Journal of the Korean Institute of Industrial Engineers
Vol. 34, No. 4, pp. 386-397, December 2008.

Modeling and Evaluating Inventory Replenishment for Short Life-cycle Products

Ching-Ho Wang[1] · Shih-Wei Lin[2] · Shuo-Yan Chou[1†] · Chun-Hsiang Tsai[1]

[1]Department of Industrial Management, Taiwan University of Science and Technology, Taipei, Taiwan, ROC
[2]Department of Information Management, Chang Gung University, Taoyuan, Taiwan, ROC

Due to the rapid advancement of technologies, a growing number of innovative products with a short life-cycle have been introduced to the market. As the life-cycles of such products are shorter than those of durable goods, the demand variation during the life-cycle adds to the difficulty of inventory management. Traditional inventory planning models and techniques mostly deal with products that have long life-cycles. The assumptions on the demand pattern and subsequent solution approaches are generally, not suitable for dealing with products with short life-cycles. In this research, inventory replenishment problems based on the logistic demand model are formulated and solved to facilitate the management of products with short life-cycles. An extended Wagner-Whitin approach is used to determine the replenishment cycle, schedules and lot-sizes.

Keywords: Inventory Replenishment, Logistic Model, Short Product Life-cycle, Life-cycle Cost Evaluation, Present Value

1. Introduction

In recent years, manufacturing industries are facing increasing and constant challenges from their competitors as well as customers, which prompts them toward continuous development, design and manufacture of new products to meet the diverse demand. With the rapid advancement of technologies, more innovative products with short life-cycles are developed and introduced to the market. Such products appear and vanish in the market at a much faster pace than conventional modeling and planning can handle. With the length of the life-cycle being compressed, the variation among different stages of the demand needs to be considered for modeling and planning inventory replenishment.

In the field of inventory management, the objective is to determine when and how to order so that the inventory cost can be minimized. In the U.S.A., businesses invested in inventory at about 15 to 20 percent of the annual GDP over the past decade (Thomas, 1995). With suppliersand buyers located in different countries or areas, the lead-time for producing goods with short life-cycles, referred to as the open-to-buy (OTB) period, is usually seven to eight months ahead of the season. Planning of demand and delivery of such products becomes even more crucial to ensure profit.

In 1970s, the energy crisis had deeply affected the world, especially in the escalation of inflation rates. During the period of 1993 to 1995 in China the annual inflation rate was approximately seventeen percent on the average (Chen, 1998). Chen also indicated that the effects of inflation and time-value of money for inventory models are critical however, they were not considered explicitly as parameters in most of the existing models. The value of product due to the influence of the time-value of money may decrease significantly. Both the time-value of money and inflation

† Corresponding author : Shuo-Yan Chou, Department of Industrial Management, Taiwan University of Science and Technology, No. 43, Sec. 4, Keelung Rd., Taipei, 106, Taiwan, R.O.C., Tel : +8862-2737-6342, Fax : +8862-2737-6344,
E-mal : sychou@mail.ntust.edu.tw
Received July 7, 2008; Revision Received August 27, 2008; Accepted August 31, 2008.

should therefore be incorporated in thedecision model to reflect the influence of inflation and discounting rates.

Traditional inventory planning models and techniques deal with products in the mature stage of their long product life-cycles. In such a stage, demand is assumed to be constant or probabilistically stable, e.g., following normal distribution. The assumption and solution approaches may not be suitable for products with short life-cycles. It is observed that in consequence of the imitation effect(Rogers, 1983; Chou *et al*., 2001), the number of buyers and the demand for a certain product are influenced by the previous turnouts. Such dynamics of demand along product life- cycles can be modeled with logistic models rather than a fixed level with normal distribution.

Taking account of the limitations of existing works as described above, this research formulates and solves an inventoryreplenishment problem based on a logistic demand model-characterizing the lifecycle of the demand-to facilitate the management of products of short life-cycles with the consideration of shortage and time discounting. The characteristics of the logistic model enable us to model demand and derive inventory policy more analytically.

2. Literature Review

One of the assumptions for the basic economic order quantity (EOQ) model is fixed demand. Many proposed inventory models extended this assumption. Donaldson (1977) proposed a replenishment policy for inventory items having a linear trend in demand without shortages over a finite time horizon. Resh *et al*. (1976), Silver (1979), Brosseau (1982), Mitra *et al*. (1984), and Goyal *et al*. (1992) also designed approximate methods for linearly time-dependent demand. Some research works (Barbosa and Friedman, 1979; Friedman, 1981) have generalized the previous one by considering different functional forms for the demand rate, such as power functions. Some research considers inventory-level-dependent demand in which the demand rate of a product is a function of the initial inventory level and dependent on the instantaneous inventory level.Chung (2003) proposed an algorithm for solving inventory models with inventory-level- dependent demand rates. Zhou and Yang (2003) developed an optimal replenishment policy for items with inventory-level-dependent demand and fixed lifetime under the LIFO policy. Balkhi and Benkherouf (2004) proposed an inventory model for deteriorating items with stock-dependent and time-varying demand rates. Readers can refer to the paper (Urban, 2005) for an excellent review of this kind of inventory demand. Since the demand rate varies with time, both the ordering quantity and the ordering cycle should vary with the passage of time so as to obtain the minimum inventory cost. Keeping the ordering cycle constant and increasing the ordering size will result in a higher inventory cost than the inventory cost of adjusting both the ordering quantity and the ordering cycle.

The cost function associated with the inventory system includes replenishment cost, inventory carrying cost and shortage cost. In the basic EOQ model, shortages are not permitted. Thus there are only two costs in the analysis of an EOQ inventory system. The purchasing cost or unit production cost is constant, and it is subsumed under this system. The realistic model is one which considers a separate inflation rate for each of its cost components. Hence, Misra (1979) divided these costs into two classes; class one consists of all those costs which increase at the inflation rate that prevails in the company, which is internal inflation rate class two consists of those that increase at the inflation rate of the general economy or of the supplier company, which is external inflation rate. Many articles have not considered inflation and time-value of money as parameters of the system (Resh *et al*., 1976; Donaldson, 1977; Barbosa and Friedman, 1979; Firedman, 1981; Brosseau, 1982; Goyal *et al*., 1992).

