RK-Butcher알고리듬의 사용에 의한 주기적 진동 문제의 수치적 시뮬레이션

Numerical Simulation of Periodic and Oscillatory Problems by Using RK-Butcher Algorithms

Daechul Park* · Devarajan Gopal** · V. Murugesh*

요 약

본 논문은 주기적 진동 문제를 연구하기 위해 Runge-Kutta(RK)-Butcher 알고리듬이 소개되었다. RK-Butcher 알고리듬을 사용하여 얻어진 시뮬레이션 결과와 고전적인 4차 RK(4) 방법을 통해 얻은 결과들을 제안한 알고리듬의 성능을 확인하기 위하여 몇몇 주기적 진동 문제들의 정확한 해와 비교하였다. RK-Butcher 알고리듬의 시뮬레이션 결과는 항상 문제의 정확한 해 RK(4) 방법보다 더 근접한 결과를 줌이 확인되었다. 정확도 측면에서 RK-Butcher 알고리듬이 RK(4) 방법과 비교해볼 때 우수함을 알 수 있다. 제안한 RK-Butcher 알고리듬은 프로그램 언어로 쉽게 구현할 수 있으며 임의 시간에 종료해도 훌륭한 근사적인 해를 얻을 수 있다. RK-Butcher 알고리듬은 짧은 시간내에 이상적인 정확한 해에 근접한 결과를 주기 때문에 궤도 와 두 물체의 문제를 연구하는데 훌륭한 수치 알고리듬으로 적용가능하다.

Abstract

In this paper, Runge-Kutta (RK)-Butcher algorithm is proposed to study the periodic and oscillatory problems. Simulation results obtained using RK-Butcher algorithms and the classical fourth order Runge-Kutta (RK(4)) methods are compared with the exact solutions of a few periodic and oscillatory problems to confirm the performance of the proposed algorithm. The simulation results from RK-Butcher algorithms are always found to be very close to the exact solutions of these problems. Further, it is found that the RK-Butcher algorithm is superior when compared to RK(4) methods in terms of accuracy. The RK-Butcher algorithm can be easily implemented in a programming language and a more accurate solution may be obtained for any length of time. RK-Butcher algorithm is applicable as a good numerical algorithm for studying the problems of orbit and two body as it gives the nearly identical solutions.

Keywords : Periodic problems, Oscillatory problems, RK(4) - method and RK-Butcher Algorithms

I. Introduction

Runge-Kutta methods have been used by many researchers [1-6] to determine numerical solutions for problems, which are modeled as initial value problems (IVPs) involving differential equations that arise in the fields of science and engineering.

* Dept. of Information and Communication Engineering, Hannam University,

** Dept. of Digital Information and Communication, Woosong University

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Although the RK method was introduced at the beginning of the twentieth century, research in this area is still very active and its applications are enormous because of its extending accuracy in the determination of approximate solutions and its flexibility.

RK methods have become very popular, both as computational techniques and research applications [1, 7-8]. The developed algorithms was used to solve differential equations efficiently and yet provide the equivalent of approximating the exact solutions by matching in terms of the Taylor series expansion.

RK algorithms have always been considered to be excellent tools for the numerical integration of differential equations (ODEs).

Butcher [2] derived the best RK pair, together with an error estimate, and this is known as the RK-Butcher algorithm. It is nominally considered to be sixth order since it requires six function evaluations (it looks like a sixth-order method but in fact is a fifth-order method). In practice, the 'working order' is close to 5(fifth order), but accuracy of the results obtained exceeds all other algorithms examined, including RK-Fehlberg, RK-centroidal mean (RKCeM) and RK-arithmetic mean (RKAM) methods.

These two algorithms are efficient for singular system problems which we encounter in Science and Engineering. Hence we have chosen RK(4) and RK-Butcher Algorithms for comparison purposes. Though these two algorithms do exist, here the emphasis is finding a suitable application in the field of Science and Engineering. Accordingly, we have identified a problem where these algorithms are elegant and efficient in the sense of implementation and error analysis. Murugesh and Murugesan [9-10] introduced the RK-Butcher algorithm in Raster and Time-multiplexing CNN simulations. Recently, Devarajan et al. [4] used the RK-Butcher algorithm for finding the numerical solution of an industrial robot arm control problem.

