On Output Feedback Tracking Control of Robot Manipulators with Bounded Torque Input

Javier Moreno-Valenzuela, Víctor Santibáñez, and Ricardo Campa

Abstract: Motivated by the fact that in many industrial robots the joint velocity is estimated from position measurements, the trajectory tracking of robot manipulators with output feedback is addressed in this paper. The fact that robot actuators have limited power is also taken into account. Let us notice that few solutions for the torque-bounded output feedback tracking control problem have been proposed. In this paper we contribute to this subject by presenting a theoretical reexamination of a known controller, by using the theory of singularly perturbed systems. Motivated by this analysis, a redesign of that controller is introduced. As another contribution, we present an experimental evaluation in a two degrees-of-freedom revolute-joint direct-drive robot, confirming the practical feasibility of the proposed approach.

Keywords: Bounded torque input, output feedback, robot control, singularly perturbed systems, trajectory tracking.

1. INTRODUCTION

In practice, it is much easier to obtain an estimation of the joint velocity from encoder position measurements than to add to the robot mechanical structure a direct velocity sensor like a tachometer, which may provide signals contaminated with noise. Thus, there have been many efforts in providing a theoretical justification to the industrial usage of estimating the joint velocities from position measurements by using numerical algorithms.

The problem of output feedback tracking control of manipulators subject to constrained torques consists in designing a control algorithm that produces joint torques within the allowed power capabilities of the robot actuators by using only joint position measurements, so that the error between the time-varying desired position and the actual position of the system goes asymptotically to zero for a set of initial conditions.

As far as we know, the first study that used saturation functions to generate a solution for the output feedback tracking control of manipulators subject to bounded torques was due to Loría and Nijmeijer [1]. They showed how the semiglobal and exponential stabilization of the closed-loop system is achieved by using large enough observer gains while bounded controls are guaranteed. Until now, only a few solutions for this problem have been proposed. Let us present a literature review: In the paper [2], a class of bounded output feedback tracking controllers was proposed, which attempted to generalize the results in [1]. The class of controllers proposed was developed by invoking Matrosov's Theorem to show the asymptotic stability of the closed-loop system. In the paper by Dixon *et al.* [3] a controller was proposed to solve the output feedback tracking control with bounded torques, which yielded semi-global stability in closed-loop. More recently, in [4], the controller in [1] was redesigned by assuming that viscous friction is present at the robot joints; then the global asymptotic stability is assured if large enough damping is presented. The work in [5] proposed three alternative approaches, including the output feedback case study, for the (semi) global tracking of robot manipulators with bounded inputs through simple extensions of "PD+" (Proportional-Derivative in the position error) control law to the bounded-input case. It is noteworthy that other results on output feedback tracking control of manipulators have been proposed, including robust versions, [6-9], but they did not consider the case of actuator limited power.

Although much effort was done to derive the cited controllers and very complex stability analyses were necessary, to the best of the authors' knowledge, no

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experimental evaluation of output feedback tracking controllers that contain saturation functions in its structure has been reported.

The objective of this study is to revisit the problem of output feedback tracking control of robot manipulators by reexaminating/redesigning the controller in [1]. The contribution of this paper is threefold:

- A new stability analysis of the controller [1], which is presented from the perspective of singularly perturbed systems.
- A redesign of the controller [1], which takes advantage of the new stability framework.
- An experimental comparison in a two degrees-of-freedom revolute-joint direct-drive robot, which shows that the proposed redesign has better tracking performance than the classical PD+ tracking controller [10] and the so-called Loría-Nijmeijer controller [1].

This paper is organized as follows. Section 2 is devoted to present some properties on hyperbolic functions, which are important in the proposed analysis. In Section 3 the robot dynamics and the control goal are given. Section 4 deals with the analysis of the Loría-Nijmeijer control by using the stability theory of singularly perturbed systems. A new controller, coming from the redesign of the Loría-Nijmeijer controller is discussed in Section 5. Section 6 is devoted to the experimental results. Finally, some concluding remarks are given in Section 7.

Notation: Throughout this paper the following notation will be adopted. $\|x\| = \sqrt{x^T x}$ stands for the Euclidean norm of vector $x \in \mathbb{R}^n$. $\lambda_{min}\{A(x)\}$ and $\lambda_{Max}\{A(x)\}$ denote, respectively, the minimum and maximum eigenvalues of a symmetric positive definite matrix $A(x) \in \mathbb{R}^{n \times n}$, for all $x \in \mathbb{R}^n$. $\|B(x)\| = \sqrt{\lambda_{Max}\{B(x)^T B(x)\}}$ stands for the induced norm of a matrix $B(x) \in \mathbb{R}^{m \times n}$, for all $x \in \mathbb{R}^n$.

