

Optimum Parameter Values for A Metal Plating Process

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금속도금공정에서의 최적 모수 값 결정

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The problem of determining the optimum metal plating thicknesses on the plane and curved surfaces of an electronic part is considered. A lower specification limit for the plating thickness is usually pre-specified. In most applications, the plating thickness on the curved surface is proportional to that on the plane surface. The proportion can be adjusted by adding chemical catalysts to the plating fluid. From the economic point of view, nonconforming items with a thickness smaller than the lower specification limit incur rejection costs, such as rework and scrap costs, while a thicker plating may incur an excessive material costs. In this article, an economic model is proposed for simultaneously determining the target plating thickness and the ratio of the plating thickness on the plane surface to that on the curved surface. An illustrative example demonstrates the applicability of the proposed model.

Keywords: Metal Plating Process, Optimum Process Target, Economic Model

1. Introduction

Metal plating is the process of coating a material with a metallic layer to ensure the proper functioning and protection of the material, and its use value is unquestionable in many application areas. Recently, metal plating has gained a focused attention especially in high-tech manufacturing industries, such as semiconductor and other electronic devices, where products are usually metal-plated to instill required electronic functions into devices. A plating process should provide an acceptable plating thickness for every incoming item to attain the desired function. Since the plating thickness varies over time and with position on the surface, the process is usually

setup so that the plating thickness may be targeted to a larger value than the lower specification limit. However, the difference between the specification limit and the target value of the plating thickness induces unnecessary material costs due to over-plating. Determination of a proper target for the plating thickness is important because the plating metals are usually quite expensive.

The problem is very similar to the canning problem or filling process except for the variation mechanism of quality characteristics. Determining the optimum process mean for the filling process has been discussed for more than 40 years. The previous works for canning problem can be basically classified into two categories. The one is mainly focused on finding the optimal target mean. See, for example, Bettes

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(1962), Hunter and Kartha (1977), Bisgaard *et al.* (1984), Golhar (1987), Boucher and Jafari (1991), Elsayed and Chen (1993), Tang and Lo (1993), Bai and Lee (1993), Al-Sultan (1994), Arcelus and Rahim (1994), Lee and Kim (1994), Chen and Chung (1996), Lee and Jang (1997), Hong and Elsayed (1999), Hong *et al.* (1999), Pfeifer (1999), Lee *et al.* (2001), Teeravaraprug and Cho (2002), and Duffuaa and Siddiqui (2003). The other deals with simultaneous determination of the process mean and variance, which includes such works as Rahim and Al-Sultan (2000), Rahim and Shaibu (2000), Kim *et al.* (2000), and Rahim *et al.* (2002). Recently, Bowling *et al.* (2004) developed a model for process target levels within the framework of a multi-stage serial production process. Lee *et al.* (2005), Hong *et al.* (2006) and Lee *et al.* (2006) investigated optimum target parameters for different production processes.

In this paper, we consider a situation where target values are needed for two quality characteristics in a metal plating process; (i) the plating thickness on the plane surface and (ii) the plating thickness on the curved surface of an electronic part. To the best of our knowledge, there are no previous studies applicable to this situation. Both of the two characteristics should satisfy the same specification limit for a later assembly of the parts into various electronic devices. But the plating thickness on curved surfaces is usually different from that on plane surfaces. <Figure 1> depicts the schematic diagram of a multi-layer printed circuit board (PCB), where metal plating is applied on the curved surface along the through hole as well as the plain surface. The mean plating thickness on plane surfaces may easily be adjusted to a target value by setting the electric current and processing time without incurring significant costs. On the other hand, the thickness on the curved surface can be adjusted expensively by adding costly chemical catalysts to the plating fluid. Therefore, the addition of catalysts to control the plating thickness on the curved surface needs to be carefully determined.

This article proposes an economic model for simultaneously determining the optimum values of the mean plating thickness on the plane surface and the amount of catalysts to control the plating thickness on the curved surface. In fact, we determine the ratio of the plating thicknesses on the plane

and curved surfaces. The ratio is supposed to be proportional to the amount of catalysts added to the plating liquid. The proposed model incorporates various economic aspects associated with metal plating processes, such as manufacturing cost, loss due to scraps, and revenue from sale. This paper is organized as follows: The profit model is first developed in section 2. The properties of the proposed model and solution procedures are discussed in section 3. An illustrative numerical example is then provided to demonstrate the proposed model. Conclusions are drawn in the last section.