In this article, we introduce the first and foremost advantages of RK-Butcher Algorithm which is converging to the near exact solution with more accuracy when applied the RK-Butcher algorithm to study the periodic and oscillatory problems.

II. RK-Butcher algorithms

The normal order of RK algorithm is the approximate number of leading terms of an infinite Taylor series which calculates the trajectory of a moving point [7]. The remainder of the infinite sum, which is excluded, is referred to as the local truncation error (LTE). These RK algorithms are forward-looking predictors, i.e. they do not use any information from preceding steps to predict the future position of a point. For this reason, they require a minimum of input data and consequently are very simple to program and use.

The general p-stage Runge-Kutta method for ordinary

solving an IVP is

$$y' = f(x,y) \tag{1}$$

with the initial condition $y(x_0) = y_0$ defined by

$$y_{n+1} = y_n + h\sum_{i=1}^{p} b_i k_i$$

where

$$k_i = f\left(x_n + c_i h, y_n + h\sum_{j=1}^{p} a_{ij}k_j\right), i = 1,2,3,\cdots,p$$

and

$$c_i = \sum_{j=1}^{p} a_{ij}, i = 1,2,\cdots,p$$

In the preceding equations c and b are p-dimensional vectors and $A(a_{ij})$ is a $p \times p$ matrix. Then the Butcher array takes the form of the lower triangular matrix:

$$\begin{array}{c|cccccc} c_1 & a_{11} & & & & & \\ c_2 & a_{21} & a_{22} & & & & \\ c_3 & a_{31} & a_{32} & a_{33} & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\ c_p & a_{p1} & a_{p2} & \cdots & & a_{p,p-1} & a_{p,p} \\ \hline & b_1 & b_2 & \cdots & & b_{p-1} & b_p \end{array}$$

The RK-Butcher algorithm of equation (1) is of the form

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf\left(x_n + \frac{h}{4}, y_n + \frac{k_1}{4}\right) \\ k_3 &= hf\left(x_n + \frac{h}{4}, y_n + \frac{k_1}{8} + \frac{k_2}{8}\right) \\ k_4 &= hf\left(x_n + \frac{h}{2}, y_n - \frac{k_2}{2} + k_3\right) \\ k_5 &= hf\left(x_n + \frac{3h}{4}, y_n + \frac{3k_1}{16} + \frac{9k_4}{16}\right) \\ k_6 &= hf\left(x_n + h, y_n - \frac{3k_1}{7} + \frac{2k_2}{7} + \frac{12k_3}{7} - \frac{12k_4}{7} + \frac{8k_5}{7}\right) \end{aligned} \tag{2}$$

5th order predictor of the RK-Butcher algorithm is

$$y_{n+1} = y_n + \frac{1}{90}(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6)$$

4th order predictor of the RK-Butcher algorithm is

$$y^*_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_4 + k_6)$$

The local truncation error estimate (EE) is

$$EE = y_{n+1} - y^*_{n+1}$$

III. NUMERICAL SIMULATION

In this section, we introduce the RK-Butcher algorithm to four different problems discussed by Simos and Aguiar [12]. The first is an inhomogeneous problem, the second is the nonlinear undamped Duffing's equation, the third is an 'almost' periodic orbit problem and finally the fourth one is a well-known two-body problem.

Even if the exact solution is not available, we are evaluating these two algorithms (RK(4) and RK-Butcher Algorithm) for comparison purposes. However the exact solution does exist even for singular system problems. Hence comparing these two algorithms with reference to the exact solution is being carried out.