2. PROPERTIES ON HYPERBOLIC FUNCTIONS

Before discussing the problem statement and the solution of the output feedback tracking control using a torque-bounded controller, let us introduce some notation and properties of hyperbolic functions. The notation $\mathrm{hypfunc}(z_i)$ denotes a hyperbolic function of $z_i \in \mathbb{R}$, e.g., $\tanh(z_i)$ and $\ln(\cosh(z_i))$, while the notation

$$\mathrm{Hypfunc}(z) = diag\{\mathrm{hypfunc}(z_1), \ldots, \mathrm{hypfunc}(z_n)\},$$

$z = [z_1, \ldots, z_n]^T \in \mathbb{R}^n$, denotes a diagonal matrix containing as elements the hyperbolic function of each element of the vector z. As example,

$$\mathrm{Sech}^2(z) = diag\{\mathrm{sech}^2(z_1), \ldots, \mathrm{sech}^2(z_n)\}.$$

In particular, the hyperbolic tangent function is defined as

$$\tanh(z_i) = \frac{e^{z_i} - e^{-z_i}}{e^{z_i} + e^{-z_i}}, \ \forall\, z_i \in \mathbb{R},$$

and it can be arranged in a vector in the following way:

$$\tanh(z) = [\tanh(z_1) \cdots \tanh(z_n)]^T.$$

Let us define the matrix

$$\Delta = diag\{\delta_1, \ldots, \delta_n\}, \ \delta_i \geq 1.$$

The following properties will be used throughout this paper:

- The Euclidean norm of $\tanh(\Delta z)$, satisfies:

$$\|\tanh(\Delta z)\| \leq \begin{cases} \lambda_{Max}\{\Delta\}\|z\|, & \forall z \in \mathbb{R}^n, \\ \sqrt{n}, & \forall z \in \mathbb{R}^n. \end{cases}$$

- The time derivative of $\tanh(\Delta z)$, $z \in \mathbb{R}^n$, is given by

$$\frac{d}{dt}\tanh(\Delta z) = \mathrm{Sech}^2(\Delta z)\Delta \dot{z}.$$

- The maximum eigenvalue of the matrix $\mathrm{Sech}^2(\Delta z)$ is one, for all $z \in \mathbb{R}^n$, i.e.,

$$\lambda_{max}\{\mathrm{Sech}^2(\Delta z)\} = 1, \ \ \forall\, z \in \mathbb{R}^n.$$

- Another important property used in this paper is

$$\frac{d}{dt}\ln(\cosh(\delta_i z_i)) = \tanh(z_i)\delta_i \dot{z}_i.$$

- There exist constant $r \geq 1$ and $k_b > 1/2$ such that, for all $\|z\| \leq r$

$$k_b \|\tanh(\Delta z)\|^2 \geq \sum_{i=1}^{n} \ln(\cosh(\delta_i z_i)) \geq \frac{1}{2}\|\tanh(\Delta z)\|^2. \quad (1)$$

- There exist large enough $r \geq 1$ and $k_a > 1$ such that, for all $\|z\| \leq r$,

$$\|z\| \leq k_a \|\tanh(\Delta z)\|. \quad (2)$$

Note that in properties (1) and (2) the value of r

can be increased if the value of k_a and k_b are also increased.

3. ROBOT DYNAMICS AND CONTROL GOAL

The dynamics in joint space of a serial-chain n-link robot manipulator considering the presence of friction at the robot joints can be written as [11-14],

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F_v\dot{q} = \tau, \tag{3}$$

where $M(q)$ is the $n \times n$ symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times n$ vector of centripetal and Coriolis torques, $g(q)$ is the $n \times 1$ vector of gravitational torques, $F_v = diag\{f_{v1},\ldots, f_{vn}\}$ is the $n \times n$ positive definite diagonal matrix which contains the viscous friction coefficients of the robot joints, and τ is the $n \times 1$ vector of applied torque inputs

The following properties are satisfied for the dynamic model (3) (see e.g., [11-14]). For robots with revolute joints, the vector of gravitational torques is bounded, i.e.,

$$k_g \geq \sup_{\forall q \in \mathbb{R}^n} \| g(q) \|. \tag{4}$$

For all $q, \dot{q}, x, y, z \in \mathbb{R}^n$, the inertia and Coriolis matrix (obtained by using Christoffel symbols) satisfy

$$C(x,y)z = C(x,z)y, \tag{5}$$

$$C(x,y+z) = C(x,y) + C(x,z), \tag{6}$$

$$\lambda_{Max}\{M(q)\} \| x \|^2 \geq x^T M(q) x \geq \lambda_{min}\{M(q)\} \| x \|^2, \tag{7}$$

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T, \tag{8}$$

$$x^T \left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q}) \right] x = 0, \tag{9}$$

$$\| C(q,\dot{q}) \| \leq k_C \| \dot{q} \|, \quad k_C > 0. \tag{10}$$

Let Ψ denotes the torque space, defined as

$$\Psi = \{\tau \in \mathbb{R}^n : -\tau_i^{Max} < \tau_i < \tau_i^{Max}\}, \quad i = 1,\ldots,n, \tag{11}$$

with $\tau_i^{Max} > 0$ the maximum torque input for the ith joint. Assume that only robot joint displacements $q(t) \in \mathbb{R}^n$ are available for measurement and that the robot torque input is constrained to be at the torque space (11). Then, the output feedback tracking control problem is to design a control input $\tau \in \Psi$ so that the joint displacements $q(t) \in \mathbb{R}^n$ converge asymptotically to the desired joint displacements $q_d(t) \in \mathbb{R}^n$, i.e.,

$$\lim_{t \to \infty} \tilde{q}(t) = 0, \tag{12}$$

where

$$\tilde{q}(t) = q_d(t) - q(t)$$

denotes the tracking error. Throughout this paper we consider that $q_d(t)$ is two times differentiable and

$$\| \dot{q}_d(t) \| \leq \| \dot{q}_d \|_M \quad \forall t \geq 0, \tag{13}$$

$$\| \ddot{q}_d(t) \| \leq \| \ddot{q}_d \|_M \quad \forall t \geq 0, \tag{14}$$

where $\| \dot{q}_d \|_M > 0$ and $\| \ddot{q}_d \|_M > 0$ denote known constants.

