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# Sound Visualization in Time Domain by Using Spatial Envelope
# 공간 포락을 적용한 시간 영역 음장 가시화[#]

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## ABSTRACT

Acoustic holography exhibits the spatial distribution of sound pressure in time or frequency domain. The obtained picture often contains far more than what we need in practice. For example, when we need to know only the locations and overall propagation pattern of sound sources, a method to show only what we need has to be introduced. One way of obtaining the necessary information is to use envelope in space. The spatial envelope is a spatially slowly-varying amplitude of acoustic waves which contains the information of sources' location. A spatial modulation method has been theoretically developed to get a spatial envelope. By applying the spatial envelope, not only the necessary information is obtained but also computation time is reduced during the process of holography. The spatial envelope is verified as an effective visualization scheme in time domain by being applied to complicated sound fields.

## 요 약

음향홀로그래피는 시간 영역 혹은 주파수 영역에서 음압의 공간 분포를 가시화하는 방법이다. 그러나, 재구성된 음압은 종종 실제적으로 우리가 원하는 음장의 정보보다 훨씬 많은 정보를 가지고 있다. 예를 들어, 우리가 소음 제어를 위해 필요로 하는 정보는 소음원의 위치와 전체적인 방사형태의 정보만으로 충분하다. 따라서, 이렇게 필요로 하는 정보만을 보여줄 수 있는 방법이 필요하다 볼 수 있다. 이를 위한 한 가지 방법으로 공간에서의 포락을 생각할 수 있다. 공간복소포락은 소음원의 위치와 느리게 변하는 전체적인 변화에 대한 정보를 가지고 있는 포락 신호를 의미한다. 공간복소포락을 이용함으로써, 우리는 음장에서 불필요한 정보를 제거하여 필요한 정보를 얻을 수 있을 뿐만 아니라, 가시화 과정에서의 연산 시간도 줄일 수 있다. 이 공간포락을 얻기 위한 공간변조 방법을 이론적으로 유도하고 제안한다. 그리고, 여러 개의 단극음원에 의해 형성되는 복잡한 음장을 제안하는 방법을 이용해 단순화시킨 결과를 통해 제안하는 방법에 대한 타당성을 검증한다.

## 1. Introduction

Various papers have been published on the acoustic pressure visualization in time domain (1~5). The visualization enables us to observe the locations and shapes of not only steady-state but time-variant sources. The methods employ Kirchhoff-Helmholtz integral equation to

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reconstruct sound field where we want to predict. Those methods, therefore, take up the ways of convolving time data or taking the inverse Fourier transform of spatial distribution of all the frequencies to predict the sound field. Consequently, not only the calculation time is long but also the distributions obtained by the methods have much more information than what we need to know: for example, sources' locations and their overall propagation patterns, which are essential information for noise control.

There are two problems on visualizing the acoustic pressure distribution by using acoustic holography. First one is processing time problem and the other one is the analysis problem(6). The latter addresses how we have to analyze the results of acoustic holography. The former is related with how the results can be obtained as fast as we can. The visualization of acoustic pressure distribution in time domain definitely requires fast calculation. The issued problems can be tackled by reducing or selecting the data that can provide the information we need. These two issues motivate us to study the spatial envelope to apply the methods to visualize sound pressure in space.

The basic concept of spatial envelope of sound field and the way to get the spatial envelope are proposed by Park and Kim(7). They defined spatial envelope conceptually in accordance with the temporal envelope and proposed modulation method in wavenumber domain to get the spatial envelope with plane waves as a simplest case. They also extended the method to several monopole sources in time domain(8,9). The proposed methods are, however, mathematically verified only for plane waves and also the results of modulation are examined analytically.

In this paper, the definition of spatial envelope is reviewed with a monopole source which conveys more general definition of spatial envelope. The modulation method is also reconsidered and developed in wave-number domain.