3.1 Inhomogeneous equation

Consider the following problem

$$y'' = -100y + 99\sin x \quad (3)$$

with initial condition $y(0) = 1$ and $y'(0) = 11$.
Its analytical (exact) solution is obtained by

$$y(x) = \cos(10x) + \sin(10x) + \sin x \quad (4)$$

Equation (3) has been solved numerically using the classical fourth order Runge-Kutta method RK(4) and RK-Butcher algorithm. The simulation results (with step size 0.1 sec) along with the exact solutions (from equation (4)) are presented in Table 1 along with the absolute errors calculated between them. This result reveals the superiority of the RK-Butcher algorithm with less complexity in implementation, at the same time the error reduction is 1000 times less than that of the RK(4) method.

Table 1. Simulation results for the inhomogeneous equation at various values of "x".

Seq. No	x (sec)	Simulation results : y				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	1.0000000	1.0000000	0.0000	1.0000000	0E-07
2	0.1	1.4816067	1.4817067	1E-04	1.4816067	0E-07
3	0.2	0.6918199	0.6920199	0.0002	0.6918200	1E-07
4	0.3	-0.5533524	-0.5536524	0.0003	-0.5533525	1E-07
5	0.4	-1.0210278	-1.0214278	0.0004	-1.0210279	1E-07
6	0.5	-0.1958365	-0.1963365	0.0005	-0.1958368	3E-07
7	0.6	1.2453975	1.2459975	0.0006	1.2453978	3E-07
8	0.7	2.0551066	2.0558066	0.0007	2.0551069	3E-07
9	0.8	1.5612134	1.5620134	0.0008	1.5612137	3E-07
10	0.9	0.2843139	0.2852139	0.0009	0.2843144	5E-07
11	1.0	-0.5416219	-0.5426219	0.0010	-0.5416224	5E-07
12	1.1	-0.1043556	-0.1054556	0.0011	-0.1043561	5E-07
13	1.2	1.2393225	1.2405225	0.0012	1.2393232	7E-07
14	1.3	2.2911729	2.2924729	0.0013	2.2911736	7E-07
15	1.4	2.1127925	2.1141925	0.0014	2.1127932	7E-07
16	1.5	0.8880915	0.8895915	0.0015	0.8880922	7E-07
17	1.6	-0.2459909	-0.2475909	0.0016	-0.2459919	1E-06
18	1.7	-0.2448940	-0.2465940	0.0017	-0.2448950	1E-06
19	1.8	0.8831813	0.8849813	0.0018	0.8831823	1E-06
20	1.9	2.0848846	2.0867846	0.0019	2.0848857	1.1E-06
21	2.0	2.2303235	2.2323235	0.0020	2.2303247	1.2E-06

3.2 Duffing's equation

Consider the nonlinear undamped Duffing equation

$$y'' + y + y^3 = B\cos(\omega x) \quad (5)$$

where B = 0.002 and $\omega = 1.01$ The analytical solution of the above equation is given by

$$y(x) = \sum_{i=0}^{3} A_{2i+1}\cos[(2i+1)\omega x] \quad (6)$$

where $A_1 = 0.200179477536, A_3 = 0.246946143 \times 10^{-3}$

$A_5 = 0.304016 \times 10^{-6}$ and $A_7 = 0.374 \times 10^{-9}$. Equation (5) has been solved numerically with boundary conditions of the form

$$y(0) = A_1 + A_3 + A_5 + A_7, y'(0) = 0$$

The simulation results obtained (with step size 1 sec) using the RK-Butcher algorithm and the RK(4) methods along with exact solutions(from equation (6)) and absolute errors calculated between them are presented in Table 2. It is inferred that the RK-Butcher algorithm gives better solution for the nonlinear undamped Duffing's equation when compared to RK(4).

