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AN ITERATIVE DISTRIBUTED SOURCE METHOD FOR THE DIVERGENCE OF SOURCE CURRENT IN EEG INVERSE PROBLEM

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ABSTRACT. This paper proposes a new method for the inverse problem of the three-dimensional reconstruction of the electrical activity of the brain from electroencephalography (EEG). Compared to conventional direct methods using additional parameters, the proposed approach solves the EEG inverse problem iteratively without any parameter. We describe the Lagrangian corresponding to the minimization problem and suggest the numerical inverse algorithm. The restriction of influence space and the lead field matrix reduce the computational cost in this approach. The reconstructed divergence of primary current converges to a reasonable distribution for three dimensional sphere head model.

1. INTRODUCTION

Electroencephalography (EEG) is one of noninvasive tools to measure the potential difference on the scalp surface, which is the result of movement of ions, the *primary current*, within activated regions in the brain. EEG has several strong aspects as a tool of exploring brain activity; for example, its time resolution is very high, i.e., on the level of a single millisecond, compared to other methods of looking at brain activity, such as PET and fMRI which have time resolution between seconds and minutes. EEG measures the brain's electrical activity directly, while other methods record changes in blood flow (e.g., SPECT, fMRI) or metabolic activity (e.g., PET) which are indirect markers of brain electrical activity.

The reconstruction of the primary current distribution using EEG data which is the electric potential measured at sensors on the scalp surface is the main goal of EEG research and called the *inverse problem* of EEG. Its solution requires the repeated simulation of the electric potential distribution in the head by a given primary current, which is called the *forward problem*.

It has been known that there are primary current distributions which induce the same EEG data [1]. This is an indication of the fact that the solution to the inverse problem is not unique.

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The non-uniqueness of the inverse problem implies that assumptions on the model as well as anatomical and physiological *a priori* knowledge should be taken into account to obtain a unique solution. In order to make the solution unique, additional conditions are imposed on the solution. e.g., small number of current dipoles or spatial smoothness of the primary current distribution.

The equivalent dipole method [2] is based on the moving dipole model. In this method primary current in the brain are approximated by a small number of current dipoles, and their locations and moments are estimated by fitting to the EEG data. In BESA (Brain Electric Source Analysis) [3] and MUSIC (Multiple Signal Classification) [4], the locations of dipoles are assumed to be fixed during certain time interval, and they are determined from the EEG data measured repeatedly during that time interval. These approaches can be easily and intuitively understood by clinicians. Moreover, since no restriction is imposed on the dipoles, this method seems adequate so long as dipoles are expected to be localized. However a key problem is the correct estimation of the number of dipoles. In these model, only the cross product of location and moment of dipole is determined.

Alternatively continuous distribution is discretized as an array of numerous dipoles sited at the activated region. Since the model allows a large degree of freedom, there are infinite numbers of primary current distribution that reconstruct the measured EEG data. In order to make the solution unique, for example, spatial smoothness of the primary current distribution is used in the Minimum Norm Method (MNM) and the Low Resolution Electromagnetic Tomography (LORETA) [5]. In such distributed source models, however, the additional constraint has no connection with the electro-physiological phenomena and it is quite difficult to evaluate the adequacy of this constraint.

One conventional iterative method, the Focal Underdetermined System Solution (FOCUSS), has another problem in convergence. The range of source distribution is getting smaller with iterations and eventually converges to a point distribution. Therefore FOCUSS is required to determine the proper number of iterations.

As one of iterative distributed source methods, the adjoint state approach in continuous case was proposed in [6] where the source currents are obtained. However the method can not avoid the intrinsic ill-posedness of the EEG inverse problem. In this paper we modify this approach to reconstruct the divergence of source current instead of source current itself and implement the method for EEG inverse problem in discrete case. The reconstructed source current converges to a reasonable distribution for three dimensional sphere head model.

2. Forward problem formulation

2.1. **Maxwell equations.** We begin with introduction of notations. Let **E** and **D** be the electric field and electric displacement, respectively, ρ the electric free charge density, ϵ the electric permeability and **J** the electric current density. By μ we denote the magnetic permeability and by **H** and **B** the magnetic field and induction, respectively.

