A COMPUTATION METHOD IN PERFORMANCE EVALUATION IN CELLULAR COMMUNICATION NETWORK UNDER THE GENERAL DISTRIBUTION MODEL

KYUNGSUP KIM[1]

[1]Computer Science and Engineering, Chungnam National University, Yooseong-gu, Daejeon, Korea
E-mail address: kskim@cse.cnu.ac.kr

Abstract. The paper considers the computation method in the performance evaluation of cellular network in the phase-type distribution assumptions that the channel holding times induced from mobility are modeled by well-fitted distributions to reflect an actual situation. When ww consider a phase-type distribution model instead of exponential distribution, the complexity of the computation increase exponential even though the accuracy is improved. We consider an efficient numerical algorithm to compute the performance evaluations in cellular networks such as a handoff call dropping probability, new call blocking probability, and handoff arrival rate. Numerical experiment shows that numerical analysis results are well approximated to the results of simulation.

1. Introduction

We consider performance evaluation problems of the call admission control in cellular networks. Various handoff priority-based call admission control (CAC) schemes have been proposed [1] [2]. A guard channel scheme proposed by Hong and Rappaport [3] is that a number of channels in each cell are reserved for exclusive use by handoff calls and remaining channels are shared by both new and handoff calls. In the queue priority scheme, when all channels are occupied, either new calls are queued while handoff calls are dropped [4], or new calls are dropped while handoff calls are queued [5] [6], or both calls are queued and rearranged [7]. In this paper, a handoff prioritization strategy with guard channel and queue is considered.

We note that there are some rough approximations in cellular system modeling in the past research literature. First, the channel holding times for new calls and handoff calls have been assumed to be independent, exponentially distributed, and have the same average values [3] [8]. It is known that the distribution of the cell dwell time induced by user's mobility should be modeled by a general distribution in order to evaluate exactly the performance metrics. Orlik and Rappaport proposed that the sum of exponential distributions (Hyper-exponential function) can

be approximated to the dwell time distribution [9] [10]. Second, there are different classes of telephone traffics with different qualities of service (QoS) in the modern communication network. For examples, the new call and handoff call has different characteristics such as channel holding times and cell dwell times [3] [8], and the real time traffic and non-real time traffics have different QoSs. Therefore, we introduce multidimensional Markov model to handle the multi-class traffics. However, if we assume more general model and increase the dimension in order to get more exact solutions, then the complexity of computation increase exponentially. We handle the numerical computation problems in computing performance evaluations in the call admission control of cellular networks

In this paper, we develop a numerical algorithm computing a performance of a CAC using a channel reservation and handoff queueing, when there are different mobility patterns of users. We note that the mobility of wireless users impacts the performance, such as the blocking probability and the mean of delay. Thus, more realistic analytical model of the mobility and service rate is needed. General distribution model has computation complexity problems due to the exponential increase of dimension. Thus we propose a novel numerical method to compute the performance values of the CAC as the values for a handoff call dropping probability, new call blocking probability, and handoff arrival rate, when new call arrival rates are known.

The remainder of this paper is structured as follows. In section 2, we consider a model of mobility, and the computation method for the $M/Ph/C/K$ queueing model. The matrix solution of the analytical model for Channel guard scheme with handoff buffer is presented and the numerical algorithm is discussed in section 3. In Section 4, the numerical results of the analytical model are verified by some numerical examples. Finally, conclusions are drawn.

2. TRAFFIC MODEL IN MOBILE NETWORKS

2.1. **Mobility Modeling.** The probability that a new call is blocked is denoted by new call blocking probability (CBP)(P_{nb}) and the probability that a handoff call is dropped is denoted by Handoff Call Dropping Probability (CDP) (P_{hd}). These quantities are most significant QoS metrics in CAC scheme. When new calls and handoff calls are competing for the usage of a finite channel resource in a cell, their claims for QoS are different. From users' point of view, a call forced to terminate during service is more annoying than the new call blocked at its start. Therefore, handoff calls are commonly given a higher priority in accessing the wireless channel. This can be realized by handoff priority-based Call Admission Control (CAC) Schemes. We note that the mobility patterns of mobile user such as slow or fast speed influence to the QoS in wireless networks. The mobility plays an important role in the performance of a cellular networks.