Table 2. Simulation results for the Duffing's equation at various values of "x"

Seq. No	x (sec)	Simulation results: y				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	0.2004294	0.2004294	0E-07	0.2004294	0E-07
2	1.0	0.3066526	0.3066526	0E-07	0.3066526	0E-07
3	2.0	0.2199634	0.2200634	1E-04	0.2199634	0E-07
4	3.0	0.0207932	0.0209932	0.0002	0.0207932	0E-07
5	4.0	-0.1036664	-0.1039664	0.0003	-0.1036664	0E-07
6	5.0	-0.0375666	-0.0379666	0.0004	-0.0375667	1E-07
7	6.0	0.1578427	0.1583427	0.0005	0.1578428	1E-07
8	7.0	0.2990128	0.2996128	0.0006	0.2990129	1E-07
9	8.0	0.2543050	0.2550050	0.0007	0.2543051	1E-07
10	9.0	0.0651066	0.0659066	0.0008	0.0651069	3E-07
11	10.0	-0.0919355	-0.0928355	0.0009	-0.0919358	3E-07
12	11.0	-0.0682613	-0.0692613	0.001	-0.06826136	6E-08
13	12	0.1123593	0.1134593	0.0011	0.11235938	8E-08
14	13	0.2815429	0.2827429	0.0012	0.2815434	5E-07
15	14	0.2809778	0.2822778	0.0013	0.2809783	5E-07
16	15	0.1111868	0.1125868	0.0014	0.1111875	7E-07
17	16	-0.0689371	-0.0704371	0.0015	-0.0689377	6E-07
18	17	-0.0905881	-0.0921881	0.0016	-0.0905889	8E-07
19	18	0.0662643	0.0679643	0.0017	0.0662653	1E-06
20	19	0.2550834	0.2568834	0.0018	0.2550844	1E-06
21	20	0.2986884	0.3000884	0.0014	0.2986894	1E-06

3.3 An orbit problem

Consider a (nearly) 'almost' periodic orbit problem studied by Stiefel and Bettis [11]. That is

$$z'' + z = 0.001e^{ix}, z(0) = 1, z'(0) = 0.995i, z \in C \quad (7)$$

The analytical solution for this is given by

$$z(x) = u(x) + iv(x), \quad u,v \in R \text{ where}$$
$$u(x) = \cos x + 0.0005x \sin x \quad (8)$$
$$v(x) = \sin x - 0.0005x \cos x$$

The true solution in equation (8) represents a perturbational motion on a circular orbit in the complex plane. Rewriting the equation (7) in the following equivalent form

$$u'' + u = 0.001\cos x, u(0) = 1, u'(0) = 0, \quad (9)$$
$$v'' + v = 0.001\sin x, v(0) = 0, v'(0) = 0.9995$$

Now, equation (9) has been solved numerically using the classical fourth order Runge-Kutta method and the RK-Butcher algorithm. The simulation results (with step size 0.1sec) along with exact solutions (from equation(8)) and the absolute errors computed between them are presented in Table 3. From Table 3 to Table 5 it is found that RK-Butcher algorithm works very well (with negligible error upto 7 decimal places) when compared to RK(4) method which yields a little bigger error.

Table 3. Simulation results for an orbit problem at various values of "x"

Seq. No	x (sec)	Simulation results: u				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	1.0000000	1.0000000	0	1.0000000	0E-07
2	0.1	0.9950091	0.9950092	1E-07	0.9950091	0E-07
3	0.2	0.9800864	0.9800865	1E-07	0.9800864	0E-07
4	0.3	0.9553807	0.9553808	1E-07	0.9553807	0E-07
5	0.4	0.9211388	0.9211389	1E-07	0.9211388	0E-07
6	0.5	0.8777024	0.8777025	1E-07	0.8777024	0E-07
7	0.6	0.8255050	0.8255053	3E-07	0.8255050	0E-07
8	0.7	0.7650676	0.7650679	3E-07	0.7650676	0E-07
9	0.8	0.6969935	0.6969938	3E-07	0.6969935	0E-07

10	0.9	0.6219623	0.6219626	3E-07	0.6219623	0E-07
11	1.0	0.5407229	0.5407234	5E-07	0.5407229	0E-07
12	1.1	0.4540861	0.4540866	5E-07	0.4540861	0E-07
13	1.2	0.3629168	0.3629173	5E-07	0.3629168	0E-07
14	1.3	0.2681249	0.2681256	7E-07	0.2681249	0E-07
15	1.4	0.1706567	0.1706574	7E-07	0.1706567	0E-07
16	1.5	0.0714850	0.0714857	7E-07	0.0714850	0E-07
17	1.6	-0.0284001	-0.0284011	1E-06	-0.0284001	0E-07
18	1.7	-0.1280018	-0.1280028	1E-06	-0.1280018	0E-07
19	1.8	-0.2263259	-0.2263260	1E-07	-0.2263259	0E-07
20	1.9	-0.3223908	-0.3223919	1.1E-06	-0.3223908	0E-07
21	2.0	-0.4152377	-0.4152389	1.2E-06	-0.4152377	0E-07