Hadley (1964)did some research on the effects of a wide range of values of the pertinent parameters, namely : fixed cost of placing an order, rate of return, the unit cost of the item and the inventory carrying cost on the inventory system. The final result did not differ significantly over a very wide range of values of the pertinent parameters, but in extreme circumstances a sizably different resultcan be obtained. Trippi and Lewin (1974)researched the present value of discounted costs of the basic EOQ model over an infinite planning horizon. In their final conclusion, the present value model seems much more robust than the classical average cost per unit time model in the context of errors in the EOQ model. Buzacott (1975) developed an EOQ model that incorporated uniform inflation for all the costs and minimized the average annual cost into

Table 1. Classification of products with short life-cycles and corresponding examples

Short product life-cycle	Examples
Recurring seasonal products	Halloween candy, snow blowers, suntan lotion, Valentine's chocolate, seasonal fruits etc.
Seasonal fashion products	spring wardrobe items, swimsuits, etc.
Fad products	music, videotapes, books/magazine, electronic watch, movie, hula hoops, office products, high-tech product (disk drives, compact disk, CPU), etc
Upgrade products	upgrades and new versions of software packages

the model, $b(t) = b_0 e^k$, where b_0 and k are the cost at time $t = 0$ and the inflation rate,respectively. He also showed that if the unit price is only changed at the beginning of each cycle, the objective function should be maximization of profit instead of minimization of cost; further, the pricing continuously increased at the inflation rate. Bierman and Thomas (1977) simultaneously considered inflation and time-value of money, with and without time discounting, in their policies. These factors have significant deviation when the net discount rate is high, but only one inflation rate for all costs. Misra (1979) presented an EOQ model that considered the time-value of money and different inflation rates for various costs associated with an inventory system. Datta and Pal (1991) studied both the effects of inflation and the time-value of money with a linear time-dependent demand rate and allowing shortages.

From the perspective of the short product life-cycle, a remarkable feature is that product demand increases and decreases rapidly during a short period of time. Such products are only sold for a limited or finite period of time, especially high-tech products, seasonal fashion products, fad products and so on (Lapide, 2001). Examples are shown in <Table 1>.

Based on research by Rogers (1983) on markets with a fixed number of potential buyers, the buyers are distributed symmetrically across the product life-cycle and can be classified into five categories. The market innovators of the product are only about 2.5% of the total buyers, which will create about 13.5% of so-called early adopters by the imitation effect. The imitation effect continues to create 34% of the early majority and the late majority, respectively, and 16% of the laggards. The shape of this distribution is similar to the curve that is used to characterize the demandin a traditional product life-cycle (Moore, 1993; Baxter, 1995), as shown in <Figure 1>.

Mathematical models have been proposed for this kind of dynamic demand change, including the model of diffusion of durables by Bass (1969), BRANDAID model by Little (1975), and the modelof channel structure by McGuire and Staelin (1983). Furthermore, Mahajan *et al*. (1990) provided a review and directions for research about new product diffusion models in marketing. Their review focused on the Bass model. Ho *et al*. (2002) and Kuman and Swaminathan (2003) provided a model of new product under supply constraints. Both of them also focused on the Bass model.

Life-cycle curves as such can be divided into two halves-the growth and the decline parts-and modeled by exponential distribution. However, the exponential distribution has a memory-less characteristic, which

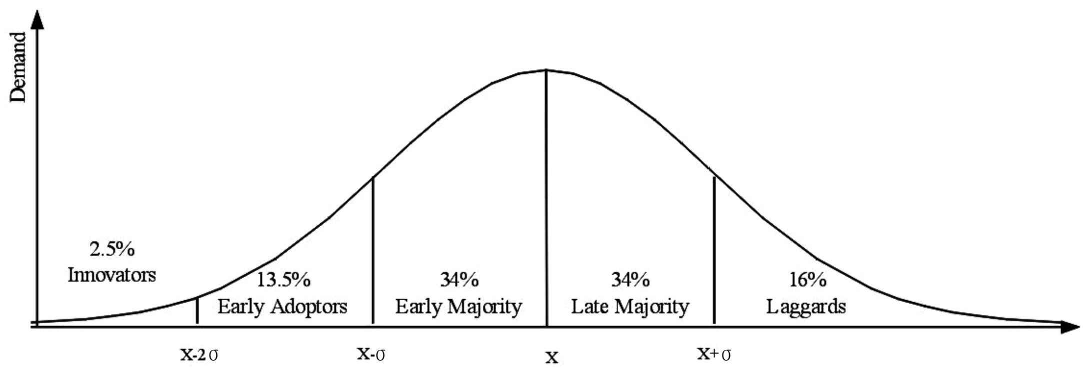


Figure 1. Imitation effects among the buyers

does not fit well with dynamic markets where demand variation between periods has a tight relationship. Instead, the logistic model which finds thousands of applications in various areas (Lilien *et al*., 1992; Urban and Hauser, 1993; Banks, 1994; Marchetti *et al*., 1996; Meyer *et al*., 1999) is adopted to model the demand over the short life-cycle of fashion-related products. The logistic model which is a special case of Bass model, consider the imitation effect and ignore the innovation effect used in Bass model. That is to say, the logistic model takes no notice of sales promotion.

In a simple and widely used exponential growth model, the growth of population $p(t)$ is proportional to the population. It can be written as a differential equation as follows :

$$\frac{dp(t)}{dt} = \alpha p(t) \quad (1)$$

which can be rewritten as

$$p(t) = e^{\alpha t + \beta} \quad (2)$$

where α and β are respectively a constant growth rate and a location (horizontal shifting) parameter. Population $p(t)$can also be considered as a particular set of people rather than general population, for example : participants in the spread of knowledge, people who fall ill during an epidemic, and buyers of certain products. A shortcoming of this model in practical application is that as t goes to infinity, the population goes to infinity.