4. REEXAMINATION OF THE LORÍA-NIJMEIJER CONTROLLER

In this Section a result related to the closed-loop stability of the Loría and Nijmeijer controller [1] is presented. The controller in [1] is given by

$$\begin{aligned}\tau = {} & M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + g(q) \\ & + F_v\dot{q}_d + K_p \tanh(\tilde{q}) + K_d \tanh(\tilde{\vartheta}),\end{aligned} \tag{15}$$

where K_p and K_d are $n \times n$ positive definite diagonal matrices. The signal $\tilde{\vartheta}$ is given by

$$\dot{q}_c = -b_f \tanh(\tilde{\vartheta}), \tag{16}$$

$$\tilde{\vartheta} = q_c + b_f \tilde{q}, \tag{17}$$

where b_f is a strictly positive constant.

A sufficient condition for the controller (15) to produce bounded torques is

$$\begin{aligned}&\max_{\forall q \in \mathbb{R}^n} \{\| M_i(q) \|\} \| \ddot{q}_d \|_M \\ &+ \max_{\forall q, \dot{q}_d \in \mathbb{R}^n} \{\| C_i(q,\dot{q}_d) \|\} \| \dot{q}_d \|_M + \max_{\forall q \in \mathbb{R}^n} \{g_i(q)\} \\ &+ f_{vi} | \dot{q}_{di} |_M + K_{pi} + K_{di} < \tau_i^{Max}, i = 1,\ldots,n,\end{aligned} \tag{18}$$

where $M_i(q)$ is the ith row of the inertia matrix $M(q)$, $C_i(q,\dot{q}_d)$ is the ith row of the matrix $C(q,\dot{q}_d)$, $g_i(q)$ the ith element of the vector $g(q)$, and $| \dot{q}_{di}(t) | \leq | \dot{q}_{di} |_M$ for all $t \geq 0$. Note that under the robot model property (4) and assumptions (13)-(14), the condition (18) holds.

The closed loop system is given by

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}} \\ -M(q)^{-1}[[C + C_d]\dot{\tilde{q}} + F_v\dot{\tilde{q}} + K_d \tanh(\tilde{\vartheta}) \end{bmatrix}$$

$$+K_p \tanh(\tilde{q})] \Big], \quad (19)$$

$$\varepsilon \frac{d}{dt}\tilde{\vartheta} = -\tanh(\tilde{\vartheta}) + \dot{\tilde{q}}, \quad (20)$$

with notation $C = C(q,\dot{q})$ and $C_d = C(q,\dot{q}_d)$, which was obtained by substituting the controller (15) into the robot dynamics (3), invoking the robot model properties (5) and (6), equations (16)-(17), and the definition

$$\varepsilon = 1/b_f. \quad (21)$$

It is noteworthy that the closed-loop system (19)-(20) has the form of a singularly perturbed system with a unique equilibrium point at the state space origin $[\tilde{q}^T\ \dot{\tilde{q}}^T \tilde{\vartheta}^T]^T = 0 \in \mathbb{R}^{3n}$.

Proposition 1: Assume that

$$\lambda_{min}\{K_d\} + \lambda_{min}\{F_v\} - k_C \| \dot{q}_d \|_M k_a^2 > 0, \quad (22)$$

where k_a is defined in (2). Then, for all initial conditions

$$\left\| \left[\tilde{q}(0)^T\ \dot{\tilde{q}}(0)^T\ \tilde{\vartheta}(0)^T \right]^T \right\| < 1$$

there always exists $\varepsilon^* > 0$ such that for all $\varepsilon^* > \varepsilon > 0$, the state space origin of system (19)-(20) is exponentially stable.

Proof: The local exponential stability of the perturbed system can be done by invoking the Theorem 9.3 in [15]. First note that when $\varepsilon = 0$ the quasi-steady state solution is

$$\tanh(\tilde{\vartheta}) = \dot{\tilde{q}},$$

which only holds if $|\dot{\tilde{q}}_i| < 1$, $i = 1,\dots,n$. Therefore, the reduced system is given by

$$\frac{d}{dt}\begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}} \\ -M(q)^{-1}[[C + C_d]\dot{\tilde{q}} + F_v\dot{\tilde{q}} + K_d\dot{\tilde{q}} \\ +K_p \tanh(\tilde{q})] \end{bmatrix}, \quad (23)$$

which is a time-varying system and the state space origin is the only equilibrium point. The exponential stability of the reduced model (23) can be shown through the following Lyapunov function

$$V(t,\tilde{q},\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \sum_{i=1}^{n} k_{pi} \ln(\cosh(\tilde{q}_i)) + \alpha\dot{\tilde{q}}^T M(q)\tanh(\tilde{q}),$$

where α is a small enough positive constant.

The properties on hyperbolic functions described in Section 2 will be invoked defining $\Delta = I$, the $n \times n$ identity matrix.

By using the robot model property (7) and the right-hand side of the hyperbolic function property (1), it can be shown that for $0 < \alpha < \alpha_1^*$, the function $V(t,\tilde{q},\dot{\tilde{q}})$ is positive definite for all $[\tilde{q}^T\ \dot{\tilde{q}}^T]^T \in \Omega_r$, where

$$\Omega_r = \left\{ [\tilde{q}^T\ \dot{\tilde{q}}^T]^T \in \mathbb{R}^{2n} : \ \| [\tilde{q}^T\ \dot{\tilde{q}}^T]^T \| \le r \right\}, \quad (24)$$

with r arbitrarily large. In addition, by using the left-hand side of the hyperbolic function property (1), we can claim that for a large enough $c_0 > 0$ the inequality

$$V(t,\tilde{q},\dot{\tilde{q}}) \le c_0 \left\| \left[\| \tanh(\tilde{q}) \|\ \ \| \tanh(\dot{\tilde{q}}) \| \right]^T \right\|^2, \quad (25)$$

holds for all $[\tilde{q}^T\ \dot{\tilde{q}}^T]^T \in \Omega_r$. The inequality (25) will be used later.