Table 4. Simulation results for an orbit problem at various values of "x"

Seq. No	x (sec)	Simulation results: v				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	0.0000000	0.0000000	0	0.0000000	0E-07
2	0.1	0.0997836	0.0997837	1E-07	0.0997836	0E-07
3	0.2	0.1985713	0.1985714	1E-07	0.1985713	0E-07
4	0.3	0.2953769	0.2953770	1E-07	0.2953769	0E-07
5	0.4	0.3892341	0.3892342	1E-07	0.3892341	0E-07
6	0.5	0.4792061	0.4792064	3E-07	0.4792061	0E-07
7	0.6	0.5643948	0.5643951	3E-07	0.5643948	0E-07
8	0.7	0.6439500	0.6439503	3E-07	0.6439500	0E-07
9	0.8	0.7170774	0.7170777	3E-07	0.7170774	0E-07
10	0.9	0.7830472	0.7830475	3E-07	0.7830472	0E-07
11	1.0	0.8412008	0.8412013	5E-07	0.8412008	0E-07
12	1.1	0.8909579	0.8909584	5E-07	0.8909579	0E-07
13	1.2	0.9318217	0.9318222	5E-07	0.9318217	0E-07
14	1.3	0.9633843	0.9633848	5E-07	0.9633843	0E-07
15	1.4	0.9853307	0.9853312	5E-07	0.9853307	0E-07
16	1.5	0.9974419	0.9974426	7E-07	0.9974419	0E-07
17	1.6	0.9995969	0.9995976	7E-07	0.9995969	0E-07
18	1.7	0.9917743	0.9917750	7E-07	0.9917743	0E-07
19	1.8	0.9740520	0.9740530	1E-06	0.9740520	0E-07
20	1.9	0.9466071	0.9466081	1E-06	0.9466071	0E-07
21	2.0	0.9097134	0.9097145	1.1E-06	0.9097134	0E-07

Table 5. Simulation results for an orbit problem at various values of "x"

Seq. No	x (sec)	Simulation results: |z|=|u+jv|				
		Exact Solutions |z|EXT	RK(4) Solutions |z|RK4	RK(4) Error |z|EXT -|z|RK4	RK-Butcher Solutions |z|RKB	RK-Butcher Error |z|EXT-|z| RKB
1	0.0	1.0000000	1.0000000	0E-07	1.0000000	0E-07
2	0.1	0.9999999	1.0000000	-1E-07	0.9999999	0E-07
3	0.2	0.9999999	1.0000000	-1E-07	0.9999999	0E-07
4	0.3	0.9999998	1.0000000	-2E-07	0.9999998	0E-07
5	0.4	0.9999999	1.0000000	-1E-07	0.9999999	0E-07
6	0.5	0.9999999	1.0000002	-3E-07	0.9999999	0E-07
7	0.6	0.9999999	1.0000004	-5E-07	0.9999999	0E-07
8	0.7	1.0000000	1.0000004	-4E-07	1.0000000	0E-07
9	0.8	0.9999999	1.0000003	-4E-07	0.9999999	0E-07
10	0.9	1.0000000	1.0000004	-4E-07	1.0000000	0E-07
11	1.0	1.0000000	1.0000007	-7E-07	1.0000000	0E-07
12	1.1	1.0000000	1.0000007	-7E-07	1.0000000	0E-07
13	1.2	1.0000001	1.0000007	-6E-07	1.0000001	0E-07
14	1.3	1.0000001	1.0000008	-7E-07	1.0000001	0E-07
15	1.4	1.0000001	1.0000007	-6E-07	1.0000001	0E-07
16	1.5	1.0000002	1.0000009	-7E-07	1.0000002	0E-07
17	1.6	1.0000002	1.0000009	-7E-07	1.0000002	0E-07
18	1.7	1.0000003	1.0000011	-8E-07	1.0000003	0E-07
19	1.8	1.0000003	1.0000013	-1E-06	1.0000003	0E-07
20	1.9	1.0000004	1.0000017	-1.3E-06	1.0000004	0E-07
21	2.0	1.0000004	1.0000019	-1.5E-06	1.0000004	0E-07