The logistic curve begins with α and $p(t)$ of the exponential model; there is a negative feedback term $\lfloor 1 - p(t)/k \rfloor$ that slows down the growth as an upper limit k is reached :

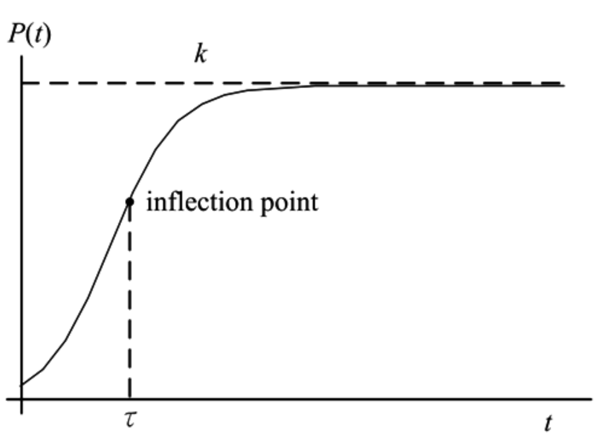


Figure 2. An example of the logistic curve

$$\frac{dp(t)}{dt} = \alpha p(t)[1 - p(t)/k] \quad (3)$$

For values of $p(t) << k$, Equation (3)closely resembles exponential growth. As the population $p(t)$ approaches k, the feedback term causes the rate of growth to reduce to zero as the population $p(t)$ approaches the limit k, producing an S-shaped curve. <Figure 2> illustrates a classical logistic growth curve.

Equation (3) can be rewritten as

$$p(t) = \frac{k}{1 + e^{-\alpha t - \beta}}, \ \alpha > 0, \ \beta < 0 \quad (4)$$

where α and β function as the growth rate parameter and the location parameter, respectively, and k gives the limit asymptotically approached by the logistic function, given that constant $k > 0$ and that the value of (- αt -) is decreasing as t becomes larger (Ostrosky and Koch, 1979). The slope of a logistic curve in the form $p(t) = k/(1 + e^{-\alpha t - \beta})$ can be determined by taking the first derivative of the curve function $p(t)$.

The growth pattern of a wide range of phenomena can be usefully approximated by logistic curves, for example, the growth of particular industries such as PCs and TVs; the growth of populations of humans, animals, insects; the mastery by students of particular modules of knowledge; technology adoption; and so on, all of which show very rapid rates of growth, followed by a decline in the rate of growth and an eventual asymptotic approach to its limit (Ostrosky and Koch, 1979).

Similarly, the demand growth of high-tech products and seasonal fashion products with short life-cyclesis modeled as population growth of humans who demand the product in question. With the accumulated demand for the product at time t denoted as $D(t)$, the rate at which the demand grows can be formulated as a differential equation :

$$dD(t)/dt = \alpha D(t)(1 - D(t)/\overline{D}),$$
$$D(0) = D_0 \quad (5)$$

where α is a fixed growth rate, $\overline{D}$ is the maximum accumulated demand and D_0 is the initial demand and the values of α and $\overline{D}$ vary with the types of products and geographical areas. If there is not any extra promotion employed or technological breakthrough in the newly introduced product, the values of α, β and $\overline{D}$ for the demand model can be estimated based on historical data of similar products.

Solving the ordinary differential equation satisfying Equation(5) results in a logistic growth function :

$$D(t)=\frac{\overline{D}}{1+e^{-\alpha t}(\frac{\overline{D}}{D_0}-1)}=\frac{\overline{D}}{1+e^{-\alpha t-\beta}} \qquad (6)$$

which gives the famous S-shaped curve as shown in <Figure 3>. The demand rate on the accumulated demand increases initially until it reaches the inflection point τ of the curve, and then begins to decrease as it approaches the maximal demand. It is also noted that β is a location parameter which shifts the logistic curve by α/β in time but does not affect its shape. It can be shown by simple derivation that $t = -\beta/\alpha$, $\beta < 0$, gives the inflection point of the logistic growth function $D(t)$.

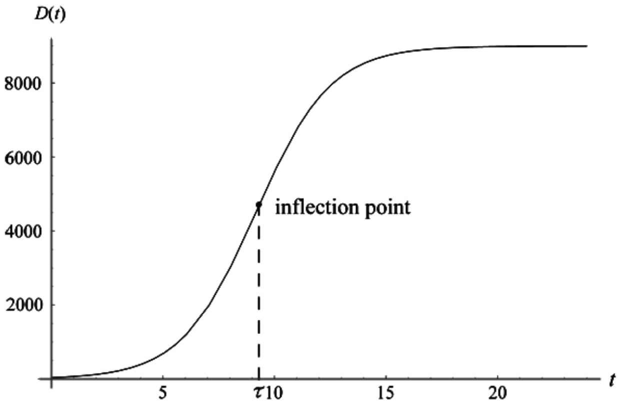


Figure 3. The logistic model for fashion-related products

These changes in the accumulated demand can be visualized much more easily with the first derivative of the logistic curve, which is a bell-shaped curve similar but not identical with the normal distribution curve as shown in <Figure 4>. The bell-shaped curve represents the demand rate at different times. The demand rate at time t, denoted as $d(t)$, can be calculated by taking the first derivative of $D(t)$.

$$d(t)=\frac{dD(t)}{dt}=\frac{\alpha\overline{D}e^{-\alpha t-\beta}}{(1+e^{-\alpha t-\beta})^2} \qquad (7)$$

The complete logistic demand model therefore contains the introduction, growth, maturity and decline of the demand for the product. The maturity of the demand begins at τ. The unique characteristic for this