## 2. Spatial Envelope

### 2.1 Definition of Spatial Envelope

The envelope of acoustic waves can be regarded as a band signal in frequency domain. The banded acoustic waves propagate in space with some wavenumber which constitute a certain band in wavenumber domain by the dispersion relation. As a result, the spatial envelope would be defined along the same line as the temporal one.

To begin with, let $p(r,t)$ is acoustic pressure induced by a monopole which consists of a slowly varying amplitude $a_S(r)$ on position $r$ and $a_T(t)$ on time, $t$. In addition, let's assume that the acoustic pressure is a real and narrow band signal, which has a center frequency $f_c$ and phase $\varphi_T$ and $\varphi_S$ on time and space respectively. Then, the pressure can be written as

$$p(r,t)=a_S(r)a_T(t)\frac{\cos\{2\pi f_c t-k_c r+\varphi_S+\varphi_T\}}{r}, \qquad (1)$$

where $k_c$ is a wavenumber corresponding to the center frequency ($k_c=2\pi f_c/c$, $c$ is the speed of propagation). Eq. (1) is also rewritten as

$$p(r,t)=\mathrm{Re}\left\{\boldsymbol{p}_{CE}(r,t)e^{-j(2\pi f_c t-k_c r)}\right\}, \qquad (2)$$

where the bold indicates complex value. Then, the $\boldsymbol{p}_{CE}(r,t)$ is obtained as

$$\boldsymbol{p}_{CE}(r,t)=\frac{a_S(r)a_T(t)e^{-j\{\varphi_S+\varphi_T\}}}{r}, \qquad (3)$$

which is the very complex envelope on time and space.

The spatial envelope can be defined from Eq. (3) by just taking the spatial part of $\boldsymbol{p}_{CE}(r,t)$.

That is,

$$p_{CE}(r) = \frac{a(r)e^{-j\varphi_s}}{r}. \tag{4}$$

The remaining temporal complex envelope corresponds to the conventional definition derived by Rice[(10)].

The spatial complex envelope is similar to the temporal one except the $1/r$ term, which is related with the location of the monopole source. In other words, the spatial envelope contains not only slow fluctuation in space but the spatial information of sources: e.g. locations, distributions, shape, etc.

### 2.2 Spatial Envelope in Wavenumber Domain

Equation (2) implies that the complex envelope can be obtained by removing the component of center frequency. To understand the properties of complex envelope and find modulation method for obtaining the spatial envelope, let's take into account the following complex envelope as a special but important example,

$$p_{CE}(r,t) = \frac{\cos(2\pi f_m t - k_m r)}{r}. \tag{5}$$

Then $p(r,t)$ is made up of two frequency components, $f_1 = f_c + f_m$ and $f_2 = f_c - f_m$. Therefore, two corresponding wavenumber spectra would be presented by 2-D Fourier transform.

The pressure field of interest is observed in familiar Cartesian coordinate to understand the phenomena of modulation easily, and the pressure filed is generated by a monopole located at $(x_1, y_1)$ to see the effect of source's spatial information. Then, the wavenumber spectra of $p(r,t)$ are expressed as

$$P(k_x,k_y,z_h;f_1) = j\pi \frac{e^{-jz_h\sqrt{k_1^2-k_x^2-k_y^2}}}{\sqrt{k_1^2-k_x^2-k_y^2}} e^{j(x_1k_x+y_1k_y)} \tag{6}$$

$$P(k_x,k_y,z_h;f_2) = j\pi \frac{e^{-jz_h\sqrt{k_2^2-k_x^2-k_y^2}}}{\sqrt{k_2^2-k_x^2-k_y^2}} e^{j(x_1k_x+y_1k_y)}. \tag{7}$$

The spectra are depicted in Fig. 1 whose left figures correspond to $f_1$ and right ones follow $f_2$.

As we can see in Fig.1, the fractional term represents the specified wavenumber, $k_1$ or $k_2$, and the remaining oscillating term contains the information of source location in common.