3.4 Two-body problem

Consider the following system of coupled differential equations, which is well known as two-body problem

$$y'' = -\frac{y}{(y^2+z^2)^{3/2}},\ z'' = -\frac{z}{(y^2+z^2)^{3/2}}, \tag{10}$$
$$y(0)=1,\ y'(0)=0,\ z(0)=0,\ z'(0)=1$$

whose analytical solution is given by

$$y(x)=\cos(x),\ z(x)=\sin(x) \tag{11}$$

The above system of equation (10) has been solved numerically using the classical fourth order Runge-Kutta method and RK-Butcher algorithm. The simulation results (with step size 0.1sec) along with exact solutions (from equation (11)) and absolute errors between them are calculated for various values of y and z and are presented in Table 6 and Table 7, respectively. Here too, the RK-Butcher registers its supremacy which is evident from this simulated results.

Table 6. Simulation results for two-body problem at various values of "x"

Seq. No	x (sec)	Simulation results: y				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	1.0000000	1.0000000	0	1.0000000	0E-07
2	0.1	0.9950041	0.9950042	1E-07	0.9950041	0E-07
3	0.2	0.9800665	0.9800666	1E-07	0.9800665	0E-07
4	0.3	0.9553365	0.9553366	1E-07	0.9553365	0E-07
5	0.4	0.9210609	0.9210610	1E-07	0.9210609	0E-07
6	0.5	0.8775825	0.8775828	3E-07	0.8775825	0E-07
7	0.6	0.8253356	0.8253359	3E-07	0.8253356	0E-07
8	0.7	0.7648421	0.7648424	3E-07	0.7648421	0E-07
9	0.8	0.6967066	0.6967069	3E-07	0.6967066	0E-07
10	0.9	0.6216098	0.6216101	3E-07	0.6216098	0E-07
11	1.0	0.5403022	0.5403027	5E-07	0.5403022	0E-07
12	1.1	0.4535959	0.4535965	6E-07	0.4535959	0E-07
13	1.2	0.3623575	0.3623580	5E-07	0.3623575	0E-07
14	1.3	0.2674986	0.2674991	5E-07	0.2674986	0E-07
15	1.4	0.1699669	0.1699674	5E-07	0.1699669	0E-07
16	1.5	0.0707369	0.0707376	7E-07	0.0707369	0E-07
17	1.6	-0.0291997	-0.0292004	7E-07	-0.0291997	0E-07
18	1.7	-0.1288447	-0.1288454	7E-07	-0.1288447	0E-07
19	1.8	-0.2272024	-0.2272031	7E-07	-0.2272024	0E-07
20	1.9	-0.3232898	-0.3232905	7E-07	-0.3232898	0E-07
21	2.0	-0.4161470	-0.4161480	1E-06	-0.4161470	0E-07

Table 7. Simulation results for two-body problem at various values of "z".