In the low frequency band (below 2000Hz) the temporal derivatives can be neglected in the Maxwell equations of electrodynamics [7]. Therefore, the electric and magnetic fields can be described by the quasi-static Maxwell equations

$$\nabla \cdot \mathbf{D} = \rho, \tag{2.1a}$$
$$\nabla \times \mathbf{E} = 0, \tag{2.1b}$$
$$\nabla \times \mathbf{B} = \mu\mathbf{J}, \tag{2.1c}$$
$$\nabla \cdot \mathbf{H} = 0. \tag{2.1d}$$

The electric field is expressed as a negative gradient of a scalar electric potential u,

$$\mathbf{E} = -\nabla u. \tag{2.2}$$

The electric current density is generally divided into two part, the so-called *primary current*, $\mathbf{J}_p$ and *secondary current* $\sigma\mathbf{E}$,

$$\mathbf{J} = \mathbf{J}_p + \sigma\mathbf{E}, \tag{2.3}$$

where σ denotes the 3×3 conductivity tensor.

2.2. **Electric forward problem.** Since the divergence of curl of a vector is zero, taking the divergence of (2.1c) and using (2.2) and (2.3) give the Possion equation

$$\nabla \cdot (\sigma\nabla u) = \nabla \cdot \mathbf{J}_p \quad \text{in } \Omega\,. \tag{2.4}$$

Let us assume that all fields of biological origin are quasi-static [8]. This allows us to establish the relationship between the electric potential u and the primary current $\mathbf{J}_p$ that is movement of ions within the dendrites of the large pyramidal cells of activated regions in the cortex of the human brain through (2.4). It describes the electric potential distribution in the head domain Ω due to the primary current $\mathbf{J}_p$ in the brain. For the forward problem which is assumed that the primary current and the conductivity distribution in the head are given, the equation has to be solved for the unknown electric potential distribution u with the boundary condition

$$\sigma\frac{\partial u}{\partial \mathbf{n}} = \sigma\nabla u \cdot \mathbf{n} = 0 \qquad \text{on } S = \partial\Omega \tag{2.5}$$

and an additional reference point with given potential, i.e.,

$$u_{ref} = 0.$$

3. Inverse problem formulation

The inverse EEG problem aims to reconstruct the primary current distribution by only use of EEG data in the conductivity head model. However, since Eq. (2.4) establishes that the electric potential field u is generated by the divergence of primary current, we reconstruct the divergence of primary current $I_p = \nabla \cdot \mathbf{J}_p$ which is referred as the current source density [9].

3.1. **Mathematical formulation.** Let N be the number of EEG sensors. We measure the difference of electric potential at several locations $x_1, x_2, \ldots, x_N$ on the scalp and at a reference location x_0. Let $m_1, m_2, \ldots, m_N$ be those measured EEG data. In the EEG problem, we want to estimate the divergence of primary current I_p to produce the electric potential as close as possible to the measured EEG data. This means that if we compute the electric potential distribution $\tilde{u}$ for such I_p with Eq. (2.4), then the error function

$$\varphi(u) = \frac{1}{2}\sum_{i=1}^{N}(u(x_i) - u(x_0) - m_i)^2$$

achieves a minimum at $\tilde{u}$. We form the Lagrangian as

$$\mathcal{L}(u, w, I_p) = \varphi(u) + \int_\Omega (\nabla \cdot (\sigma \nabla u) - I_p) w \, dx,$$

where w is the Lagrangian multiplier. We then use the integration by part and the boundary condition (2.5) to obtain

$$\mathcal{L}(u, w, I_p) = \varphi(u) - \int_\Omega \sigma \nabla u \cdot \nabla w \, dx - \int_\Omega I_p w \, dx \, . \tag{3.1}$$

This form of the Lagrangian is suitable for computing Gateaux derivatives with respect to I_p. From the Gateaux derivative of the Lagrangian with respect to I_p in (3.1), we obtain

$$\frac{\partial \mathcal{L}}{\partial I_p} = -w.$$

This equation says that in order to find the divergence of primary current that yields a potential distribution that minimizes $\varphi(u)$, we should follow the descent direction w at every point x of the interested region.