Let λ_n be the arrival rate for new calls and λ_h be the arrival rate for handoff call. λ_h depends on λ_n. The cell dwell times are modeled as exponential distributions in some past literature, but real fast user's ones are not exponentially distributed [8]. We can reasonably assume that the holding channel times of the static user is an exponential distribution. The average cell dell times for high speed users depend on the speed of the users. The users are moving in a random movement pattern in a cellular network. We need a general fitting model for approximating the
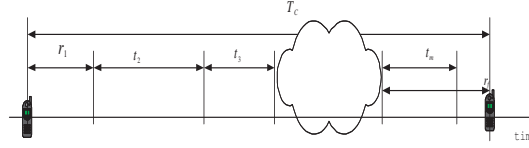
FIGURE 1. The time diagram for call holding time and cell dwell time

measured data for fitting. The distribution of a Hyper-Erlang random variable X is defined by

$$f(x) = \sum_{i=1}^{p} \alpha_i \frac{(\mu_i x)^{r_i-1}}{(r_i-1)!} \mu_i \exp(-\mu_i x) \tag{1}$$

where the mean rate μ is $\mu = \sum_{i=1}^{p} \alpha_i r_i \mu_i$ for some given p.

We discuss how the distributions of the cell dwell time and the call holding time influence the distributions of the new and handoff call channel holding times in Figure 1. Let T_c denote the lifetime of the call holding that is the length from the instant of admission by the base station to the instant when the connection is terminated in the cell or in another cell after several more handovers. If the call holding time has an exponential distribution, denoted by $f_c(x)$, then the residual call length of the handover also has the same distribution, due to the memoryless property. Figure 1 shows diagram for our study similar to that in [11]. Let t_i for $2 \leq i \leq m$ be the typical cell dwell time in a cell for a handoff user, r_1 be a cell dwell time in the first cell for a new call. r_f is a residual call holding time. We assume that cell dwell time are generally distributed as (1) depending on the mobility of a mobile user

The number of handoff times H that a mobile crosses different boundaries during a call holding time is a random variable depending on the cell size, call holding time and mobility parameter [12]. If the density function of the independent identical distribution calling holding times has a general distribution, then the handoff rate for a nonblocking call is given by

$$E[H] = \frac{(1-P_{nb})}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\mu[1-f^*(s)]}{s^2[1-(1-P_{hd})f^*(s)]} f_c^*(-s)ds, \tag{2}$$

where $f_c(t)$ is defined by the distribution of call holding time and $f(t)$ is defined by the distribution of cell dwell time with mean rate μ (refer to [11]). Then the handoff rate λ_h is

$$\lambda_h = \lambda_n(1-P_{nb})E[H]. \tag{3}$$

We can see that the handoff call arrival rate λ_h depends on the user mobility and the new call arrival rate (λ_n) (3).

2.2. $M/Ph/C/K$ **Modeling.** We model call admission control of a single cell by using $M/Ph/C/K$ model. New call and handoff arrival processes are assumed Poisson processes with arrival rates λ_n and λ_h, respectively. Cell dwell times are modeled by the server with phase-type distribution. A phase-type distribution has finite states. For k processes, we require a $k+2$ component vector $(n; n_q, n_k, n_{k-1}, \cdots, n_1)$ to describe the state of a single cell behavior where n is the total number of customers in the system, the remaining i components n_i for $0 \leq i \leq k$ represent

the distribution of customers in various phases of service, and n_q is the number of customers in the queue. A lexicographic set $\mathcal{M}_n$ is defined by

$$\mathcal{M}_n = \{(n; n_q, n_k, \cdots, n_1)\} \tag{4}$$

where $n - n_q = \sum_{i=1}^{k} n_i$. Here, $n_q = 0$ if $n \leq C$. $n_q = n - C$ if $n > C$. Let us define a lexicographic ordering relation on the set $\mathcal{M}_n$. Let us define the ordering relation $(\prec)$ of the lexicographic labeling by $(n; n_q, n_k, \cdots, n_1) \prec (n; n'_q, n'_k, \cdots, n'_1)$ if it satisfies one of the following statements:

(1) there is a first j from the left side such that $n_j < n'_j$, and $n_i = n'_i$ for all $i > j$ if $0 \leq n \leq C$ $(n_q = n'_q = 0)$

(2) there is a first j from the left side such that $n_j < n'_j$, $n_q = n'_q$ and $n_i = n'_i$ for all $i > j$ if $n \geq C$.