On the other hand, the time derivative of $V(t,\tilde{q},\dot{\tilde{q}})$ is given by

$$\begin{aligned}\dot{V}(t,\tilde{q},\dot{\tilde{q}}) = & -\dot{\tilde{q}}^T C_d\dot{\tilde{q}} - \dot{\tilde{q}}^T F_v\dot{\tilde{q}} - \dot{\tilde{q}}^T K_d\dot{\tilde{q}} \\ & - \alpha\tanh(\tilde{q})^T [C_d - C^T]\dot{\tilde{q}} - \alpha\tanh(\tilde{q})^T F_v\dot{\tilde{q}} \\ & - \alpha\tanh(\tilde{q})^T K_d\dot{\tilde{q}} - \alpha\tanh(\tilde{q})^T K_p\tanh(\tilde{q}) \\ & + \alpha\dot{\tilde{q}}^T M(q)\mathrm{Sech}^2(\tilde{q})\dot{\tilde{q}},\end{aligned}$$

where the robot model properties (8)-(9) were used. By using the hyperbolic function property (2), the robot model property (10), the inequality

$$\| \dot{q} \| \ \le \ \| \dot{\tilde{q}} \| + \| \dot{q}_d \|_M,$$

with $\| \dot{q}_d \|_M$ defined in (13), and some other properties on hyperbolic functions (see Section 2) we can write an upper bound on $\dot{V}(t,\tilde{q},\dot{\tilde{q}})$ as follows

$$\begin{aligned}\dot{V}(t,\tilde{q},\dot{\tilde{q}}) \le & \ k_C \| \dot{q}_d \|_M k_a^2 \| \tanh(\dot{\tilde{q}}) \|^2 \\ & - \lambda_{min}\{F_v\} \| \tanh(\dot{\tilde{q}}) \|^2 \\ & - \lambda_{min}\{K_d\} \| \tanh(\dot{\tilde{q}}) \|^2 \\ & + \alpha\sqrt{n} k_C k_a^2 \| \tanh(\dot{\tilde{q}}) \|^2 \\ & + \alpha 2k_C \| \dot{q}_d \|_M k_a \| \tanh(\tilde{q}) \| \| \tanh(\dot{\tilde{q}}) \| \\ & + \alpha k_a \lambda_{Max}\{F_v\} \| \tanh(\tilde{q}) \| \| \tanh(\dot{\tilde{q}}) \| \\ & + \alpha k_a \lambda_{Max}\{K_d\} \| \tanh(\tilde{q}) \| \| \tanh(\dot{\tilde{q}}) \| \\ & - \alpha\lambda_{min}\{K_p\} \| \tanh(\tilde{q}) \|^2 \\ & + \alpha\lambda_{Max}\{M(q)\} k_a^2 \| \tanh(\dot{\tilde{q}}) \|^2,\end{aligned} \quad (26)$$

which is satisfied for all $\| [\tilde{q}^T \dot{\tilde{q}}^T]^T \| \leq r$. Finally, the upper bound (26) can be rewritten in the matrix form construction

$$\dot{V}(t,\tilde{q},\dot{\tilde{q}}) \leq -\begin{bmatrix} \| \tanh(\tilde{q}) \| \\ \| \tanh(\dot{\tilde{q}}) \| \end{bmatrix}^T Q \begin{bmatrix} \| \tanh(\tilde{q}) \| \\ \| \tanh(\dot{\tilde{q}}) \| \end{bmatrix}, \tag{27}$$

where Q is a symmetric matrix with elements

$$Q_{11} = \alpha \lambda_{min}\{K_p\},$$
$$Q_{12} = -\frac{\alpha k_a}{2}[2k_C \| \dot{q}_d \|_M + \lambda_{Max}\{F_v\} + \lambda_{Max}\{K_d\}],$$
$$Q_{21} = -\frac{\alpha k_a}{2}[2k_C \| \dot{q}_d \|_M + \lambda_{Max}\{F_v\} + \lambda_{Max}\{K_d\}],$$
$$Q_{22} = \lambda_{min}\{K_d\} + \lambda_{min}\{F_v\} - k_c \| \dot{q}_d \|_M k_a^2 - \alpha k_a^2[\sqrt{n}k_C + \lambda_{Max}\{M(q)\}].$$

By Sylvester's Theorem, the matrix Q is positive definite if condition (22) is satisfied and

$$0 < \alpha < \alpha_2^*,$$

with α_2^* small enough. The positive definiteness of Q implies that $\dot{V}(t,\tilde{q},\dot{\tilde{q}})$ is negative definite for all $[\tilde{q}^T \; \dot{\tilde{q}}^T]^T \in \Omega_r$. Summarizing, we can always find a constant

$$0 < \alpha < \min\{\alpha_1^*, \alpha_2^*\}$$

such that $V(t,\tilde{q},\dot{\tilde{q}})$ is positive definite and $\dot{V}(t,\tilde{q},\dot{\tilde{q}})$ is negative definite into the set Ω_r implying the local asymptotic stability of the state space origin of the reduced model (23).