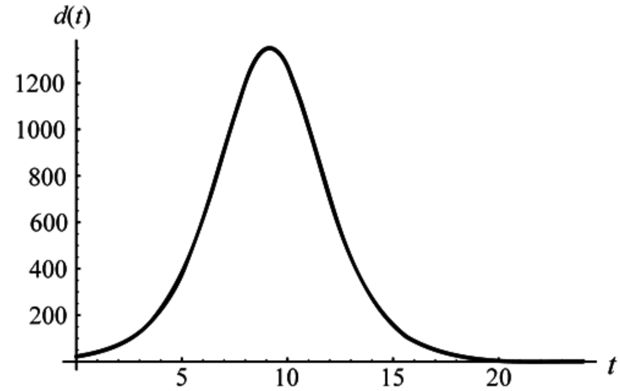


Figure 4. The first derivative of the logistic curve

model is that it represents the short life-cycle of high-tech products, seasonal fashion products and so on and provides a fundamental yet meaningful basis to model demands and analyze subsequent impact.

3. Mathematical Model

The developed mathematical model of the inventory replenishment problem is based on the following assumptions :

- The system operates for a prescribed time period H.
- The inventory is continuously reviewed.
- The lead time is zero. Because the demand model is a deterministic one, that is, with a known demand, a contract can be made between the manufacturer and the distributor/retailer. Under the contract, the manufacturer would follow the delivery or replenishment schedule to manufacture the required amount of products and deliver them to the distributor/retailer by the due date. In other words, the replenishment does not happen when the products are exhausted; instead it follows the planned schedule derived through the optimization procedure in this study. The lead time for the replenishment can thus be assumed to be zero.
- A single item is considered, whose accumulated demand, $D(t)$, and demand rate, $d(t)$, follows the logistic demand, which is a continuous function.
- The internal and the external inflation rates can be summarized by a unique inflation rate, i r represents the discount rate, and $r-i > 0$.
- Both initial and final inventories are zero. Moreover,

shortages are allowed and fully backordered. The replenishment quantity is replenished instantaneously, as shown at point p in <Figure 5>. For the case where backorders are not allowed, the shortage cost can be set at a very high value, which will prevent the occurrence of the shortage. Thus, the proposed model is applicable for the system with or without shortages. For the case where customers are willing to suspend their purchases until a later time, for example, for certain fashion athletic shoes, the distributor/retailer has to provide the customers with certain incentives to retain such demand. The burden of such incentives is a main contributor to the shortage cost.

- The inventory level, shortage level, replenishment quantity, logistic demand model, and net discount rate are assumed to be continuous variables and the other variables are assumed to be discrete variables.
- The total cost function includes the fixed purchasing/setup cost, purchasing/production cost, holding cost, and shortage cost.

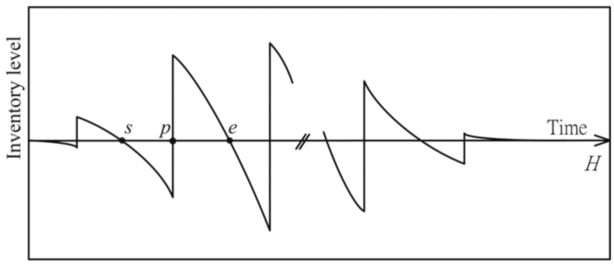


Figure 5. Graphical representation of the fluctuation of inventory levels

The notations used throughout this study are defined as follows :

$d(t)=\dfrac{\alpha\bar{D}e^{-\alpha t-\beta}}{(1+e^{-\alpha t-\beta})^2}$, denotes the demand rate in the logistic model, where t is time, $\bar{D}$ is the maximum accumulated demand, α is the fixed growth rate, β is the location parameter;

R = net discount rate of inflation. $PW(P_t)=P_t(e^{-r}e^{-i})^t=P_te^{-Rt}$, where P_t is the value of P at time t and r-$i>0$;

F_0 = fixed purchasing/setup cost for each replenishment

P_0 = unit purchasing/production cost for each of the products

H_0 = unit holding cost, i.e., cost per unit and per time period

S_0 = unit shortage cost, i.e., cost per unit and per time period

p = time at the replenishing point

s = starting time of the replenishment cycle at which shortage is about to occur

e = ending time of the replenishment cycle at which inventory level reaches zero;

$I(t)$ = inventory level at time t

$S(t)$ = shortage level at time t

$Q(p)$ = replenishment quantity at various replenishment point p.

First, consider the shortage level, $S(t)$, at time t, $s \leq t \leq p$. It is utterly depleted by the effect of the logistic model during the period for which inventory level is negative. The variable state of $S(t)$ with respect to t is determined by the following first-order differential equation :

$$\begin{cases}\dfrac{d}{dt}S(t)=\dfrac{\alpha\bar{D}e^{-\alpha t-\beta}}{(1+e^{-\alpha t-\beta})^2},\ s\leq t\leq p,\\ S(s)=0.\end{cases} \tag{8}$$

Solving the differential equation satisfying Equation (8) results in a function :

$$S(t)=\int_s^t\frac{\alpha\bar{D}e^{-\alpha u-\beta}}{(1+e^{-\alpha u-\beta})^2}du,\ s\leq t\leq p. \tag{9}$$

Similarly, the inventory level, $I(t)$, at time t, $p\leq t\leq e$ is represented by the following differential equation :

$$\begin{cases}\dfrac{d}{dt}I(t)=-\dfrac{\alpha\bar{D}e^{-\alpha t-\beta}}{(1+e^{-\alpha t-\beta})^2},\ p\leq t\leq e,\\ I(e)=0.\end{cases} \tag{10}$$

Solving the differential equation satisfying Equation (10) results in a function :

$$I(t)=\int_p^e\frac{\alpha\bar{D}e^{-\alpha u-\beta}}{(1+e^{-\alpha u-\beta})^2}du-\int_p^t\frac{\alpha\bar{D}e^{-\alpha u-\beta}}{(1+e^{-\alpha u-\beta})^2}du=\int_t^e\frac{\alpha\bar{D}e^{-\alpha u-\beta}}{(1+e^{-\alpha u-\beta})^2}du,\ p\leq t\leq e. \tag{11}$$