2. The Profit Model

Consider the production process of an electronic device where copper plating is applied to a panel consisting of many parts and containing several holes for connecting to various electrical circuits. Let X_1 and X_2 be the dimensions of plating thickness on the plain surface and in the hole, respectively. An outgoing item is judged conforming if both X_1 and X_2 are greater than or equal to the lower specification limit L . From the past experience, it is known that the dimension of plating thickness on the plane surface of the panel is linearly proportional to but not the same as that on the curved surface in the hole. This may be formally expressed as

$$X_2 = \alpha X_1 + \epsilon \quad (1)$$

where α ($0 < \alpha < 1$) represents the linear coefficient of X_1 with respect to X_2 . X_1 and ϵ are assumed to be independently normally distributed with $X_1 \sim N(\mu, \sigma^2)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$. Then X_2 is also normally distributed with mean $\alpha\mu$ and standard deviation $\alpha\sigma/\rho$, where ρ denotes the correlation coefficient between X_1 and X_2 . Note that the correlation coefficient ρ is given by

$$\rho = \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1)Var(X_2)}} = \frac{\alpha\sigma}{\sqrt{\alpha^2\sigma^2 + \sigma_\epsilon^2}}.$$

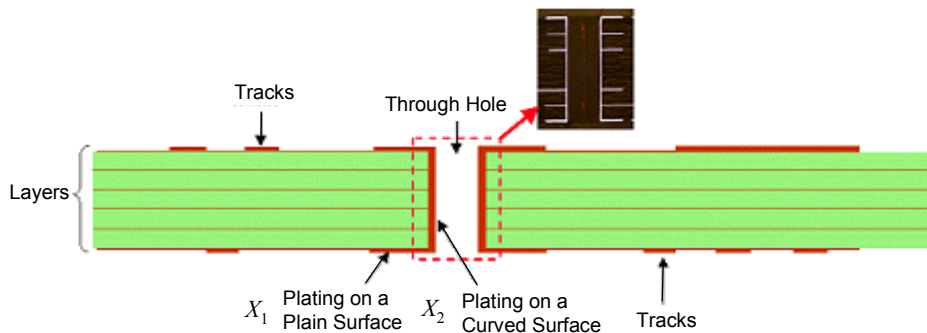


Figure 1. Cross-Sectional Diagram of a Multi-Layer PCB (<http://www.polarinstruments.com>)

For brevity, let $X = (X_1 - L)/\sigma$, $Y = (X_2 - L)/\sigma$, $l = L/\sigma$, and $\tau = (\mu - L)/\sigma$. It can then be shown that

$$X \sim N(\tau, 1), \tag{2a}$$

$$Y \sim N\left(\alpha\tau - (1 - \alpha)l, \frac{\alpha^2}{\rho^2}\right), \tag{2b}$$

$$Y|_{X=x} \sim N\left(\alpha x - (1 - \alpha)l, \frac{\alpha^2(1 - \rho^2)}{\rho^2}\right). \tag{2c}$$

Note that the lower specification is zero for both X and Y , and the decision parameters are τ and α .

Let A be the fixed cost component per item including the inspection cost, and let B and C be the unit material cost components proportional to X and Y , respectively, for metal plating. In fact, the unit material cost of X and Y will be the same if the plane surface area is the same as the curved surface area. Hence, C/B represents the ratio of the curved surface area to the plane surface area. The ratio α of Y to X can be adjusted by changing the viscosity of plating fluid with the addition of chemical catalysts. Field studies indicate that α is proportional to the quantity of the catalysts in our region of interest. These chemical catalysts or additives are usually quite expensive, and thus the adjustment requires an additional cost of $D\alpha$. Consequently, the manufacturing cost related to the metal plating process will be

$$MC = A + BX + CY + D\alpha \tag{3}$$

Suppose an income R is attained from the sale of each conforming item and a scrap cost S is incurred for each non-conforming item. Then, the profit function for the metal plating process is written by

$$P = \begin{cases} R - MC, & X \geq 0, Y \geq 0, \\ -MC - S, & \text{otherwise.} \end{cases} \tag{4}$$