The cardinality of a lexicographic ordering set $\mathcal{M}_n$ for n customers is defined by m_n ($|\mathcal{M}_n| = m_n$). Accordingly, the cardinality of $\mathcal{M}_n$ can be computed by

$$m_n = \begin{cases} \frac{(n+k-1)!}{n!(k-1)!}, & \text{when } 1 \leq n \leq C; \\ \frac{(C+k-1)!}{C!(k-1)!}, & \text{when } K \geq n > C. \end{cases} \tag{5}$$

where $n!$ is the factorial of n and K is the maximal number of customers in the system. This gives the number of the total elements of the state space, $T = 1 + \sum_{n=1}^{K} m_n$. There is an one-to-one mapping such that $(n; n_q, n_k, n_{k-1}, \cdots, n_1)$ corresponds to $[n, l]$ for $1 \leq l \leq m_n$ with the same order. The state probability $p_{n; n_k, n_{k-1}, \cdots, n_1}$ represents the probability for which there are n customers in system and n_i customers in the phase i of service for $i = 1, \cdots k$, where $n_k + n_{k-1} + \cdots + n_1 = \min\{n, C\}$. If there is no ambiguity, we use the simplified notation $p_{[n,l]} = p_{n; n_q, n_k, n_{k-1}, \cdots, n_1}$. The vector-valued balanced equation can be written by

$$\begin{aligned} D_0 \vec{p}_0 &= V_0 \vec{p}_0 + W_0 \vec{p}_1 \\ D_n \vec{p}_n &= U_n \vec{p}_{n-1} + V_n \vec{p}_n + W_n \vec{p}_{n+1} \\ D_K \vec{p}_K &= U_K \vec{p}_{K-1} + V_K \vec{p}_K, \end{aligned} \tag{6}$$

where $\vec{p}_n = \begin{bmatrix} p_{[n,1]} & p_{[n,2]} & \cdots p_{[n,m_n]} \end{bmatrix}^t$ is a column vector with dimension m_n, m_n is the number of the possible states when there are n customers [13]. V_n is a matrix that presents a transition from an internal state of a server into another internal state without changing the numbers of customers in the queue and system. W_n is denoted a transition matrix induced by a customer departure from the system, U_n is a transition matrix induced by a new customer arrival. D_n is a diagonal matrix whose (i, i) entry equals the sum of all the entries in the i-th columns of the matrices U_{n+1}, V_n and W_{n-1}, i.e.

$$D_n(i, i) = \sum_{j=1}^{m_{n-1}} U_n(j, i) + \sum_{j=1}^{m_n} V_n(j, i) + \sum_{j=1}^{m_{n+1}} W_n(j, i).$$

Refer to the global state description of $M/Ph/C$ in Chapter 6 [14] to know the detail description. The normalization equation is satisfied as follows: $\sum_{n=0}^{N} \sum_{i=1}^{m_n} p_{[n,i]} = 1$. The steady state probabilities p_n can be computed by $\sum_{i=1}^{m_n} p_{[n,i]}$.

3. Analysis of Call admission control Scheme

3.1. **CAC with Guard channel and Handoff Queueing.** We will analyze the performance of a CAC scheme with handoff queueing and guard channels. We assume that the channel holding times for new calls and handoff calls are independent and have different distribution [3] [12] [9]. The one-dimensional Markov chain model for CAC schemes assuming that cell dwell times of new calls and handoff calls are identically distributed may not be appropriate. Therefore, the multi-dimensional Markov chain model is needed.

Let us consider the channel holding time. There are two kinds of channel holding times : a new call channel holding and handoff call channel holding time. Let t_{nh} and t_{hh} denote the new call channel holding time and the handoff call channel holding time, respectively. The new call channel holding time is $t_{nh} = \min\{T_c, r_1\}$ and the handoff call channel holding time is $t_{hh} = \min\{r_f, t_m\}$. We separated calls into new calls and handoff calls when considering the channel holding time. We need to consider the channel holding time for merged traffic of new calls with rate λ_n and handoff calls with rate λ_h. We assume that the distributions of channel holding times is approximated by hyper-Erlang distribution. We study the multi-dimensional Markov chain under the assumption that some random variable, such as dwell time may be modeled by the hyper-Erlang distribution. We study the multi-dimensional Markov chain under the assumption that some random variable, such as cell dwell time may be modeled by the Hyper-Erlang distribution [12] [9]. We develop an algorithm that computes the blocking probability of new calls and the dropping probability of handoff calls. We develop an algorithm that computes the blocking probability of new calls and the dropping probability of handoff calls.