The wavenumber spectra of the complex envelope, $p_{CE}(r,t)$, are obtained similarly with Eq. (6) and (7). The different thing is the wavenumbers in the fractional terms, namely, $k_m$ or $-k_m$ instead of $k_1$ or $k_2$. Figure 2 shows us the spectra of spatial envelope. By the comparison between Fig. 1 and Fig. 2, the modulation method of spatial distribution has to meet the following requirements: the wavenumbers are changed into corresponding

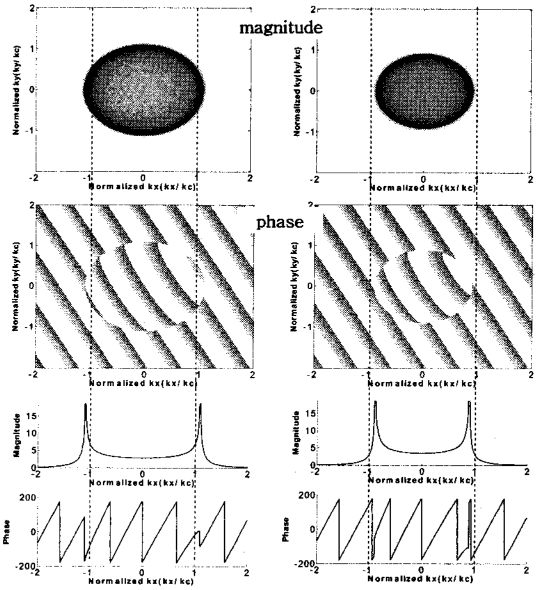

**Fig. 1** Wavenumber spectra with respect to each frequency, which are normalized with respect to the center wavenumber, $k_c$; the left spectra are according to $f_1$ and the right ones are $f_2$

low wavenumbers constituting the complex envelope but the information of source location must not be affected.

## 3. Spatial Modulation Method

The analogy between the temporal and spatial complex envelope indicates that the spatial modulation would also have the same form as the temporal one does. In other words, as the frequencies are moved by the complex modulation, the wavenumber spectra are also changed to match the shifted frequencies without regard to the source's spatial information. This means that the fractional term related with wavenumber should be changed; $k_1$ should be $k_m$ and $k_2$ are changed to $-k_m$.

Besides, it is also noteworthy that the phase of wavenumber spectra corresponding minus frequency, $-f_m$, should have minus phase in contrast to the positive one.

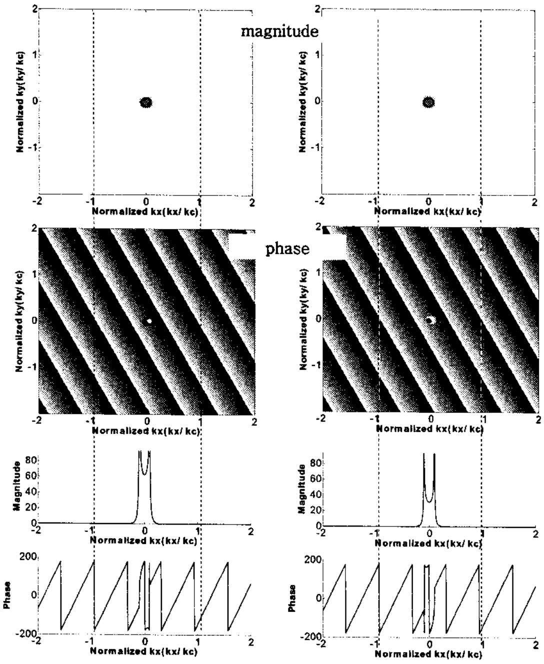

Fig. 2 Wavenumber spectra of spatial envelope with respect to each frequency, which are normalized with respect to the center wavenumber, $k_c$; the left spectra are according to $f_m$ and the right ones are $-f_m$

The plain method to change the wavenumber regardless of the information of source location is to multiply Eq. (6) and (7) by inverse fractional term consisting of $k_1$ or $k_2$ and by the term related with low wavenumber, $k_m$ or $-k_m$ as well in each spectrum as depicted in Fig. 3.