At the replenishment point p, the replenishment quantity is computed as

$$Q = S(p) + I(p) = \int_s^p \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + \int_p^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt = \int_s^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt. \tag{12}$$

The present worth of the fixed purchasing/setup cost at the replenishment point p can be expressed as

$$FC = F_0 e^{-Rp}. \tag{13}$$

The present worth of the purchasing/product cost at the replenishment point p can be expressed as

$$PC = P_0 e^{-Rp} Q = P_0 e^{-Rp} \int_s^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt. \tag{14}$$

The present worth of the holding cost over the period $[p, e]$ can be expressed as

$$HC = H_0 \int_p^e e^{-Rt} I(t) dt = H_0 \int_p^e e^{-Rt} \int_t^e \frac{\alpha \bar{D} e^{-\alpha u - \beta}}{(1+e^{-\alpha u-\beta})^2} du dt = -\frac{H_0}{R}\left(e^{-Rt} \int_t^e \frac{\alpha \bar{D} e^{-\alpha u - \beta}}{(1+e^{-\alpha u-\beta})^2} du \right)\Bigg|_p^e - \frac{H_0}{R} \int_p^e e^{-Rt} \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt = \frac{H_0}{R} \int_p^e (e^{-Rp} - e^{-Rt}) \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt. \tag{15}$$

The present worth of the shortage cost over the period $[s, p]$ can be expressed as

$$SC = S_0 \int_s^p e^{-Rt} S(t) dt = S_0 \int_s^p e^{-Rt} \int_s^t \frac{\alpha \bar{D} e^{-\alpha u - \beta}}{(1+e^{-\alpha u-\beta})^2} du dt = -\frac{S_0}{R}\left(e^{-Rt} \int_s^t \frac{\alpha \bar{D} e^{-\alpha u - \beta}}{(1+e^{-\alpha u-\beta})^2} du \right)\Bigg|_s^p + \frac{S_0}{R} \int_s^p e^{-Rt} \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt = \frac{S_0}{R} \int_s^p (e^{-Rt} - e^{-Rp}) \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt. \tag{16}$$

The WW approach (Wagner and Whitin, 1958) is a dynamic programming model, which can be used to solve the replenishment schedule and cycle over the planning horizon so that the total cost is minimized. The optimal sequence replenishment schedule can be determined by the WW approach over the planning horizon as given by Equation (17).

$$W_e = \min\{W_s + F(s, p^*, e), 0 \le s < e \le H\}$$

$$W_0 = 0 \tag{17}$$

In the interval $s \le t \le e$, the present worth of the total variable cost for each cycle, defined as the sum of fixed purchasing/setup, purchasing/production, holding and shortage costs, and is given by

$$F(s,p,e) = F_0 e^{-Rp} + P_0 e^{-Rp} \int_s^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + \frac{H_0}{R} \int_p^e (e^{-Rp} - e^{-Rt}) \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + \frac{S_0}{R} \int_s^p (e^{-Rt} - e^{-Rp}) \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt. \tag{18}$$

It is noted that Equation (18) is one-dimensional. For given s and e, the optimal replenishment point p can be obtained from the following equation

$$\frac{\partial F(s,p,e)}{\partial p} = 0,$$

which with full expansion equals

$$-RF_0 e^{-Rp} - RP_0 e^{-Rp} \int_s^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt - H_0 e^{-Rp} \int_p^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + S_0 e^{-Rp} \int_s^p \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt = 0. \tag{19}$$

If the cost function is convex, the following sufficient condition should satisfy Equation (18)so that the minimum cost is guaranteed in the interval $[s, e]$.

$$\frac{\partial^2 F(s,p^*,e)}{\partial p^2} > 0$$

where

$$R^2 F_0 e^{-Rp} + R^2 P_0 e^{-Rp} \int_s^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + RH_0 e^{-Rp} \int_p^e \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt - H_0 e^{-Rp} \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} - RS_0 e^{-Rp} \int_s^p \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} dt + S_0 e^{-Rp} \frac{\alpha \bar{D} e^{-\alpha t - \beta}}{(1+e^{-\alpha t-\beta})^2} > 0. \tag{20}$$

The minimum present worth of the total cost can be determined by a recursive procedure in the forward method over a prescribed time period H. Then, W_H, minimal present worth of the total cost can be found in the last procedure. The optimal replenishment schedule and cycle can be determined by tracking backward from time H to 0. <Figure 6> profiles the solution procedure for Equation (18), in which T_e and $p^*_{s,e}$ are defined as follows :

T_e = the starting point of the last replenishment cycle from time zero to time e (i.e., $[T_e, e]$), $T_e = 0, 1, 2, \cdots, e-1$, and $e = 1, 2, \cdots, H$,

$p^*_{s,e}$ = the optimal replenishment schedule in cycle $[s, e]$.

```
Step 1  s = 0;
        for (e = 1e ≤ H ; e++) {
            Obtain p* from Equation (19)
            Calculate F(s, p,* e) from Equation (18)
            W_e = F(s, p,* e), p*_s,e = p* and T_e = s
        }
Step 2  for (e = 2 ; e ≤ He++) {
            for (s = 1 ; s ≤ e-1 ; s++) {
                Obtain p* from Equation (19)
                Calculate F(s, p*, e) from Equation (18)
                If W_e ≥ W_s + F(s, p*, e)
                  W_e = W_s + F(s, p*, e), p*_s,e = p* and T_e = s
            }
        }
Step 3   e = H
        while (e ≠ 0) {
            Replenishment cycle = [T_e, e];
            Replenishment schedule = p*_Te,e
            Accumulated total cost = W_e
            e = T_e
        }
```

Figure 6. The solution procedure for the WW approach

4. Numerical Examples

The proposed dynamic programming technique was implemented on a PC with a Pentium IV 1400 MHz CPU, and developed withMathematica 5.0 and Visual Basic 6.0. The solution procedure is illustrated by three examples. The first case tries to model the inventory replenishment for personal digital assistants (PDA), in which shortage and time discounting effects are taken into consideration. The second case models after swimsuits but does not allow shortage due to the characteristics of the product. Other than an extremely high shortage cost, the same data set as the first case is used to demonstrate in contrast the effect of the shortage on the replenishment schedule. The third case considers fashion athletic shoes with a demand model where demand grows at a sharper pace at the beginning of the lifecycle and the demand occurs in a more compact pattern.