The conclusion of exponentially stability of the state space origin of the system (23) comes from using inequalities (27) and (25), by writing

$$\dot{V}(t,\tilde{q},\dot{\tilde{q}}) \leq -\frac{\lambda_{min}\{Q\}}{c_0} V(t,\tilde{q},\dot{\tilde{q}}),$$

which holds for all $[\tilde{q}^T \; \dot{\tilde{q}}^T]^T \in \Omega_r$.

On the other hand, the boundary layer model is

$$\frac{d}{d\sigma}\tilde{\vartheta} = -\tanh(\tilde{\vartheta}) + \dot{\tilde{q}}, \tag{28}$$

where $\sigma = t/\varepsilon$ and $\dot{\tilde{q}}$ is interpreted as a constant. Let us define the function

$$W(\tilde{\vartheta}_i) = \frac{1}{2}[\tanh(\tilde{\vartheta}_i) - \dot{\tilde{q}}]^2,$$

whose derivative with respect to the scaled time $\sigma = t/\varepsilon$ is given by

$$\frac{d}{d\sigma}W(\tilde{\vartheta}_i) = -\text{sech}^2(\tilde{\vartheta}_i)[\tanh(\tilde{\vartheta}_i) - \dot{\tilde{q}}]^2.$$

Then, for a region $|\tilde{\vartheta}_i| \leq \eta$, and $|\dot{\tilde{q}}_1| < 1$, we have that

$$\frac{d}{d\sigma}W(\tilde{\vartheta}_i) \leq -\text{sech}^2(\eta)W(\tilde{\vartheta}_i),$$

which by the comparison lemma [15] implies that

$$[\tanh(\tilde{\vartheta}(\sigma)) - \dot{\tilde{q}}] \to 0$$

with exponential convergence rate as the scaled time $\sigma = t/\varepsilon$ increases.

We have proven that the reduced model (23) is exponentially stable in an arbitrarily large region of initial conditions $[\tilde{q}(0)^T \; \dot{\tilde{q}}(0)^T]^T \in \Omega_r$, while the boundary layer system (28) is exponentially stable for any bounded initial condition $|\tilde{\vartheta}_i(0)| \leq \eta$, but subject to the condition

$$|\dot{\tilde{q}}_i| < 1.$$

Then, by Theorem 9.3 in [15] there exists ε^* such that $\varepsilon^* > \varepsilon > 0$ guarantees exponential stability of the closed-loop system (19)-(20) for all initial conditions

$$\left\| \begin{bmatrix} \tilde{q}(0)^T & \dot{\tilde{q}}(0)^T & \tilde{\vartheta}(0)^T \end{bmatrix}^T \right\| < 1,$$

which completes the proof of Proposition 1. □

5. REDESIGN OF THE LORÍA-NIJMEIJER CONTROLLER

In this Section we present a redesign of the controller (15)-(17). Motivated by the stability theory of singularly perturbed systems, we look for obtaining a design with a larger region of attraction than the one in the proof of Proposition 1. To this aim, consider the controller

$$\begin{aligned} \tau = {} & M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + g(q) \\ & + F_v\dot{q}_d + K_p \tanh(\Delta_p\tilde{q}) + K_d \tanh(\Delta_d\tilde{\vartheta}), \end{aligned} \tag{29}$$

where K_p, K_d are $n \times n$ diagonal positive definite diagonal matrices,

$$\Delta_p = \text{d}iag\{\delta_{p1}, \ldots, \delta_{pn}\},$$
$$\Delta_d = \text{d}iag\{\delta_{d1}, \ldots, \delta_{dn}\},$$

δ_{pi}, $\delta_{di} \geq 1$. The controller (29) is used along with the following linear filter to obtain $\tilde{\vartheta}$

$$\dot{q}_c = -b_f \tilde{\vartheta}, \tag{30}$$

$$\tilde{\vartheta} = q_c + b_f \tilde{q}. \tag{31}$$

Two differences are remarked with respect to the controller (15), (16), and (27): The incorporation of the gains Δ_p and Δ_d used into the saturation functions (hyperbolic tangent function), and the use of a linear filter to obtain the signal $\tilde{\vartheta}$.

By substituting the controller (29) into the robot dynamics (3) and using equations (30)-(31) the closed loop system can be written as

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}} \\ -M(q)^{-1}\Big[[C + C_d]\dot{\tilde{q}} + F_v \dot{\tilde{q}} + K_d \tanh(\Delta_d \tilde{\vartheta}) + K_p \tanh(\Delta_p \tilde{q}) \Big] \end{bmatrix}, \tag{32}$$

$$\varepsilon \frac{d}{dt}\tilde{\vartheta} = -\tilde{\vartheta} + \dot{\tilde{q}}, \tag{33}$$

and ε defined in (21). The following statement concerns the stability of the closed-loop system (32)-(33).

Proposition 2: Consider the condition

$$\frac{\lambda_{min}\{K_d\}}{\lambda_{Max}\{\Delta_d\}} + \frac{\lambda_{min}\{F_v\}}{\lambda_{Max}\{\Delta_d\}^2} - k_C \| \dot{q}_d \|_M k_a^2 > 0, \tag{34}$$

with k_a as defined in (2). Then, for all initial conditions

$$\left[\tilde{q}(0)^T \ \dot{\tilde{q}}(0)^T \ \tilde{\vartheta}(0)^T \right]^T \in S_r$$

with

$$S_r = \left\{ \| [\tilde{q}^T \ \dot{\tilde{q}}^T \tilde{\vartheta}^T]^T \| \le r \right\}$$

and r arbitrarily large, there always exists $\varepsilon^* > 0$ such that, for all $\varepsilon^* > \varepsilon > 0$, the state space origin of system (32)-(33) is exponentially stable.