Seq. No	z (sec)	Simulation results: z				
		Exact Solutions	RK(4) Solutions	RK(4) Error	RK-Butcher Solutions	RK-Butcher Error
1	0.0	0.0000000	0.0000000	0E-07	0.0000000	0E-07
2	0.1	0.0998334	0.0998334	0E-07	0.0998334	0E-07
3	0.2	0.1986693	0.1986693	0E-07	0.1986693	0E-07
4	0.3	0.2955202	0.2955202	0E-07	0.2955202	0E-07
5	0.4	0.3894183	0.3894183	0E-07	0.3894183	0E-07
6	0.5	0.4794255	0.4794255	0E-07	0.4794255	0E-07
7	0.6	0.5646424	0.5646425	1E-07	0.5646424	0E-07
8	0.7	0.6442177	0.6442178	1E-07	0.6442177	0E-07
9	0.8	0.7173561	0.7173562	1E-07	0.7173561	0E-07
10	0.9	0.7833269	0.7833270	1E-07	0.7833269	0E-07
11	1.0	0.8414710	0.8414711	1E-07	0.8414710	0E-07
12	1.1	0.8912073	0.8912074	1E-07	0.8912073	0E-07
13	1.2	0.9320391	0.9320392	1E-07	0.9320391	0E-07
14	1.3	0.9635582	0.9635583	1E-07	0.9635582	0E-07
15	1.4	0.9854497	0.9854500	3E-07	0.9854497	0E-07
16	1.5	0.9974949	0.9974952	3E-07	0.9974949	0E-07
17	1.6	0.9995735	0.9995738	3E-07	0.9995735	0E-07
18	1.7	0.9916647	0.9916650	3E-07	0.9916647	0E-07
19	1.8	0.9738475	0.9738478	3E-07	0.9738475	0E-07
20	1.9	0.9462999	0.9463002	3E-07	0.9462999	0E-07
21	2.0	0.9092973	0.9092976	3E-07	0.9092973	0E-07

Ⅳ. Conclusions

The simulated results of the periodic and oscillatory problems using RK-Butcher algorithm is very close to these exact solutions of the problem when compared to that of the classical fourth order Runge-Kutta method. From the Tables 1-7, one can observe that for most of the time intervals, the absolute error is less or negligible error (upto 7 decimal places) in RK-Butcher algorithm when compared to that of the classical fourth order Runge-Kutta method (RK(4)) (which yields a little error) along with the exact solutions. Hence, RK-Butcher algorithm is more suitable for studying the periodic and oscillatory problems and especially it is recommended for studying the problems of orbit and two-body problems since it gives an almost identical solution with negligible error upto designated significant digits. That is, the solution we get is always found to be the exact one or with least error possible. Secondly, it always found to be stable and converges faster than other algorithms. Finally, the software implementation is simple and can be extended for any length of time.

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Daechul Park

He received the B.S. degree in Electronics Engineering from Sogang University, Korea, in 1977, the M.S. Degree and the Ph.D. degree in Electrical and Computer Engineering from the University of New Mexico, Albuquerque, NM, USA, in 1985 and 1989, respectively. Since 1989 he had worked in ETRI, Korea as a member of senior technical staff til 1993. Since 1993, he has been with the Department of Information and Communication Engineering, Hannam University, Daejeon, where he is a currently a professor. His research interests include image processing, mobile multimedia communication and signal processing algorithm development.

Devarajan Gopal

He received the BE in electronics and communication from College of Engineering, Guindy, Madras University, the M.Tech. in communication and radar engineering from Indian Institute of Technology, New Delhi, and the PhD at Cochin University of Science and Technology, India. He joined the Defence Research and Development Organization in the Government of India and served as a Scientific Officer / Scientist for about two decades. He was associated with different universities in India and Ethiopia for curriculum revision at the undergraduate and graduate levels. He has held various positions at Madras University and Addis Ababa University. Currently, he is working as a Professor in the Department of Digital Information and Communication at Woosong University, Korea. His fields of interest are in digital signal processing, computer networks, and broadband communication.

V. Murugesh

He obtained his Bachelor of Science in Computer Science and Master of Computer Applications degree from Bharathiar University, Coimbatore, India during 1992 and 1995, respectively and completed his PhD in Computer Science from Bharathidasan University, Tiruchirappalli, India during 2006. He has held various positions at National Institute of Technology, Tiruchirappalli, India and Sona College of Technology, Salem, India. Currently, he is working as Assistant Professor in the Department of Information and Communication Engineering at Hannam University, Daejeon, Republic of Korea. His fields of interest are in Neural Network based Image Processing and Scientific computing.