Case 1 : A new model of PDA is introduced to the market. The maximal accumulated demand $\overline{D}$ for the model is estimated to be 9,000 for the market, with the planning horizon H equal to 18 months. It is assumed that when the shortage occurs, customers will wait for certain time periods to get the PDA only if price discounts are awarded. The price discount is compounded to the unit shortage cost S_0 which is set to be \$30. Other parameter values for this case are assigned where $\alpha = 0.6$, $\beta = -5.5$, $R = 1\%$ per time period, F_0 = \$15,000 per replenishment, P_0 = \$300, and H_0 = \$3 during the planning horizon. <Table 2> gives the detailed data of the replenishment schedule and the costs for this case. In the optimal schedule, the present worth of the minimal total cost is \$2,564,492, the total quantity over the planning horizon is 8,918, the average unit cost (AUC) is \$287.56, and the optimal number of replenishment cycles is four.

Table 2. The resulting replenishment schedule for Case 1

Cycle	[s, e]	p^*	$F(s, p^*, e)$	W_e	Lot-size
1	[0, 6]	3.2233	355,575	355,575	1,134
2	[6, 9]	6.7858	898,539	1,254,114	3,104
3	[9, 11]	9.3373	697,013	1,951,127	2,477
4	[11, 18]	11.4283	613,365	2,564,492	2,203

Case 2 : A new style of swimsuit is introduced to the market. The maximal accumulated demand $\overline{D}$ for the style is estimated to be 9,000 for the market, with the planning horizon H equalto 18 weeks. It is however assumed that when the shortage occurs, the customer's sale will be lost, that is, shortages are not allowed. The shortage cost S_0 is thus assigned with a very large value, in this case NT\$10,000. Other parameter values for this case are assigned where $\alpha = 0.6$, $\beta = -5.5$, R = 1% per time period, F_0 = NT\$15,000 per replenishment, P_0 = NT\$300, and H_0 = NT\$3 during the planning horizon. <Table 3> gives the detailed data of the replenishment schedule and the costs for this case. In the optimal schedule, the present worth of the minimal total cost is $2,581,890, the total quantity over the planning horizon is 8,919, the average unit cost (AUC) is $289.48, and the optimal number of replenishment cycles becomes five.

Table 3. The resulting replenishment schedule for Case 2

Cycle	[s, e]	p^*	$F(s, p^*, e)$	W_e	Lot-size
1	[0, 5]	0.000	215,563	215,563	646
2	[5, 7]	2.000	373,516	589,079	1,245
3	[7, 9]	3.000	677,623	1,266,702	2,348
4	[9,11]	4.000	699,323	1,966,025	2,477
5	[11, 18]	5.000	615,865	2,581,890	2,203

While case one requires four replenishment cycles, case two requires five. Interestingly that for case one, even with the shortage cost compounded from the occurrence of the shortage, itsaverage unit cost remains lower than that of case two. It is in fact understandable as the inventory holding cost for case two becomes much larger due to the no-shortage constraint. From a different perspective, because the solution space for inventory models allowing shortages is larger than that for inventory models disallowing shortages, the AUC of the former is always smaller than or equal to that of the latter.

Case 3 : A new kind of fashion athletic shoes is introduced to the market. The maximal accumulated demand $\overline{D}$ for the kind of fashion athletic shoes is estimated to be 9,000 for the market, with the planning horizon H equalto 18 weeks. It is assumed that when the shortage occurs, customers will wait for certain time periodsto get the fashion athletic shoes only if price discounts are awarded. The price discount is compounded to the unit shortage cost S_0 which is set to be $15. Other parameter values for this case are assigned where $\alpha = 0.8$, $\beta = -3.5$, $R = 1\%$ per time period, $F_0 = \$7,500$ per replenishment, $P_0 = \$150$, and $H_0 = \$1.5$ during the planning horizon. <Table 4> gives the detailed data of the replenishment schedule and the costs for this case. In the optimal schedule, the present worth of the minimal total cost is $1,302,972 of which the total quantity over the planning horizon is 8,736, the average unit cost (AUC) is equal to $149.15, and theoptimal number of replenishment cycles is three. This case illustrates the resulting of a replenishment schedule with a fewer number and more non-uniform replenishment cycles.

Table 4. The resulting replenishment schedule for Case 3

Cycle	[s, e]	p^*	$F(s, p^*, e)$	W_e	Lot-size
1	[0, 3]	1.1396	306,895	306,895	1,984
2	[3, 5]	3.4214	498,488	805,383	3,354
3	[5, 18]	5.3845	497,589	1,302,972	3,398

A sensitivity analysis is performed based on case one, where uncertain factors are studied for the effects of the variations on the replenishment plans. Two parameters, α and β, characterizing the demand pattern are taken into consideration. The sensitivity analysis of the logistic model of Case 1was performed with respect to two parameters to determine if the proposed optimization procedure is sensitive to the departure from the estimates. Theaccumulated demand and the demand rate under various α and β values, while the rest of parameter values unchanged, are shown in <Figure 7>.