Let $\Phi(\cdot)$ and $\phi(\cdot)$ be the standard normal distribution and density functions, respectively. Denote $\Pr[X \geq 0, Y \geq 0]$ by $Q(\tau, \alpha)$. Then, the expected profit per item is given by

$$EP = (R + S)Q(\tau, \alpha) - EMC - S \tag{5a}$$

where

$$Q(\tau, \alpha) = \Phi(\tau) - \Phi\left(-\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) + \int_{-\infty}^0 \phi(x - \tau)\Phi\left(-\frac{\rho[x - (1/\alpha - 1)l]}{\sqrt{1 - \rho^2}}\right)dx, \tag{5b}$$

$$EMC = A + B\tau + C\{\alpha\tau - (1 - \alpha)l\} + D\alpha. \tag{5c}$$

See Appendix 1 for detailed derivation of (5b). Given ρ , l and the cost parameters, the objective is to find (τ^*, α^*) that maximizes the expected profit EP .

3. The Optimal solution

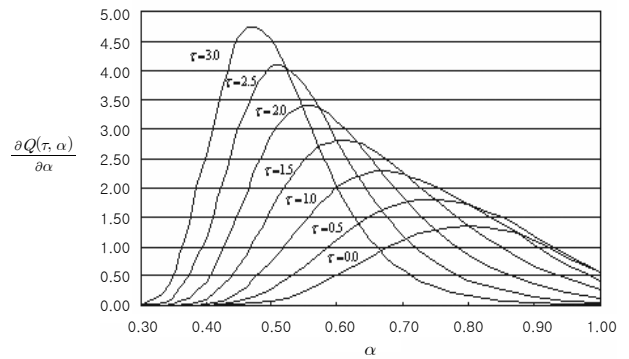
It can easily be shown that maximizing EP is equivalent to maximizing the transformed expected profit

$$TEP = Q(\tau, \alpha) - (b\tau + c\alpha\tau + d\alpha) \tag{6}$$

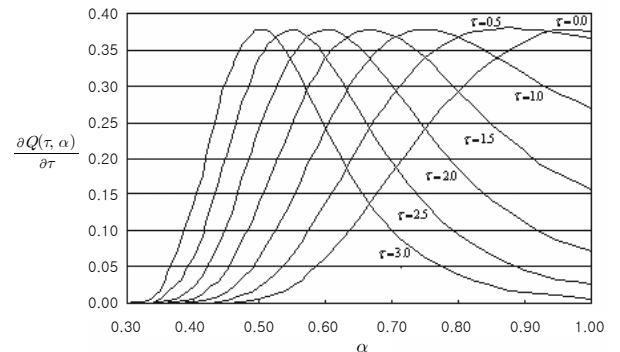
where $b = B/(R + S)$, $c = C/(R + S)$ and $d = (Cl + D)/(R + S)$. Let $G(\tau, \alpha)$ and $H(\tau, \alpha)$ be the first derivatives of TEP with respect to τ and α , respectively, i.e., $G(\tau, \alpha) = \partial TEP/\partial \tau$ and $H(\tau, \alpha) = \partial TEP/\partial \alpha$. Then

$$G(\tau, \alpha) = \frac{\partial Q(\tau, \alpha)}{\partial \tau} - (b + c\alpha), \tag{7a}$$

$$H(\tau, \alpha) = \frac{\partial Q(\tau, \alpha)}{\partial \alpha} - (d + c\tau). \tag{7b}$$



(a) $\frac{\partial Q(\tau, \alpha)}{\partial \alpha}$ with $\rho = 0.95$ and $l = 3$



(b) $\frac{\partial Q(\tau, \alpha)}{\partial \tau}$ with $\rho = 0.95$ and $l = 3$

Figure 2. Graphs of $\frac{\partial Q(\tau, \alpha)}{\partial \alpha}$ and $\frac{\partial Q(\tau, \alpha)}{\partial \tau}$