Figure 2 shows an example for $C = 6$, $M = 2$, $N = 1$ and $B = 3$. Let p_{n_1,n_2} denote the steady-state probability that there are n_1 new calls and n_2 handoff calls in the cell. Let us consider the two-dimensional steady-state probability $p(n_1, n_2)$ occurring in the $M/Ph/C/N$ queueing system. The Poisson interarrival and Hyper-Erlang distributions in $M/Ph/C/N$ queueing system can model two-dimensional Markov chain as a generalized version of two-dimensional Markov chain introduced in [15]. We assume that each base station has a finite buffer size B. Let C be the total number of channels in a cell and M and N be the number of channels only assigned for new calls and handoff calls, respectively. There are $C - M - N$ shared channels that can be used by either type of call. All channels are employed in a first-come first-serve manner. The queue model can be described by a two-dimensional (i, j) Markov chain, where i and j denote the numbers of existing new calls and handoff calls in a cell, respectively. The two-dimensional state space is given by

$$\mathcal{S} = \{(i,j) | 0 \le i < M, 0 \le j \le C - M + B \quad \text{or}$$
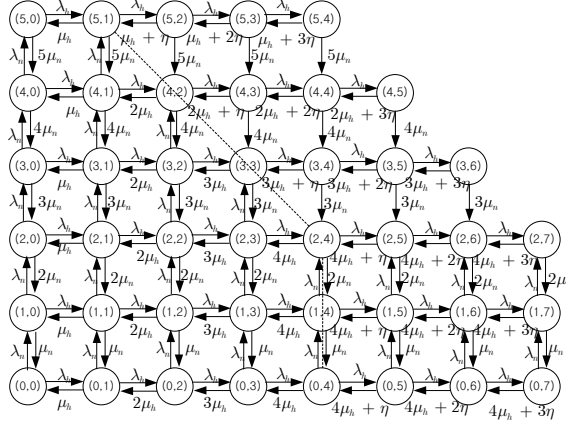$$M \le i \le C - N, 0 \le j \le C - i + B\}.$$

FIGURE 2. Two dimension Markov Chain

The state space $\mathcal{S}$ of the two-dimensional Markov chain can be divided into three parts as follows:

$$
\begin{aligned}
\mathcal{S}_1 &= \{(i,j)|i+j < C, 0 < i < C - N, 0 < j < C - M\} \\
\mathcal{S}_2 &= \{(i,j)|i \leq M, C - M \leq j \leq C - M + B\} \\
\mathcal{S}_3 &= \{(i,j)|i+j \geq C, M \leq i \leq C - N\}.
\end{aligned}
$$

Let us define a vector $\vec{n}_i = (n_{i,q}, n_{i,k} \cdots, n_{i,1})$. For c servers, we require a $k+2$ component vector to describe the state of the system, which is defined by $(n_i; \vec{n}_i) = (n_i; n_{i,q}, n_{i,k}, \cdots, n_{i,1})$ in (4) where $n_i = n_{i,q} + \sum_{j=1}^{k} n_{i,j}$ for $i = 1, 2$. The new call and handoff states can be re-ordered by lexicographical labeling. Let p_{n_1,n_2} denote the state probability that there are n_1 new calls and n_2 handoff calls. The probability p_{n_1,n_2} can be divided into the vector-valued probability $\vec{p}_{n_1,n_2}$ by lexicographical ordering. The transition diagram for the new and handoff call bounding schemes with buffers for handoff is modeled by the two-dimensional Markov chain.