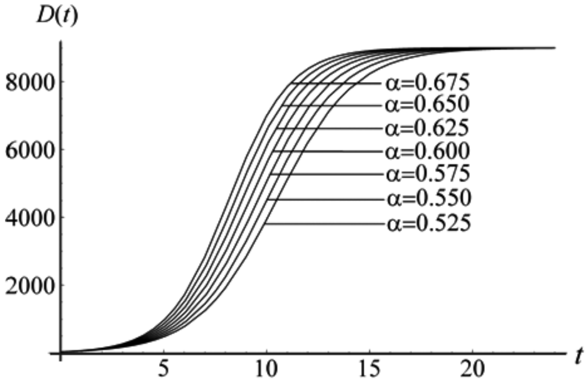
The optimal values of AUC (average unit cost) for the fixed $\Phi = \{\overline{D}, \alpha, \beta, R, P_0, F_0, H_0, S_0\}$ is denoted by AUC_0. The sensitivity of the optimum average unit cost (SAC) can be obtained by computing $[(AUC/AUC_0)-1]*100$. <Table 5> shows the results of the sensitivity analysis for varying α and β.

Table 5. Numerical sensitive analysis with respect to α and β.

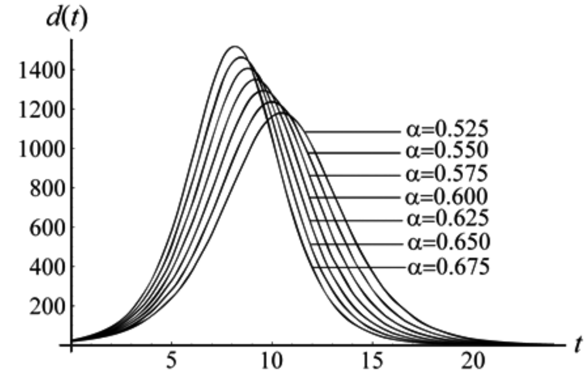
varying α	AUC^*	SAC(%)	varying β	AUC^*	SAC(%)
0.525	294.54	2.43	-6.25	290.07	0.87
0.550	290.57	1.05	-6.00	288.60	0.36
0.575	288.31	0.26	-5.75	287.57	0.01
0.600	287.56	0.00	-5.50	287.56	0.00
0.625	289.05	0.52	-5.25	289.54	0.69
0.650	291.81	1.48	-5.00	293.27	1.98
0.675	295.59	2.79	-4.75	298.67	3.86

The main findings observedfrom the above sensitivity analysis are :

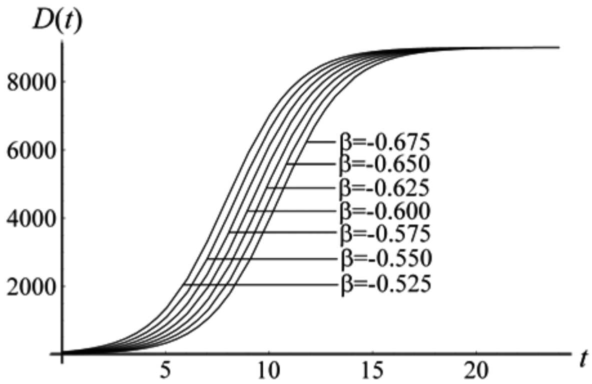
(1) Varying α from 0.525 to 0.675 ($\pm$ 12.5% of the original value 0.6), the percentage change in AUC ranges from 0.26% to 2.79%. This indicates that the AUC from the optimal replenish-



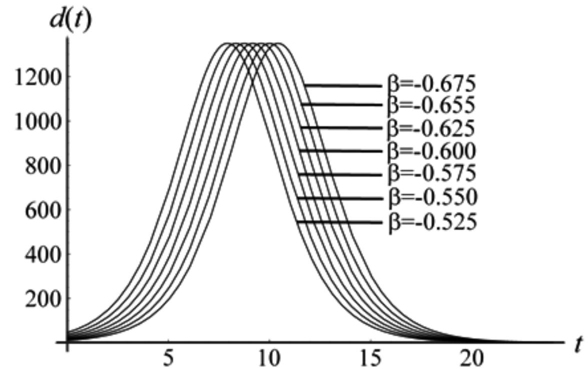
(1) Accumulated demand with various α



(2) Demand rate with various α



(3) Accumulated demand with various β



(4) Demand rate with various β

Figure 7. Theaccumulated demand and the demand rate under various α and β values

ment policy is not sensitive to the variation of α, that is, a demand model where demand grows at a sharper or a smoother pace than the prediction. In other words, if the actual parameter value of α is differentfrom the expected value, the value of ACU-the solution obtained by the WW approach-will not have a large variation.

(2) Varying β from -6.25 to -4.75 ($\pm$ 13.6% of the original value -5.5), the percentage change in AUC ranges from 0.01% to 3.86%. This indicates that the AUC from the optimal replenishment policy is not sensitive to the variation of β. That is, the value of ACU is not sensitive to the variation of the model where demand starts at an earlier or a later point in time than the predicted.

5. Conclusion and Future Research

Traditional inventory planning models and techniques mostly deal with products that have long life-cycles. The traditional assumption of constant demand for durable products is not suitable for modeling products with short life-cycles and subsequent solution approaches. In reality, the constant demand assumption does not hold for high-tech products, seasonal products and fad products. In consequence of the imitation effect, the number of buyers and the demand for the product are influenced by the previous turnouts, which can be modeled by the logistic model. In this research, inventory replenishment problems based on the logistic demand model are formulated and solved to facilitate the management of products with short life-cycles. The logistic model bears properties that match closely with products of short life-cycles. Nevertheless, the model remains applicable for representing products of both long and short life-cycles. Historical data of the product or of a similar product can be used to estimate the parameters, α and β, for the model.

By using the WW approach, the replenishment quantity is restricted to the accumulated demand in some consecutive integral periods. If the replenishment quantity is restricted to be the accumulated demand of integral periods, the WW approach has already obtained the optimal solution. However, if the replenishment quantity is not restricted to be the accumulated demand of integral periods, by using meta- heuristics such as SA and GA approaches, the resulting replenishment quantity may be allocated so that the beginning and the ending periods for the replenishment cycle are fractional. Thus, if such a solution is to be obtained, the requirement may be better fulfilled by using meta-heuristics.