Using the integration by part again with imposed boundary condition for w such that $\sigma \nabla w \cdot \mathbf{n} = 0$ yields the form that is suitable for taking Gateaux derivatives with respect to u:

$$\mathcal{L}(u, w, I_p) = \varphi(u) + \int_\Omega \nabla \cdot (\sigma \nabla w) u \, dx - \int_\Omega I_p w \, dx \, .$$

Taking the Gateaux derivatives of $\mathcal{L}$ with respect to the function u, we obtain

$$\delta \mathcal{L} = \sum_{i=1}^{N}(u(x_i) - u(x_0) - m_i)(\delta u(x_i) - \delta u(x_0)) + \int_\Omega \nabla \cdot (\sigma \nabla w) \delta u \; dx \, .$$

This must be equal to 0 for all variations δu of u, hence the adjoint state equation is

$$\begin{aligned} \nabla \cdot (\sigma \nabla w) + f &= 0 \quad \text{in } \Omega, \\ \nabla w \cdot \mathbf{n} &= 0 \quad \text{on } S, \end{aligned} \tag{3.2}$$

where $f(x) = \sum_{i=1}^{N}(u(x_i) - u(x_0) - m_i)(\delta(x - x_i) - \delta(x - x_0))$ with the usual Dirac delta function $\delta(x)$.

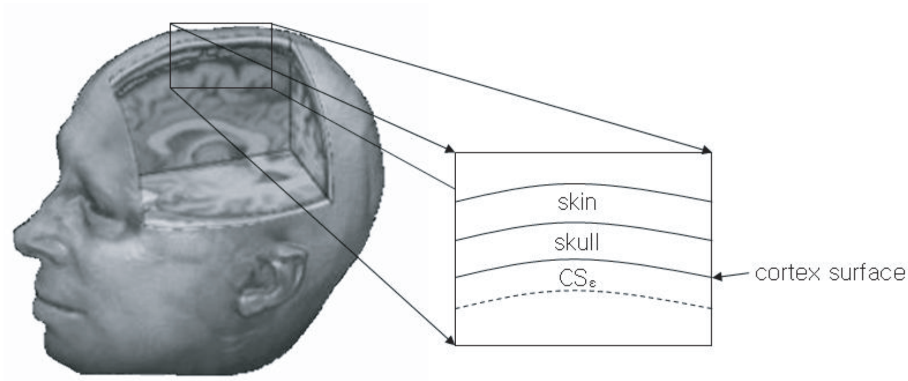


FIGURE 1. Head model

3.2. **Restriction to influence space.** Let the influence space be the activated region where the primary current can be produced. Physically, the primary current cannot be produced at the region out of the brain and the signal from deep part of the brain is not detected with EEG sensor. Therefore we consider only the cortex surface region CS_ϵ inside the cortex surface whose distance from the cortex surface is less than ϵ as described in Fig. 1, where ϵ depends on the measurability of EEG machine. Then the Lagrangian form (3.1) is expressed with characteristic function χ_{cs_ϵ} as

$$\mathcal{L}(u, w, I_p) = \varphi(u) - \int_\Omega \sigma \nabla u \cdot \nabla w \, dx - \int_\Omega I_p \chi_{\mathrm{cs}_\epsilon} w \, dx \,. \tag{3.3}$$

As considering the restricted influence space, from (3.3), we have

$$\frac{\partial \mathcal{L}}{\partial I_p} = -w \chi_{cs_\epsilon}. \tag{3.4}$$

3.3. **Algorithm: the adjoint state approach.** We give a description for the algorithm to solve the inverse EEG problems that is the constrained minimization problem as follows:

(1) Start with an estimate $I_p^{(0)}$ and set $i = 0$.
(2) Solve the state equation (2.4) with $I_p = \nabla \cdot \mathbf{J}_p$ to obtain $u^{(i)}$.
(3) Solve the adjoint state equation (3.2) with $u^{(i)}$ to obtain $w^{(i)}$.
(4) With $u^{(i)}$ and $w^{(i)}$, determine the gradient $\frac{\partial \mathcal{L}}{\partial I_p}$ by (3.4) and $I_p^{(i+1)}$ using the steepest descent method.
(5) If $I_p^{(i+1)}$ is not close to $I_p^{(i)}$, then go to step 2 and set $i = i + 1$, else stop.