Define $p_{n_1,n_2}(l_1, l_2)$ by $p_{n_1,n_2}(l_1, l_2) = p_{n_1;\vec{n}_1,n_2;\vec{n}_2}$ where $\vec{n}_i$ is l_i-th elements of $\mathcal{M}_{n_i}$ for $i = 1, 2$. Then, the state vector $\vec{p}_{n_1,n_2}$ can be defined by

$$\vec{p}_{n_1,n_2} = \begin{bmatrix} p_{n_1,n_2}(0,0) \\ p_{n_1,n_2}(0,1) \\ \vdots \\ p_{n_1,n_2}(0,m_{n_2}) \\ p_{n_1,n_2}(1,0) \\ p_{n_1,n_2}(1,1) \\ \vdots \\ p_{n_1,n_2}(m_{n_1}, m_{n_2}) \end{bmatrix}. \tag{7}$$

The state vector $\vec{p}_{n_1,n_2}$ can be defined such that $s = m_{n_1}s_1 + s_2$ element is $p_{n_1,n_2}(s_1, s_2)$. Then state probability p_{n_1,n_2} is the sum of all the elements of the state vector probability $\vec{p}_{n_1,n_2}$ since the probability p_{n_1,n_2} can be divided into the vector-valued probability $\vec{p}_{n_1,n_2}$ by lexicographical ordering. In Figure 2, we can see that $\mathcal{S}$ can be divided into $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$. First, compute the transition matrix of state $(i, j) \in \mathcal{S}_1$. We define U_I, W_I, V, I_U, I_W for convenience) from $\mathcal{M}_{n_1} \otimes \mathcal{M}_{n_1}$ to $\mathcal{M}_{\bar{n}_1} \otimes \mathcal{M}_{\bar{n}_2}$ as follows:

$$\begin{aligned} U_I(n_1, n_2) &\triangleq U_{n_1} \otimes I_{m_{n_2}} \\ W_I(n_1, n_2) &\triangleq W_{n_1} \otimes I_{m_{n_2}} \\ V(n_1, n_2) &\triangleq V_{n_1} \otimes I_{m_{n_2}} + I_{m_{n_1}} \otimes V_{n_2} \\ I_U(n_1, n_2) &\triangleq I_{m_{n_1}} \otimes U_{n_2} \\ I_W(n_1, n_2) &\triangleq I_{m_{n_1}} \otimes W_{n_2} \end{aligned} \tag{8}$$

where U_{n_j}, V_{n_j} and W_{n_j} for $j = 1, 2$ are defined by (6), I_m is an $m \times m$ identity matrix, $\otimes$ is a Kronecker product, and (n_1, n_2) is a feasible state in $\mathcal{S}$. Then, the global balanced equation can be written by

$$\begin{aligned} D(n_1, n_2)\vec{p}_{n_1,n_2} = U_I(n_1, n_2) \cdot \vec{p}_{n_1-1,n_2}+ \\ V(n_1, n_2)\vec{p}_{n_1,n_2} + W_I(n_1, n_2) \cdot \vec{p}_{n_1+1,n_2} \\ + I_U(n_1, n_2) \cdot \vec{p}_{n_1,n_2-1}+ \\ I_W(n_1, n_2) \cdot \vec{p}_{n_1,n_2+1} \end{aligned} \tag{9}$$

where $D(n_1, n_2) = \{\lambda_n + \lambda_h + n_1\mu_n + n_2\mu_h\}I_{m_{n_1} \cdot m_{n_2}}$, the negative new call state probability $\vec{p}_{-1,n_2}$ are zero m_{n_2} vectors, and the negative handoff call state probability $\vec{p}_{n_1,-1}$ are zero m_{n_1} vectors. Second, for (i, j) included inside the region of $\mathcal{S}_2$ (not a boundary value), we can derive U_I, W_I, V, I_U, I_W. $D(n_1, n_2)$ is computed by $D(n_1, n_2) = \{\lambda_n + \lambda_h + n_1\mu_n + (C - M)\mu_h + (n_2 - C + M)\eta\}I$. Third, for (i, j) included inside of the region $\mathcal{S}_3$, we note from Fig. 2 that there are no arrows in the places of new call arrival rates because new calls are blocked. Thus, U_I's are zero matrices. For W_I, after a new call is served, the remaining

capacity is used by handoff calls, because there are handoff calls waiting in the queue. Thus, the number of handoff calls instantly receiving service is added by 1 in the first service phase. W_I is computed by

$$W_I(n_1, n_2) \triangleq W_{n_1} \otimes \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}.$$