This study focus on the assumption of the deterministic nature of the model, in which the point that real world demand, lead time and shortage back ordered being non-deterministic arewell taken. In the future, the probabilistic nature of real world will be considered in the developed model. Also the sales promotion can be taken into consideration in the developed model.

References

Balkhi ZT, Benkherouf L. (2004), On an inventory model for deteriorating items with stock dependent and time-varying demand, *Computers and Operations Research*, **31**(2), 223-240.

Banks RB. (1994), Growth and diffusion phenomena : mathematical frameworks and applications, Springer : Berlin.

Barbosa LC and Friedman M. (1979), Inventory lot size models with vanishing market, *Journal of the Operational Research Society*, **30**(12), 1129-1132.

Bass FM. (1969), A new product growth for model consumer durables, *Management Science*, **15**(5), 215-227.

Baxter M. (1995), Product design : practical methods for the systematic development of new products. Chapman and Hall : London.

Brosseau LJA. (1982), An inventory replenishment policy for the case of a linear decreasing trend in demand, *INFOR Journal*, **20**(2), 252-257.

Bierman Jr., H and Thomas J. (1977), Inventorydecisions under inflationary conditions, *Decision Sciences*, **8**(1), 151-155.

Buzacott JA. (1975), Economic order quantities with inflation. Operational Research Quarterly, **26**(3), 553-558.

Chen J-M. (1998), An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting, *International Journal of Production Economics*, **55**(1), 21-30.

Chou S-Y, Chang S-L., and Yang W-D. (2001), Optimal multiple delivery schedule for demand in logistic model. International Journal of Production Economics, **73**(3), 241-249.

Chung K-J. (2003), An algorithm for an inventory model with inventory-level-dependent demand rate, *Computers and Operations Research*, **30**(9), 1311-1317.

Datta TK, Pal AK. (1991), Effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate and shortages, *European Journal of Operational Research*, **52**(3), 326-333.

Donaldson WA. (1977), Inventory replacement policy for a linear trend in demand-an analytical solution, *Operational Research Quarterly*, **28**(3), 663-670.

Friedman MF. (1981), Power-form demand and cost functions in inventory lot size models, *Computers and Operations Research*, **8**(3), 159-167.

Goyal SK, Morin D., and Nebebe F. (1992), The finite horizon trended inventory replenishment problem with shortages, *Journal of the Operational Research Society*, **43**(12), 1173-1178.

Hadley G. (1964), A comparison of order quantities computed using the average annual cost and the discounted cost, *Management Science*, **10**(3), 472-476.

Ho T-H, Savin S and Terwiesch C. (2002), Managing demand and sales dynamics in new product diffusion under supply constraint, *Management Science*, **48**(2), 187-206.

Kumar S. and Swaminathan JM. (2003), Diffusion of innovations under supply constraints, *Operation Research*, **51**(6), 866–879

Lapide L. (2001), A simple approach for short product lifecycle forecasting, *The Journal of Business Forecasting Method and Systems*, **20**, 18-20.

Lilien GL, Kotler P. and Moorthy KS. (1992), Marketing models, Prentice-Hall : NJ.

Little J. (1975), BRANDAID : A marketing-mix model, Part I : Structure; Part II : Implementation, *Operations Research*, **23**, 628-673.

Mahajan V., Muller E., and Bass FM. (1990), New product diffusion models in marketing : a review and directions for research, *Journal of Marketing*, **54**, 1-26.

MarchettiC, Meyer PS., and Ausubel JH. (1996), Human population dynamics revisited with the logistic model : How much can be modeled and predicted. Technological Forecasting and Social Change, **52**, 1-30.

McGuire T, Staelin R. (1983), An industrial equilibrium analysis of downstream vertical integration, *Marketing Science*, **2**(2), 161-192.

Meyer PS, Yung JW. and Ausubel JH. (1999), A primer on logistic growth and substitution : the mathematics of the loglet lab software, *Technological Forecasting and Social Change*, **61**(3), 247-271.

Misra RB. (1979), A note on optimal inventory management under inflation, *Navel Research Logistics Quarterly*, **26**(1), 161-165.

Mitra A., Cox JF., and Jesse Jr. RR. (1984), A note on determining order quantities with a linear tread in demand, *Journal of the Operational Research Society*, **35**(2), 141 -144.

Moore WL. (1993), Pessemier EA. Product planning and management : designing and delivering value. McGraw–Hill : New York.

Ostrosky AL. (1979), Koch JV. Introduction to mathematical economics. Houghton Mifflin : Boston.

Resh M., Friedman M., and Barbosa LC. (1976), On a general solution of the deterministic lot size problem with time–proportional demand, *Operations Research*, **24**(4), 718-725.

Rogers EM. (1983), Diffusion of innovations, 3rd Edition. The Free Press : New York.

Silver EA. (1979), Simple inventory replenishment decision rule for a linear trend in demand, *Journal of the Operational Research Society*, **30**(1), 71-75.

Thomas J. (1995), Down but not out. Distribution, 6-12.

Trippi RR. and Lewin DE. (1974), A present value formulation of the classical EOQ problem, *Decision Science*, **5**(1), 30-35.

Urban GL., Hauser JR., Design and marketing of new products (1993), Prentice-Hall : NJ.

Urban TL. (2005), Inventory models with inventory-level-dependent demand : A comprehensive review and unifying theory, *European Journal of Operational Research*, **162**(3), 792-804.

Wagner HM. and Whitin TM. (1958), Dynamic version of the economic lot size model, *Management Science*, **5**(1), 89-96.

Zhou Y-W. and Yang S-L. (2003), An optimal replenishment policy for items with inventory-level-dependent demand and fixed lifetime under the LIFO policy, *Journal of the Operational Research Society*, **54**(6), 585-593.