Eventually, the spatial complex envelope is obtained as,

$$p_m(x,y,z;t)=\frac{1}{2}\left\{\frac{e^{-jk_m r}}{r}e^{j2\pi f_m t}+\frac{e^{jk_m r}}{r}e^{-j2\pi f_m t}\right\}=\frac{\cos(2\pi f_m t-k_m r)}{r} \tag{8}$$

where $r=\sqrt{(x-x_1)^2+(y-y_1)^2+z_h^2}$ and subscript $m$ denotes that the value is modulated. From Eq. (8), the complex envelope on space and time is obtained correctly as we previously defined in Eq. (5), and described in Fig. 4.

## 4. Acoustic Holography in Time Domain

The conventional holographic reconstruction process is performed by multiplying wavenumber spectra by propagators in wavenumber

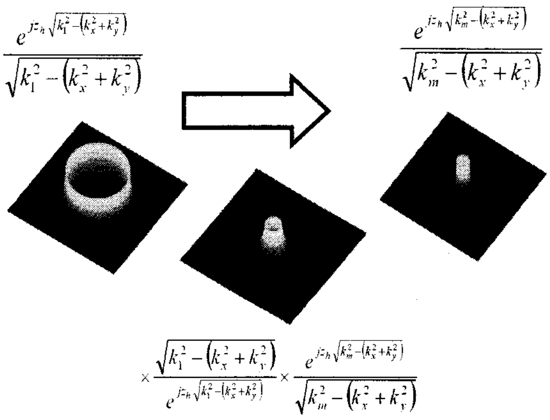

Fig. 3 Modulation process in wavenumber domain for the $k_1$ component; all the spectra are normalized with respect to $k_1$

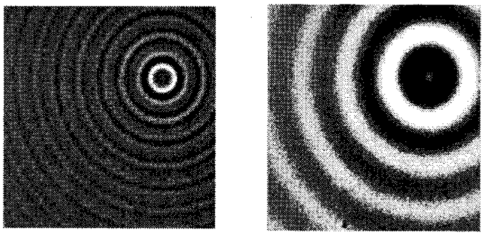
(a) (b)

**Fig. 4** Sound field; (a) initial sound field composed of band signal, (b) its envelope sound field

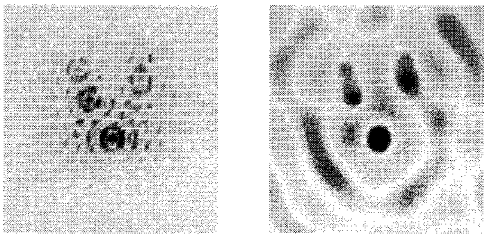
(a) (b)

**Fig. 5** Holographic reconstructed sound fields; (a) initial sound field composed of band signal radiated from 5 monopole sources, (b) its envelope field

domain with respect to each frequency(11).

In spite of that the modulation process is added to the holographic procedure, the calculation time is reduced because of decreased time data by temporal modulation. The reduced calculation time is directly proportional to the reduced temporal data. Therefore, the temporal envelope contributes to the reduction of the calculation time problem of acoustic holography. Furthermore, the analysis problem is successfully dealt by spatial envelope, which envisages overall fluctuation of acoustic waves in space.

The holographic reconstructed spatial envelope in time domain is obtained after inverse temporal Fourier transform of selected band signal, which are spatially modulated in wavenumber domain. To observe the merits of envelope sound field, let's compare the results of conventional holography and proposed method with the pressure field, which is composed of several monopole sources. Fig. 5 shows that the results at an instance.

## 5. Conclusions

We introduced complex envelope analysis to improve holographic reconstructed sound field for fast calculation and effective analysis. We defined the spatial envelope which contains spatially slow fluctuation and sources' spatial information. The spatial modulation method also derived in wavenumber domain and verified the method with the simplest but important example: a monopole source. Furthermore, it is verified that the proposed method can be applied to the holographic reconstructed process and effective to calculation time problem by applying temporal envelope and analysis problem of holography by applying spatial envelope.

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