$D(n_1, n_2)$ is computed by $D(n_1, n_2) = \{\lambda_h + n_1\mu_n + (C - n_1)\mu_h + (n_2 - C + n_1)\eta\}I$. Finally, we should carefully consider the boundary states, such as $\mathcal{S}_1 \cap \mathcal{S}_2$, $\mathcal{S}_1 \cap \mathcal{S}_3$, $\mathcal{S}_2 \cap \mathcal{S}_3$ and other boundaries of $\mathcal{S}_i$ for $i = 1, 2, 3$. In order to handle the total state probability easily, we define a vector $\vec{P}_n$ such that

$$\vec{P}_n = \begin{pmatrix} \vec{p}_{0,n} \\ \vec{p}_{1,n-1} \\ \vdots \\ \vec{p}_{n-1,1} \\ \vec{p}_{n,0} \end{pmatrix}$$

where the total number of new calls and handoff calls is n. Then we can obtain a global equation, as follows: $\mathcal{U}_n\vec{P}_{n-1} + (\mathcal{V}_n - \mathcal{D}_n)\vec{P}_n + \mathcal{W}_n\vec{P}_{n+1} = 0$, for $0 \leq n \leq K$, where $\mathcal{U}_n$, $\mathcal{V}_n$, $\mathcal{W}_n$ and $\mathcal{D}_n$ are defined by

$$\mathcal{U}_n = \begin{bmatrix} I_U(0,n) & 0 & \cdots & 0 \\ U_I(1,n-1) & I_U(1,n-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_I(n,0) \end{bmatrix}$$

$$\mathcal{W}_n = \begin{bmatrix} I_W(0,n) & W_I(0,n) & \cdots & 0 \\ 0 & I_W(1,n-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_I(n,0) \end{bmatrix}$$

$$\mathcal{V}_n = diag[V(0,n), V(1,n-1), \cdots, V(n,0)]$$

$$\mathcal{D}_n = diag[D(0,n), D(1,n-1), \cdots, D(n,0)].$$

$\mathcal{D}_n$ is a diagonal matrix whose (i,i) entry equals the sum of all the entries in the i-th columns of the matrices $\mathcal{W}_{n-1}$, $\mathcal{V}_m$ and $\mathcal{U}_{n+1}$. The above global equation can be written by the following

transition probability matrix

$$
\mathcal{Q} = \begin{bmatrix}
\mathcal{A}_0 & \mathcal{W}_0 & \cdots & 0 & 0 \\
\mathcal{U}_1 & \mathcal{A}_1 & \cdots & 0 & 0 \\
0 & \mathcal{U}_2 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & \mathcal{A}_{K-1} & \mathcal{W}_{K-1} \\
0 & 0 & \cdots & \mathcal{U}_K & \mathcal{A}_K
\end{bmatrix},
$$

in which we set $\mathcal{A}_i = (\mathcal{V}_i - \mathcal{D}_i)$.

In order to handle the total state probability easily, we define a vector $\vec{P}_n$ such that $\vec{P}_n = \begin{bmatrix} \vec{p}_{0,n} & \vec{p}_{1,n-1} & \cdots & \vec{p}_{n-1,1} & \vec{p}_{n,0} \end{bmatrix}$ where the total number of new calls and handoff calls is n. Then, we can obtain a global balance equation, as follows:

$$
\mathcal{U}_n \vec{P}_{n-1} + (\mathcal{V}_n - \mathcal{D}_n)\vec{P}_n + \mathcal{W}_n \vec{P}_{n+1} = 0, \tag{10}
$$

for $0 \leq n \leq K$. The detail derivation is omitted for the space saving. We solve the following equations

$$
\begin{aligned}
\mathcal{Q}\vec{P} &= 0 \\
E\vec{P} &= 1
\end{aligned} \tag{11}
$$

where $E = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$. Using the results, let us compute the new call dropping probability, the terminated handoff call probability, and queueing delay as QoS metrics. A new call arrival is blocked when it arrives at the state $(i, j) \in \mathcal{S}_3$. Therefore, the new call blocking probability P_{nb} is the sum of the conditional state probabilities when a new call arrives in the state $(i, j) \in \mathcal{S}_1$, such as $P_{nb} = \sum_{(i,j)\in\mathcal{S}_3} p_{i,j}$. The dropping probability of a handoff call can be calculated as the fraction of the incomplete handoff calls whose mobile leave the handoff area prior to their coming into the first queue position and getting a channel. The dropping probability $P_{hd|(i,j)}$ is defined by $P_{hd} = \sum_{(i,j)\in\mathcal{HD}} p_{i,j}$ where $\mathcal{HD} = \{(i,j) \in \mathcal{S} | j = C - M + B, \text{or } i + j = C\}$.

