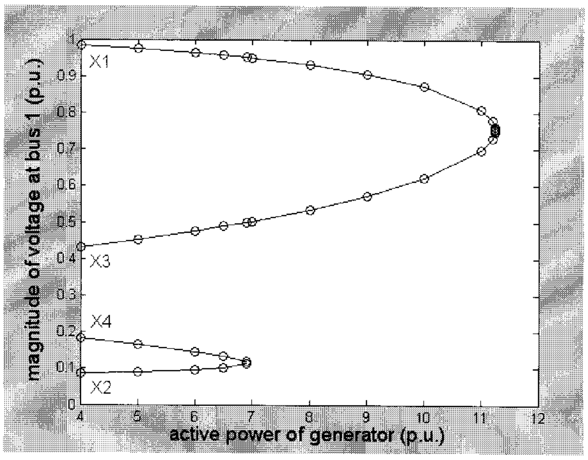

Fig. 3. PV curve for case 2

5.3 Case 3

Fixing the injecting active power of generator, P_m, at 10, the load model of bus 1 is $P=2Q=aV$. Figure 4 shows the variation of the equilibrium points when the loading condition increases. In this case, 2 *EP*s exist, and $x1$

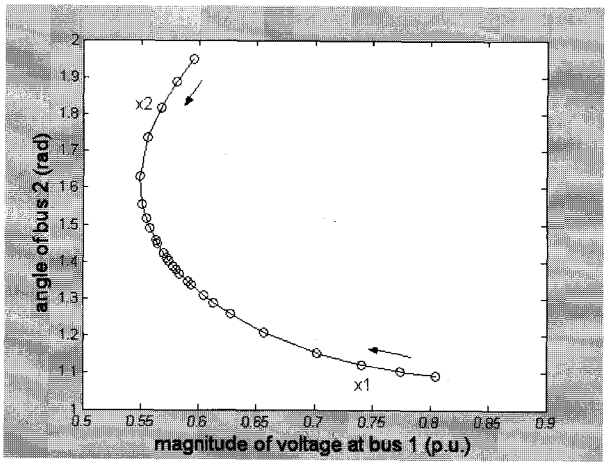

Fig. 4. Equilibrium point behaviors for case 3

corresponds to the *SEP*, $x2$ is characterized by large angle and low voltage, which identifies that the power system may lose both the angular and voltage stabilities.

It can be see from Figure 4 that as the loading condition increases, $x1$ and $x2$ coalesce and the power system loses angle and voltage stability.

Analyzing the dynamic behavior of the power system by *NF* under the condition of a=9.96, the states vector is $[\theta_1\ \theta_2\ V_1]^T$, the eigenvalues are λ_1=-16.9562, λ_2=-12.3242, λ_3=-0.2482, and the largest magnitude of h_{kl}^j is $|h_{13}^1| = 36.8873$, showing a strong nonlinearity resulting from the modes corresponding to λ_1 and λ_3. Table 4 presents the linear participation factors of the two modes for comparison.

Table 4. Linear participation factors for λ_1 and λ_3 under condition (P_m=10, a=9.96)

K	1	2	3
p_{k1}	-0.0727	0.1106	0.9621
p_{k3}	0.0006	0.8971	0.1022

5.4 Analysis and Discussions

Both the voltage and the angular stability modes of the 3 bus power system are presented above. For case 1, it can be found from Figure 2 that in the vicinity of the critical point, only an *EP* with small angle and low voltage, $x2$, exists on the lower part of the *PV* curve, representing a possible voltage stability. Moreover, dynamic mode 2, corresponding to λ_2=-0.2507, is the dominant mode characterized by voltage stability. Analysis by *NF* indicates that the strong nonlinearity in the vicinity of the critical point is resulted from mode 2. The result obtained by the method proposed in this paper also shows that the power system tends to have a voltage stability problem. The strong nonlinearity is mainly resulted from the interaction between the voltage sub-modes.

For case 2, it can be found from Figure 3 that in the vicinity of the critical point, only an *EP* with large angle and high voltage, $x3$, exists on the lower part of the *PV* curve representing a possible angular stability. Moreover, dynamic mode 3, corresponding to λ_3=-0.2505, is the dominant mode characterized by angular stability. Analysis by *NF* indicates that the strong nonlinearity in the vicinity of the critical point is resulted from mode 3. The result obtained by the method proposed in this paper also shows that the power system tends to experience angular stability problem. The strong nonlinearity is mainly resulted from the interaction between the angular sub-modes.

For case 3, it can be found from Figure 4 that there exists only an *EP* with large angle and low voltage, $x2$, representing a possible angular and voltage stability. Moreover, mode 1 and mode 3, corresponding to λ_1 and λ_3, are respectively characterized by voltage stability and angular stability. Analysis by *NF* shows that the strong nonlinearity in the vicinity of the critical point is resulted from mode 1 and mode 3. The result obtained by the method proposed in this paper also shows that the power system tends to have both the angular stability and the voltage stability problem. The strong nonlinearity is mainly resulted from the interaction between the angular and the voltage sub-modes.

6. Conclusions

Normal form method, as one of the most effective approaches used to analyze the performance of power systems, is presented in the paper. The presented approach is used to analyze the stability of a simple power system. The following conclusions are obtained from the analysis:

A. By use of the normal forms of vector fields, the conventional eigen analysis method is extended to cope with the nonlinear system. As a result, a new effective way to study the power system stability and natures at strong nonlinear condition is obtained.
B. The 2nd order analytical solutions, yielded by the normal form method, contain more information about the natures of power systems.
C. Power system stability sub-modes interact with each another, especially when the power system presents strong nonlinear characteristics. Such kind of interactions cannot be investigated by *SSA*.
D. It is all the sub-modes and their interaction that causes modern power systems to present very complicated character.

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