The main advantage of this adjoint state approach is to find I_p instead of $\mathbf{J}_p$. It is well known that finding $\mathbf{J}_p$ is severely ill-posed. On the contrary, finding I_p gives more stable results.

4. Numerical aspects

For the numerical solution of the method, we choose a finite-dimensional subspace with dimension M, the number of nodes in Ω and a standard finite element basis $\phi_1,\ldots,\phi_M$. Applying the finite element techniques to Eq. (2.4) with $I_p = \nabla \cdot \mathbf{J}_p$ yields an $M \times M$ system of linear equations

$$AU = \tilde{I}_p\,. \tag{4.1}$$

Eq. (3.2) yields a similar $M \times M$ system of linear equations to (4.1).

4.1. **Lead field matrix.**

In each iteration of the proposed algorithm, we solve the state equation (2.4) for u and the adjoint state equation (3.2) for w. Additionally, in the modified steepest decent method to determine how far we go to the negative gradient direction we solve the state equation at least twice [10]. Therefore, we solve the state equation three times and the adjoint equation once in each iteration of the algorithm.

To reduce the computational cost, we suggest computing the *lead field matrix* [11] denoted by B, which maps $\tilde{I}_p$, the vector corresponding to I_p to U_{EEG}, the vector of electric potential at the sensors,

$$B\tilde{I}_p = U_{EEG}. \tag{4.2}$$

When the restriction matrix R is a mapping from the potential vector onto the sensors such that

$$RU = U_{EEG} \tag{4.3}$$

with only one nonzero entry with the value 1 in each row, B is computed as

$$B = RA^{-1}.$$

by the relations (4.1) and (4.3). To find B, since we A is symmetric, we solve

$$AB^T = R^T\,. \tag{4.4}$$

If the lead field matrix B is precomputed, then the estimated electric potential U_{EEG} at EEG sensors, which is needed in the state equation and the steepest decent method to determine the line search is computed by (4.2) instead of solving the linear system (4.1). It means that we only need to solve the adjoint state equation for w in each iteration of the inverse method.

4.2. **Numerical inverse algorithm.**

The proposed method for inverse EEG problem to reconstruct the divergence of the primary current is described by following steps:

(1) Compute the stiffness matrix A and the lead field matrix B with a given conductivity.
(2) Start with an estimated divergence of primary current $\tilde{I}_p^{(0)}$ and set $i = 0$.
(3) Obtain U_{EEG} from (4.2) with $\tilde{I}_p^{(i)}$.
(4) Solve the adjoint state equation (3.2) to obtain w with U_{EEG} and EEG data.
(5) Determine a new value $\tilde{I}_p^{(i+1)}$ with the line search α and the gradient $-w$ using the steepest decent method:

$$\tilde{I}_p^{(i+1)} = \tilde{I}_p^{(i)} + \alpha w.$$

Here we compute U_{eeg} from (4.2) to find the line search α; see [10].

TABLE 1. Computation time (sec) of with/without B

Computation		with B	without B
Precomputation	Meshing	21.1	
	A	0.6	
	B	0.3	0
Inverse iteration (# of iterations)		803.2 (121)	2023.1 (121)
Total time		825.0	2044.7

(6) If $\tilde{I}_p^{(i+1)}$ is not close to $\tilde{I}_p^{(i)}$, then go to step 3 and set $i = i + 1$, else stop.

In the inverse algorithm, the step 1 is called the precomputation step and the steps 2 to 6 are called the inverse iteration.