The functional forms of the first and second derivatives of $Q(\tau, \alpha)$ are given in Appendix 2. Numerical studies over the reasonable range of (τ, α) indicate that $\partial Q(\tau, \alpha)/\partial \tau$ and $\partial Q(\tau, \alpha)/\partial \alpha$ are nonnegative. <Figure 2> depicts the graphs of $\partial Q(\tau, \alpha)/\partial \tau$ and $\partial Q(\tau, \alpha)/\partial \alpha$ when $\rho = 0.95$ and $l = 3$. Since both $b + c\alpha$ and $d + c\tau$ are positive, there

may exist a value of (τ, α) which makes the values of either $G(\tau, \alpha)$ or $H(\tau, \alpha)$ equal to zero. If there exists (τ^2, α^2) that makes both $G(\tau, \alpha)$ and $H(\tau, \alpha)$ equal to zero, it is necessary to see whether the Hessian matrix evaluated at (τ^*, α^*) is negative definite. If not, one may compare the values of TEP evaluated at (τ, α) , where either $G(\tau, \alpha)$ or $H(\tau, \alpha)$ is equal to zero, to obtain the optimum solution.

Suppose that there exists (τ^*, α^*) satisfying $G(\tau, \alpha) = 0$ and $H(\tau, \alpha) = 0$. If the following matrix evaluated at (τ^*, α^*) is negative definite, then at least the local maximum is guaranteed :

$$\begin{bmatrix} \frac{\partial^2 TEP}{\partial \tau^2} & \frac{\partial^2 TEP}{\partial \tau \partial \alpha} \\ \frac{\partial^2 TEP}{\partial \tau \partial \alpha} & \frac{\partial^2 TEP}{\partial \alpha^2} \end{bmatrix} \quad (8)$$

where $\frac{\partial^2 TEP}{\partial \tau^2} = \frac{\partial^2 Q(\tau, \alpha)}{\partial \tau^2}$, $\frac{\partial^2 TEP}{\partial \alpha \partial \tau} = (\frac{\partial^2 Q(\tau, \alpha)}{\partial \tau \partial \alpha}) - c$ and $\frac{\partial^2 TEP}{\partial \alpha^2} = \frac{\partial^2 Q(\tau, \alpha)}{\partial \alpha^2}$.

It requires a tedious iterative procedure to obtain the solution (τ^*, α^*) of Equation (7a) and Equation (7b). The graphs of $G(\tau, \alpha)$ and $H(\tau, \alpha)$ may help find the numerical value of (τ^*, α^*) . These graphs may be prepared in advance for typical values of ρ and l . The procedure of finding the numerical value of (τ^*, α^*) is as follows :

- i) First find values of (τ, α) that satisfy $G(\tau, \alpha) = 0$.
- ii) Next find the values of (τ, α) that satisfy $H(\tau, \alpha) = 0$.
- iii) When there exists a value of (τ, α) at which $G(\tau, \alpha) = 0$ and $H(\tau, \alpha) = 0$, check if matrix (8) evaluated at this value is negative definite. If this optimality condition is satisfied, we have the optimal solution and stop. Otherwise, continue to the next step.
- iv) List up the values of (τ, α) that satisfy either $G(\tau, \alpha) = 0$ or $H(\tau, \alpha) = 0$, and calculate TEP for these values. Select the value of (τ, α) which gives the largest TEP . Denote this value by (τ^o, α^o) .
- v) Select four points $(\tau^o -, \alpha^o -)$, $(\tau^o -, \alpha^o +)$, $(\tau^o +, \alpha^o -)$ and $(\tau^o +, \alpha^o +)$ in the neighborhood of (τ^o, α^o) . Calculate and compare TEP for the five points including (τ^o, α^o) . Set the value of (τ, α) with maximum TEP as (τ^o, α^o) .
- vi) Repeat the search procedure v) until no improvement in TEP is realized.

4. Numerical example

In this section, an illustrative numerical example is provided to demonstrate the proposed model and perform numerical

analyses to understand its properties. Consider a process where copper plating is performed on a panel that consists of several electronic parts. Each part contains holes for later connection with various electrical circuits. The plating thickness should be greater than or equal to the lower specification limit $12\mu\text{m}$ for both on the plane surface and on the curved surface in the hole to implement desired electrical characteristics. Let the plating thickness on the plane surface is normally distributed with the standard deviation $4\mu\text{m}$. Suppose the correlation coefficient between the plating thickness on the plane surface and that on the curved surface is $\rho = 0.95$. And we have the revenue of $R = \$8.0$ for each conforming part and suffer a loss of $S = \$2.0$ for each scrapped part. The unit material cost for the plating process is $B = \$1.0$. The area of the curved surface in the hole is one-tenth of the plane surface, i.e., $C = \$0.1$ and $C/B = C/B = 0.1$. The cost of chemical catalysts for adjusting the ratio α is $\$2.7$. The main objective is to determine the target plating thickness on the plane surface and the ratio α so that the expected total profit would be maximized.