Proof: This time, the quasi-steady state solution is

$$\tilde{\vartheta} = \dot{\tilde{q}}$$

and the reduced system is given by

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}} \\ -M(q)^{-1}\Big[[C + C_d]\dot{\tilde{q}} + F_v \dot{\tilde{q}} + K_d \tanh(\Delta_d \dot{\tilde{q}}) + K_p \tanh(\Delta_d \tilde{q})\Big] \end{bmatrix}, \tag{35}$$

whose unique equilibrium is the state space origin $[\tilde{q}^T \ \dot{\tilde{q}}^T]^T = 0$.

By using the Lyapunov function

$$V(t,\tilde{q},\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \sum_{i=1}^{n} \frac{k_{pi}}{\delta_{pi}} \ln(\cosh(\delta_{pi}\tilde{q}_i)) + \alpha \dot{\tilde{q}}^T M(q) \tanh(\Delta_p \tilde{q}),$$

with α a small enough strictly positive constant, δ_{pi}, $i = 1,\ldots,n$, the elements of the diagonal matrix Δ_p, the properties on hyperbolic functions described in Section 2, and the condition (34), the local exponential stability of the state space origin of the slow dynamics (35) can be proven. The proof of this claim follows similar steps that those in the analysis of the system (23) in the proof of Proposition 1. Specifically, a region of attraction of the trajectories of the slow dynamics (35) is the set Ω_r defined in (24), with r arbitrarily large.

On the other hand, the boundary layer model is

$$\frac{d}{d\sigma}\tilde{\vartheta} = -\tilde{\vartheta} + \dot{\tilde{q}},$$

where $\sigma = t/\varepsilon$ and $\dot{\tilde{q}}$ is interpreted as a constant. It becomes obvious that

$$[\tilde{\vartheta}(\sigma) - \dot{\tilde{q}}] \to 0$$

with exponential convergence rate as the scaled time $\sigma = t/\varepsilon$ increases with no restriction in the initial condition $\tilde{\vartheta}(0)$.

Therefore, by Theorem 9.3 in [15], there are sufficient conditions to claim that there exists ε^* such that

$$\varepsilon^* > \varepsilon > 0$$

guarantees the local exponential stability of the closed-loop system (32)-(33) for all initial conditions

$$\left\| \left[\tilde{q}(0)^T \ \dot{\tilde{q}}(0)^T \ \tilde{\vartheta}(0)^T \right]^T \right\| \le r.$$

with r arbitrarily chosen. □

It is noteworthy that in the paper by Burkov [16], the local exponential stability result of output feedback tracking controllers for mechanical systems was studied within the framework of the stability of singularly perturbed systems. However, the torque-bounded case was not included there.

The simplicity of the proposed analysis contrasts with the one presented previously. Specifically, in [1], the analysis of the closed-loop system trajectories is carried out by using a complex Lyapunov function, deriving an explicit bound for the observer gain b_f and a semiglobal stabilization result, while in our analysis the exponential stability of the overall closed-loop dynamics is proved by decomposing it in a slow and fast dynamics, and then the exponential stability of each dynamics is shown. The advantage of this procedure can be used to easily generate new exponentially stable torque-bounded controllers. Roughly speaking, this procedure consists in designing a controller/observer being able to produce a closed-loop system with exponentially stable slow and fast dynamics.

6. EXPERIMENTAL RESULTS

A direct-drive arm with two vertical rigid links (see Fig. 1) is available at the Control Laboratory of the *Instituto Tecnológico de La Laguna*, which was designed and built at the Robotics Laboratory of CICESE Research Center. High-torque, brushless direct-drive motors operating in torque mode are used to drive the joints without gear reduction. This is important from the control systems point of view, because the dynamics in this type of robots is highly nonlinear.

A motion control board based on a TMS320C31 32-bit floating-point microprocessor from Texas Instruments is used to execute the control algorithm. The control program is written in C programming language and executed in the control board at $h = 2.5$ ms sampling period. The maximum torque limits are τ_1^{Max} =150Nm and τ_2^{Max} =15Nm for motor 1 and 2, respectively. See the papers [17,18], for a description of the robot dynamic model.

The desired position trajectory $q_d(t)$ used in all experiments is given by

$$q_d(t) = \begin{bmatrix} 45[1-e^{-2.0t^3}]+10[1-e^{-2.0t^3}]\sin(7.50t) \\ 60[1-e^{-1.8t^3}]+125[1-e^{-1.8t^3}]\sin(1.75t) \end{bmatrix} \text{degrees.} \quad (36)$$

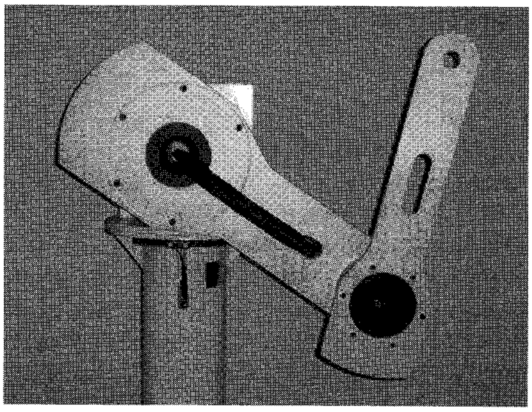

Fig. 1. Experimental robot arm.