As a stop condition for the inverse iteration in the step 6, we use the relative error of the estimated divergecne of primary current:

$$\frac{\|\tilde{I}_p^{(i+1)} - \tilde{I}_p^{(i)}\|}{\|\tilde{I}_p^{(i)}\|} < 10^{-3}. \tag{4.5}$$

The inverse method with the precomputed B is more efficient than the inverse method without B. Because with B, we additionally solve (4.4) in the step 1, but in the steps 3 and 5 we obtain U_{EEG} by only the product of B and $\tilde{I}_p$ instead of solving (4.1). Table 4.2 shows the computational costs of two methods with $h = 1/32$ and the stop condition (4.5).

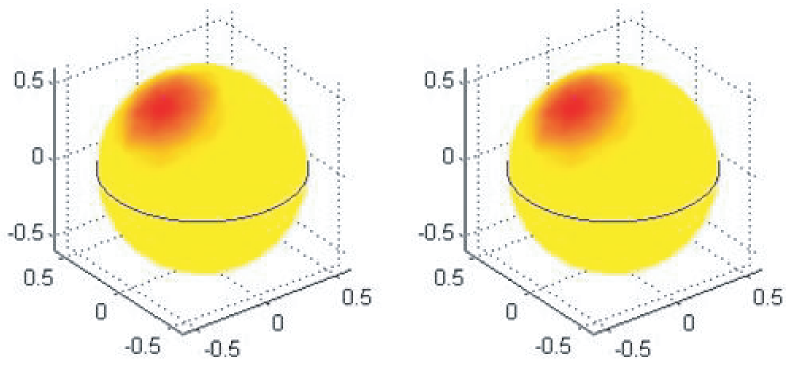
4.3. **Numerical simulation.** The numerical simulation is performed with a three dimensional sphere domain divided into three layers with radius 0.6, 0.8, 1.0 and the conductivities 0.33, 0.01, 0.43 representing the brain, the skull, and the scalp, respectively; see Fig. 1. The cortical surface is the boundary of brain. The sphere head model is approximated by meshes with 2,199 nodes and 11,415 tetrahedrons [12]. We employ the finite element method to implement the method in discrete case and the modified steepest decent method as an update algorithm [10]. Let we further assume that EEG sensors directly correspond to nodes on the boundary of the head model.

Let N_b and N_ϵ be the number of nodes on boundary and nodes on CS_ϵ for $\epsilon = 1/20$, respectively. If we assume that we measured EEG data on all boundary nodes and the influence space is small enough to make N_ϵ is less than N_b then the number of EEG data is greater than the number of unknowns. Therefore it becomes an over-determined problem which gives a unique solution in the least squares sense.

The threshold process is defined as $\tilde{I}_p = 0$ if

$$\tilde{I}_p < \max(\tilde{I}_p) - \frac{(100-\beta)(\max(\tilde{I}_p) - \min(\tilde{I}_p))}{100}$$

where β is the given percentage of threshold.



(a) (b)

FIGURE 2. (a) True I_p (b) Reconstructed $\tilde{I}_p$ with threshold 85%

The computation time for inverse iterations with the stop condition depends on the initial estimated $\tilde{I}_p$. If any *a priori* information for initial guess is not given, we start with $\tilde{I}_p = 0$ for all nodes in the influence space. When the divergence of true primary current distribution in the cortex surface is given as Fig. 2 (a), computational time from all zero distribution to the result in Fig. 2 (b) is 63 seconds with 50 iterations. The result with 85% threshold is shown in Fig. 2 (b). The node with the maximum value in Fig. 2 (b) is corresponding to the node with the maximum value in Fig. 2 (a). The numerical tests are performed using MATLAB 2008a on Microsoft Windows XP with Intel Core 2 Duo (2.3G MHz CPU clock rate) and 2G RAM.

5. CONCLUSION

We have introduced a new iterative distributed source method for EEG inverse problem using the Lagrangian approach. It reconstructs the divergence of source current instead of source current itself. The restriction of influence space is required to determine focal and realistic solution and we suggest the computation of the lead field matrix in the numerical algorithm. In the numerical simulation with the sphere model, it was shown that the reconstructed solution converges to a reasonable distribution. The approach proposed in this paper is able to be applied to other inverse problems.

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