The problem can be simplified to finding (τ^*, α^*) that maximizes TEP with $\rho = 0.95$, $l = 3$, $b = 0.1$, $c = 0.01$, and $d = 0.3$. Finding the optimal solution to the problem involves a great deal of computational resources mainly due to the complex nature of evaluating the TEP function. The model may efficiently be solved using popular mathematical software such as MathCAD and Matlab. The maximum expected profit of 0.4551 may be obtained at $(\tau^*, \alpha^*) = (1.9052, 0.9333)$ where the first derivatives with respect to τ and α are zero. Further, the Hessian matrix evaluated at (τ^*, α^*) is negative semi-definite, which guarantees the maximum over the region of interest. The surface plot of TEP is depicted in <Figure 3>.

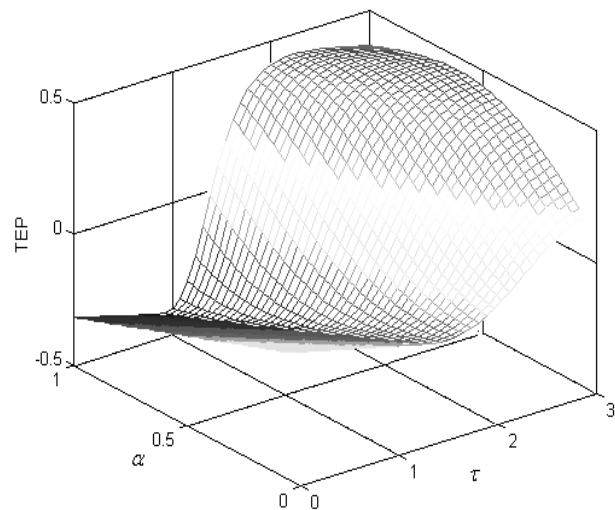
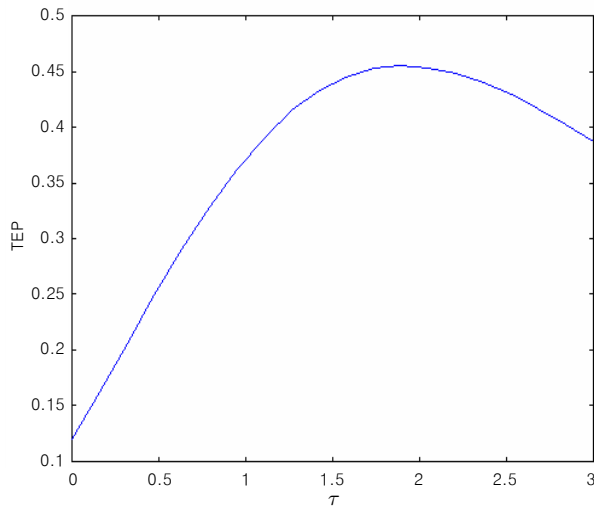
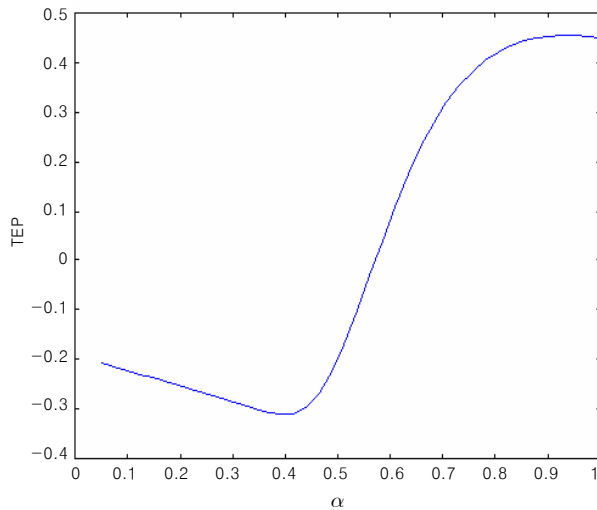


Figure 3. Surface plot of TEP function

The behavior of TEP function with respect to τ evaluated at $\alpha = \alpha^*$ and that with respect to α at $\tau = \tau^*$ are shown in <Figure 4>.