With regard to the desired joint position (36), it is easy to show that its components satisfy $\| \dot{q}_d(t) \| \leq \| \dot{q}_d \|_M$ and $\| \ddot{q}_d(t) \| \leq \| \ddot{q}_d \|_M$ for all $t \geq 0$.

An important characteristic of the trajectory in (36) is that the desired position $q_d(t)$, velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$ are null in $t = 0$; thus the closed-loop system trajectories will not present rude transients if the robot starts at rest. It is noteworthy that the execution of the proposed trajectory $q_d(t)$ in (36) demanded a 75% of the torque capabilities, which was estimated through numerical simulation and verified with the experiments.

The time evolution of the position error $\tilde{q}$ reflects how well the control system performance is. The performance criterion considered in this paper was the Root Mean Square (RMS) value of the velocity error, computed on a trip of time T, that is,

$$RMS[\dot{\tilde{q}}] = \sqrt{\frac{1}{T}\int_0^T \| \dot{\tilde{q}}(\sigma) \|^2 \, d\sigma} \text{ degrees/s.} \quad (37)$$

In practice, the discrete implementation of the criterion (37) leads to

$$RMS[\dot{\tilde{q}}] = \sqrt{\frac{1}{T}\sum_{k=0}^{i} \| \dot{\tilde{q}}(kh) \|^2 \, h} \quad \text{degrees/s,}$$

where $h = 2.5$ms is the sampling period and $T = 10$s.

We have tested three controllers with the aim of evaluating the performance of the proposed redesign. The main goal of the experimental evaluation was to assess the tracking performance of the controllers

- PD+ [10], which is a well-known passivity-based scheme,
- Loría and Nijmeijer in equation (15)-(17), and
- the new design in equation (29)-(31).

Let us first describe the results concerning the PD+ control [10], which is written as

$$\begin{aligned} \tau = {} & M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) \\ & + F_v\dot{q} + K_d\dot{\tilde{q}} + K_p\tilde{q}, \end{aligned} \quad (38)$$

where $\tilde{q} = q_d - q$ denotes the tracking error and the joint velocity measurements $\dot{q}$ are approached via simple numerical differentiation, i.e.,

$$\dot{q}_i(hk) = \frac{q_i(hk) - q_i(h[k-1])}{h}, \quad (39)$$

where h is the sampling period and k is the discrete time. It is well known that the approach (39) is very common in many robot control platforms to obtain an estimation of the velocity measurements. The controller was tested using the following proportional

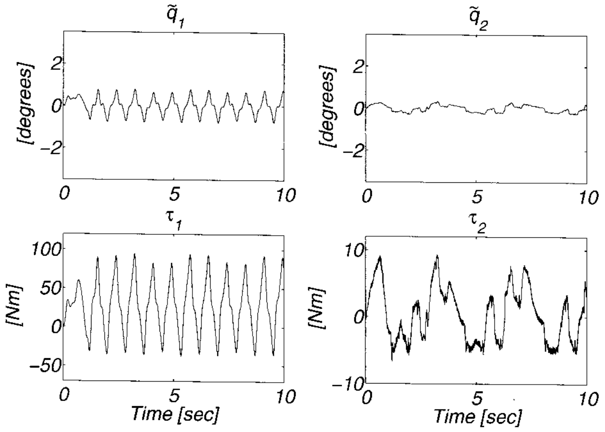

Fig. 2. Performance of the PD+ controller.

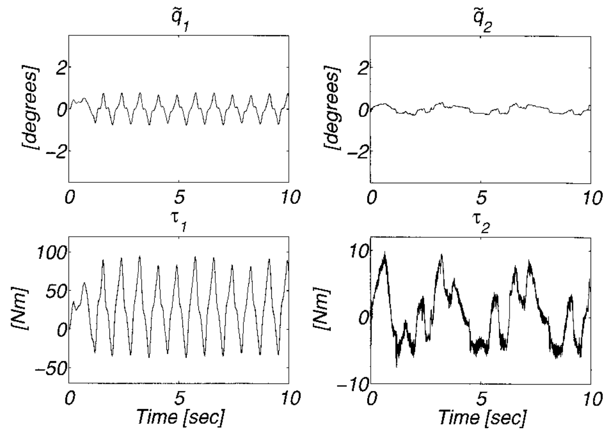

Fig. 3. Performance of the Loría-Nijmeijer controller.

and derivative control gains

$$K_p = diag\{3500,1000\}\, 1/s^2, \quad K_d = diag\{45,15\}\, 1/s. \tag{40}$$

Let us notice that the gains in (40) were chosen by trial and error until obtaining a reasonable performance in the tracking of the desired joint position $q_d(t)$, i.e., a relatively small bound of the maximum values of $\tilde{q}_1(t)$ and $\tilde{q}_2(t)$. Fig. 2 shows the time evolution of tracking errors $\tilde{q}_1(t)$, $\tilde{q}_2(t)$, and applied torques $\tau_1(t)$, $\tau_2(t)$ for this controller.

Further improvement could have been obtained in the tracking performance, but paying the price of a noisy control action which would excite other dynamics such as vibrating modes of the mechanical structure.

Since all the tested controllers have a proportional-derivative structure, we have used the same numerical value of the gains in (40) for the implementation of the Loría and Nijmeijer controller in (15) and the new controller in (29).

With respect to output feedback tracking control, the Loría and Nijmeijer controller, described in equations (15) -(17), was implemented with the gains in (40) and

$$b_f = 600.0\, [1/s] \tag{41}$$

in the nonlinear filter (16)-(17). The results of the experiment are depicted in Fig. 3.