We consider several methods for computing stationary probability distributions for large Markov Chain. There are two methods, direct and iterative, for solving linear systems. Iterative methods are the most commonly used methods for obtaining the stationary probability vector from the infinitesimal generator. In iterative method, the involved operations do not alter the form of the matrix and thus compact storage, which minimize the amount memory required to store the matrix, may be conveniently implemented, since the matrices involved are large and sparse. With direct methods, the elimination of one non-zero element of the matrix during the reduction phase often results in the creation of several non-zero elements in the position which contained zero. A successful direct method must incorporated a means of overcoming these difficulties.

When we use a good initial approximation, we should expect to compute the real solution in relatively few iterations. This is especially beneficial when a series of related experiments is being conducted and there is a little change. But iterative methods have the disadvantage

that they often require a long time to converge to the desired solution. Direct methods can be recommended if they obtain the solution in less time. However, we solve the linear system by direct method keeping sparse matrix.

3.2. **Fixed point algorithm.** We want to compute the CAC parameters as the values for a handoff call dropping probability P_{hd}, new call blocking probability P_{nb}, and handoff arrival rate λ_h, when new call arrival rates λ_n are known. These values can not computed by using local information in a single cell, but need the global information. However, it is impossible to know the global information, because the total cellular system is very large and dynamics. The values for λ_h depends on the integration on each the drop or blocking probability of the total cellular system. So, we believe that a local value of λ_h measured in single cell is not a steady state value and is a dynamical value depending on instant state. Therefore, the values for P_{hd}, P_{nb}, λ_h, and λ_n should be predicted by an iterative method under the simplified model similar to [16] [3]. Beginning with an proper initial guess for the unknowns, the equations are solved numerically using an iterative method. This section shows how to use an iterative technique to compute P_{nb}, and P_{hd} using the equations derived in Sec. 3.1. The iterative algorithm is as follows:

Algorithm 1 (Fixed point algorithm). *Compute P_{nb}, and P_{hd}:*

- *Input parameters: the new call arrival rate λ_n, the number of channels C, and the mean and derivation of the cell dwell time.*
- *Output values: the handoff call arrival rate λ_h, the new call blocking probability P_{nb}, and the handoff dropping probability P_{hd}.*
  (1) *Select initial values for P_{nb} and P_{hd}.*
  (2) *Compute the handoff call rate λ_h as (3) (the instant value can directly be measured).*
  (3) *Update old values:*

$$P_{nb,old} \leftarrow \alpha P_{nb,old} + (1 - \alpha)P_{nb}$$
$$P_{hd,old} \leftarrow \alpha P_{hd,old} + (1 - \alpha)P_{hd},$$

  where $0 \leq \alpha < 1$.
  (4) *Compute the new-call-blocking and handoff-call-dropping probabilities (P_{nb} and P_{hd}, respectively) by using the results in Sec. 3.1.*
  (5) *If $|P_{nb,old} - P_{nb}|$ and $|P_{hd,old} - P_{hd}|$ are larger than the given thresholds, then go to step 2. Otherwise, go to the final step*
  (6) *The values for λ_h, P_{nb} and P_{hd} converge.*

In Step 3, α is an exponential moving average factor. The convergence rate depends on α.

4. NUMERICAL RESULTS

In this section, the numerical computation results obtained with our analytical model are discussed. We compare the performance QoS metrics of CAC with guard channel and finite
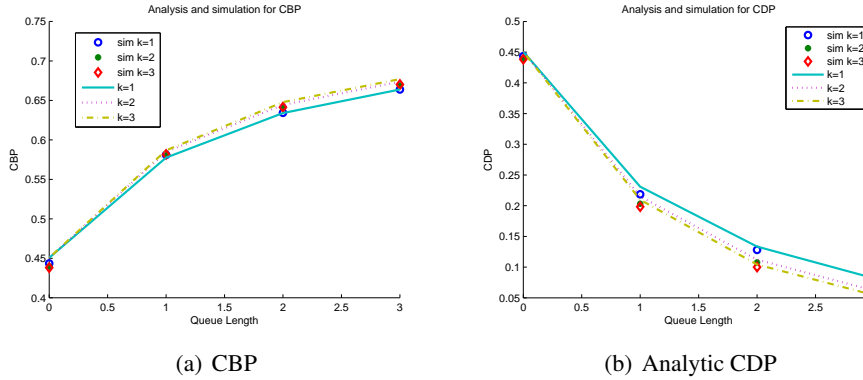
(a) CBP (b) Analytic CDP