(a) TEP with respect to τ at $\alpha = \alpha^*$



(b) TEP with respect to α at $\tau = \tau^*$

Figure 4. Behavior of TEP Function

Finally, sensitivity analyses have also been conducted to investigate the effects of process parameters on the expected total cost. The behavior of TEP with respect to the correlation coefficient ρ is depicted in <Figure 5>. It is intuitive that the expected total profit may increasingly be accrued as the correlation between X and Y is stronger. On the other hand, Figure 6 depicts the behavior of TEP with respect to the lower specification limit. The result confirms our expectation that a larger amount of plating material may be required as the lower specification limit l becomes greater, and thus the expected total profit may decrease. Furthermore, the location of optimal solution (τ^*, α^*) has also been observed for various values of ρ and l . It is worth noting that the correlation coefficient has an insignificant effect on the location of optimal solution. Consequently, the parameters have only affected the expected total profit without having a significant impact on the location of optimal solution.

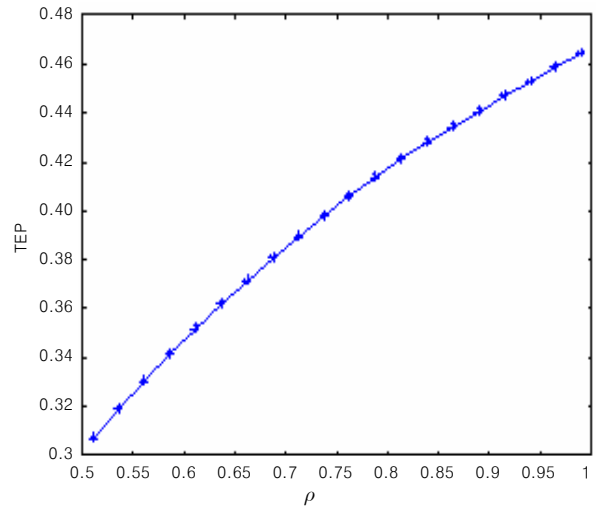


Figure 5. Sensitivity analysis with respect to ρ evaluated at (τ^*, α^*)

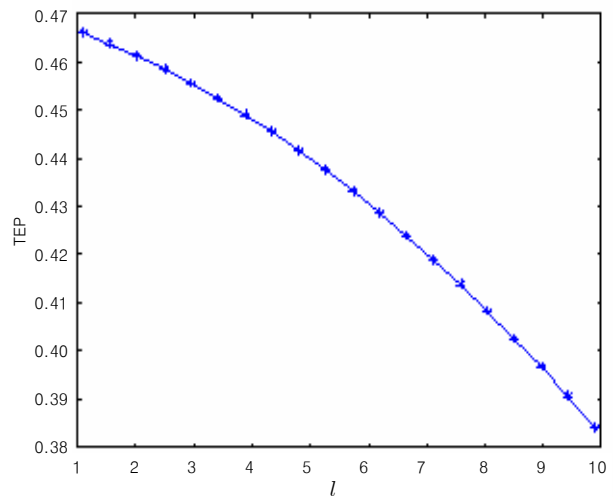


Figure 6. Sensitivity analysis with respect to l evaluated at (τ^*, α^*)

5. Conclusion

This article investigates the problem of determining the optimum parameter values of metal plating thickness on the surfaces of an electronic part. One of the main concerns associated with metal plating processes is to determine the optimum plating thickness on the surfaces. In addition, it is also desirable to make the plating thickness uniform over the surface to be plated. Proposed is an economic model for determining the most profitable parameter values for plating thickness and uniformity. It is interesting to note that an increased expected profit has been realized for a stronger correlation between dimensions of plating thickness on the plain surface and curved surface. Furthermore, it may be noticed,

from the expression of correlation coefficient, that the correlation may become stronger by reducing the variations due to random errors. Consequently, it is important to focus our attention on how to reduce the variations due to random errors to gain a higher profit.

Determining the optimum process target values has long been studied in the context of canning or filling problems. The problem studied in this paper may also be seen as a variation of canning problems. However, the profit model needs to be constructed in a different way for different manufacturing processes so that the specific nature of corresponding process may be reflected. For instance, the proposed model in this paper incorporates economic impacts of the uniformity of plating thickness over the surface to be plated. Similarly, economic models to determine the optimum process settings need to be further investigated for other manufacturing processes by integrating various economic aspects unique to the process under study.