Finally, we have implemented the controller redesigned in (29), which uses the linear filter (30)-(31). In order to keep a fair comparison with respect to the PD+ and Loría and Nijmeijer controllers, we used the numerical values of K_p and K_d in (40), b_f in (41), and

$$\Delta_p = diag\{1.5, 1.5\}\, [\text{dimensionless}],$$
$$\Delta_d = diag\{1.0, 1.0\}\, [\text{dimensionless}].$$

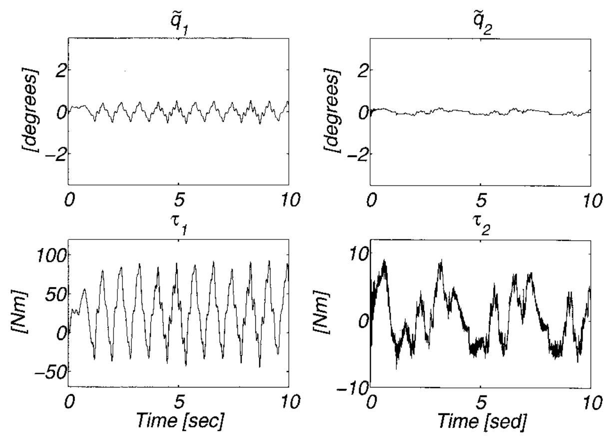

Fig. 4. Performance of the Loría-Nijmeijer redesigned controller.

Table 1. Performance of the controllers: PD+, Loría and Nijmeijer (LN), and Loría and Nijmeijer redesigned (LNR).

Index	PD+	LN	LNR
$\max\{\lvert\tilde{q}_1(t)\rvert\}$ deg	0.78	0.79	0.58
$\max\{\lvert\tilde{q}_2(t)\rvert\}$ deg	0.34	0.34	0.21
$RMS[\tilde{q}]$ deg	0.418	0.419	0.267

The results are illustrated in Fig. 4.

All the tested controllers assure theoretically that the position error $\tilde{q}(t)$ must vanish as time increases. In practice, Figs. 2 to 4 reveal an steady state oscillatory behavior. This is due to several factors such as uncompensated Coulomb friction and the discrete controller implementation.

Table 1 summarizes the information about the tracking performance of the three schemes. In addition, Fig. 5 shows a bar chart of the $RMS[\tilde{q}]$ value computed for the three tested controllers. The performance of the PD+ controller (38) and the Loría and Nijmeijer scheme (15) is very similar. The reason is that in the situation of a very small tracking error

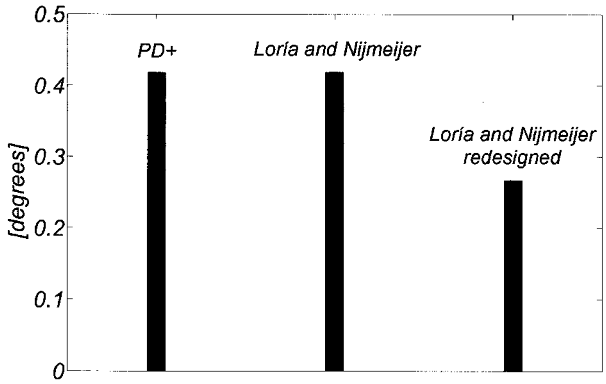

Fig. 5. Bar chart of the $RMS[\tilde{q}]$ value computed for the three tested controllers.

$\tilde{q}$ the structure of these controllers is very similar. In addition, the best performance of the three controllers was obtained with the Loría and Nijmeijer redesigned controller (29) because it presented the smallest values of $\max\{|\tilde{q}_1(t)|\}$, $\max\{|\tilde{q}_2(t)|\}$ and $RMS[\tilde{q}]$. This is also appreciated in the Fig. 5, which depicts the bar chart of the $RMS[\tilde{q}]$ value.

The comparison reveals that all the controllers work efficiently since the tracking errors are relatively close to the performance of the PD+ controller (38), although the Loría and Nijmeijer redesigned controller (29), which incorporates a gain into the saturation function (hyperbolic tangent function) and uses a linear filter, presents the lowest tracking error $\tilde{q}(t)$. The explanation is that the redesigned controller (29) incorporates the extra parameters Δ_p and Δ_d whose numerical value has effect in increasing the slope of the profile of the hyperbolic tangent function in the proximity of the origin.

7. CONCLUDING REMARKS

In this paper the output feedback tracking control of robot manipulators subject to constrained torques was studied. We presented a new stability analysis based on singularly perturbed system theory for a controller already proposed in the literature, the so-called Loría and Nijmeijer controller. In addition, a redesign of this controller was also proposed, consisting in using gains into the saturation functions and incorporating a linear observer. An experimental study was also provided, showing that the proposed redesign has better performance than a traditional trajectory tracking controller, the PD+ scheme, and the Loría and Nijmeijer algorithm.

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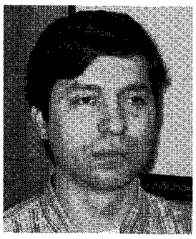

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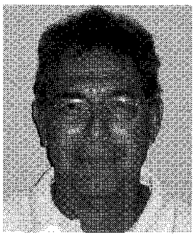

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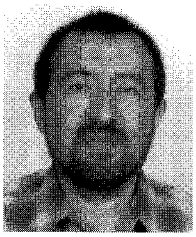

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