FIGURE 3. The loss probabilities(CBP and CDP) for Guard channel with respect to queue length
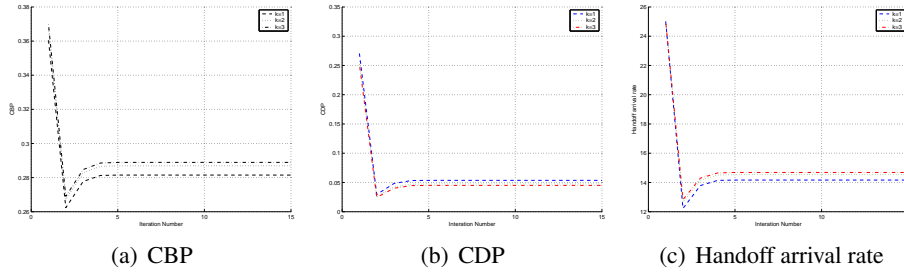

(a) CBP (b) CDP (c) Handoff arrival rate

FIGURE 4. The CDP and CBP probabilities with respect to Erlang Index k

queueing for various parameter settings to find the critical parameters of the performance under the assumption that the cell dwell time distribution is an Erlang distribution as a special case.

Figure 3 illustrates the effects of the change of Erlang index on the new-call-blocking and handoff-call-dropping probabilities, depending on queue length with respect to each Erlang index. We set $\lambda_n = 10$, $\lambda_h = 10$, $C = 3$, $M = 0$, $N = 0$, $\mu_h = \mu_n = 5$, and $0 \leq B \leq 5$. We compare the different Erlang distributions with same mean $\frac{1}{\mu}$, but different variances $\frac{1}{(k\mu^2)}$, when guard channel schemes with queue is used. We can see that there is some differences in the blocking and dropping probabilities for different Erlang Indices. We can verify that the results of our analysis are almost equal to the results derived by the event-driven simulation in Fig 3. Here, arrival process in each cell is generated with identical independent distribution. However, it is known as Erlang loss that the steady state probabilities for an $M/G/C/C$ is the same as those of an $M/M/C/C$ with the same arrival process and the channel number [16]. Here, we can also see that there are some differences of the dropping and blocking probabilities for $M/E_k/C/K$ with respect to the Erlang index k.

We consider the effects of both the mobility and traffic types on the network performance for different Erlang Indexes. We set $\lambda_n = 20$, $C = 6$, $M = 2$, $N = 3$, $\mu_h = \mu_n = 5$, $\eta = 2.5$, and $B = 5$. In this example, the handoff arrival rate λ_h is computed by using (3). Figure 4 shows differences of the new-call-blocking probability and the handoff-dropping-probability, with respect to k. λ_h, P_{nb}, and P_{hd} are computed by Algorithm 1. In numerical experiment, we can see that α is closely related to the convergence of the algorithm. When $\alpha = 0$, algorithm does not converge but rather, it oscillates. Thus, in order to prevent the divergence of the algorithm, we use the exponential moving average filter. Figure 4 also shows the convergence of P_{hd}, P_{hb}, and λ_h. There is a trade-off between the convergence and the stability of the algorithm in choosing α.

5. CONCLUSION

We have developed an analytical model for a cellular system that utilizes CAC with guard channel and handoff queueing under the assumption that cell dwell time has a phase-type distribution. We have made CAC performance analysis reflected mobility effect. We have proposed a numerical algorithm to compute the QoS metrics, such as the new-call-blocking probability and the handoff-call forced-terminated probability when the distribution of channel holding times is an Erlang distribution. The complexity of the computation increases exponentially as the dimension of phase increase. By numerical experiment results, we have verified that there should be a tradeoff between the exactness of the performance model and the computational complexity. We have verified the analysis reliance by using event-driven simulation. In future works, the multidimensional mobility should be researched.

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