Appendix 1. Derivation of (5b)

Let Z_1 and Z_2 be standardized random variables of X and Y , respectively. Then Z_1 and Z_2 have a standard bivariate normal distribution with correlation coefficient ρ . And

$$\begin{aligned} Q(\tau, \alpha) &= \Pr[X \geq 0, Y \geq 0] \\ &= \Pr\left[Z_1 \geq -\tau, Z_2 \geq -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right] \\ &= 1 - \Pr[Z_1 \leq -\tau] - \Pr\left[Z_2 \leq -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right] \\ &\quad + \Pr\left[Z_1 \leq -\tau, Z_2 \leq -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right] \\ &= \Pr[Z_1 \leq \tau] - \Pr\left[Z_2 \leq -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right] \\ &\quad + \Pr\left[Z_1 \leq -\tau, Z_2 \leq -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right] \\ &= \Phi(\tau) - \Phi\left(-\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}\right) \\ &\quad + \Psi\left(-\tau, -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}; \rho\right), \end{aligned}$$

where $\Psi(\cdot, \cdot; \rho)$ is the standard bivariate normal distribution function with correlation coefficient ρ . Note that the standard bivariate normal density function $\psi(u, v; \rho)$ can be written as

$$\psi(u, v; \rho) = \phi(u) \frac{1}{\sqrt{2\pi(1-\rho^2)}} \exp\left[-\frac{1}{2(1-\rho^2)}(v-\rho u)^2\right]$$

By changing u and v with appropriate variables and taking integration of $\psi(u, v; \rho)$, it can be shown that

$$\Psi\left(-\tau, -\rho\left\{\tau - \left(\frac{1}{\alpha} - 1\right)l\right\}; \rho\right)$$

$$= \int_{-\infty}^0 \phi(x-\tau) \Phi\left(-\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx$$

Appendix 2. Functional forms of the first and second derivatives for $Q(\tau, \alpha)$

$$\begin{aligned} \frac{\partial Q(\tau, \alpha)}{\partial \tau} &= \phi(\tau) + \rho \phi\left(\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) \\ &\quad + \int_{-\infty}^0 (x-\tau) \phi(x-\tau) \Phi\left(-\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx, \end{aligned}$$

$$\begin{aligned} \frac{\partial Q(\tau, \alpha)}{\partial \alpha} &= \frac{\rho l}{\alpha^2} \left\{ \phi\left(\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) \right. \\ &\quad \left. - \frac{-1}{\sqrt{1-\rho^2}} \int_{-\infty}^0 \phi(x-\tau) \phi\left(\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx \right\}, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 Q(\tau, \alpha)}{\partial \tau^2} &= \\ &= -\tau \phi(\tau) - \rho^3 \left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right] \phi\left(\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) \\ &\quad - \int_{-\infty}^0 [1 - (x-\tau)^2] \phi(x-\tau) \Phi\left(-\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 Q(\tau, \alpha)}{\partial \tau \partial \alpha} &= \\ &= -\frac{\rho^3 l}{\alpha^2} \left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right] \phi\left(\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) - \frac{\rho l}{\alpha^2 \sqrt{1-\rho^2}} \\ &\quad \int_{-\infty}^0 (x-\tau) \phi(x-\tau) \Phi\left(-\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 Q(\tau, \alpha)}{\partial \alpha^2} &= -2 \frac{\rho l}{\alpha^3} \left\{ \phi\left(\rho\left[\tau - \left(\frac{1}{\alpha} - 1\right)l\right]\right) \right. \\ &\quad \left. - \frac{1}{\sqrt{1-\rho^2}} \int_{-\infty}^0 \phi(x-\tau) \phi\left(\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx \right\} \\ &\quad - \frac{\rho^3 l^2}{\alpha^4} \left\{ - (1-\rho^2)^{2/3} \int_{-\infty}^0 \left[x - \left(\frac{1}{\alpha} - 1\right)l\right] \right. \\ &\quad \left. \phi(x-\tau) \phi\left(\frac{\rho[x-(1/\alpha-1)l]}{\sqrt{1-\rho^2}}\right) dx \right\}. \end{aligned}$$

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