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OPTIMZATION OF A PIN FIN BASED ON THE INCREASING RATE OF HEAT LOSS

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ABSTRACT. A pin fin is optimized based on the increasing rate of heat loss by using a two-dimensional analytic method. The optimum heat loss, corresponding optimum thermal resistance and fin length are presented as a function of the fin base thickness, convection characteristic numbers ratio, fin outer radius and ambient convection characteristic number. One of the results shows that both the optimum heat loss and fin length decrease linearly whereas the optimum thermal resistance increases very slightly with increase of the fin base thickness.

1. INTRODUCTION

Extended surfaces or fins play an important role to enhance the heat transfer in many engineering application and many kind of fins have been studied [1-3]. A pin fin is one of popular used fins and many studies about it have been reported. For example, Laor and Kalman [4] presented a theoretical-numerical analysis of spine fins and etc. in which those fins are subject to a temperature-dependent heat transfer coefficient. Su and Hwang [5] developed the analytic transient solutions using the Laplace transformation and separation of variables methods when the base is subject to a step change in temperature, and the heat dissipation is convected from the lateral surface and the fin tip to the surroundings in a cylindrical pin fin.

Optimization of fins is an interesting area of study for fins. One of the representative optimization procedure is to find the maximum heat loss for given fin material. For this kind of optimization of fins, Kang and Look [6] presented the optimum procedure for a thermally asymmetric convective and radiating annular fin. Yu and Chen [7] discussed the optimization of rectangular profile circular fins with variable thermal conductivity and convective heat transfer coefficients.

For optimization of a pin fin, Yeh [8] investigated analytically the optimum dimensions of rectangular fins and cylindrical pin fins considering temperature dependent heat transfer coefficients and heat transfer from the fin tip. Kang [9] presented an optimum procedure of a pin fin for fixed fin volumes. Chung and Iyer [10] presented an extended integral approach to determine the optimum dimensions for rectangular longitudinal fins and pin fins by incorporating traverse heat conduction.

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In those papers for a pin fin optimization, fin base temperature is given as a constant for the boundary condition and the effect of fin base thickness is not considered. In this optimization of a pin fin, inside wall temperature is given and the fin base temperature is varied with the variation of fin base thickness and outer radius. Under these conditions, a pin fin optimized based on the increasing rate of heat loss with increase of the fin length by using a two-dimensional analytic method.

2. 2-D ANALYTIC METHOD

2.1. **Temperature.** Two-dimensional governing differential equation under steady state condition for a pin fin as shown in Fig. 1 is given as a dimensionless form by Eq. (1).

$$\frac{\partial^2\theta}{\partial R^2}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{\partial^2\theta}{\partial X^2}=0 \tag{1}$$

Four boundary conditions are required to solve the governing differential equation and these conditions are given as Eq. (2) through Eq. (5).

$$\left.\frac{\partial\theta}{\partial R}\right|_{R=0}=0 \tag{2}$$

$$-\left.\frac{\partial\theta}{\partial X}\right|_{X=L_b}=\frac{1-\theta\,|_{X=L_b}}{L_b} \tag{3}$$

$$\left.\frac{\partial\theta}{\partial X}\right|_{X=L_e}+M_e\,\theta\,|_{X=L_e}=0 \tag{4}$$

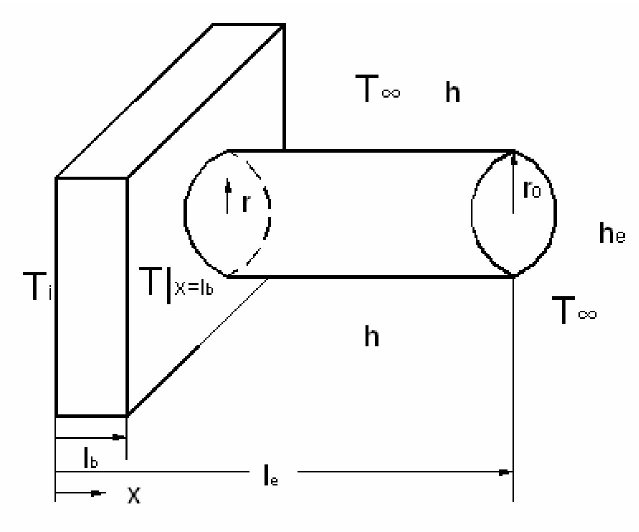

FIGURE 1. Schematic diagram of a pin fin

$$\left.\frac{\partial\theta}{\partial R}\right|_{R=R_o} + M\,\theta\,\big|_{R=R_o} = 0 \tag{5}$$

There is no heat transfer through all surfaces for which R=0 since the shape of the fin and ambient conditions are symmetric and this boundary condition is written by Eq. (2). Fin base boundary condition is represented by Eq. (3) and it means that heat conduction from inner wall to the fin base and heat conduction through the fin base are the same. Equation (4) is a fin tip boundary condition and it means physically that heat conduction through the tip is equal to the heat transfer by convection from the tip surface. Finally, one more boundary condition is given by Eq. (5) and physically it explains that heat conduction through pin fin outer radius surface is equal to heat convection from that surface.

By solving Eq. (1) with Eqs. (2)-(4), the equation for temperature profile within a pin fin is given as Eq. (6).

$$\theta(X,R) = \sum_{n=1}^{\infty} \frac{R_o\{\cosh(\lambda_n X) + g_2(\lambda_n)\sinh(\lambda_n X)\} J_1(\lambda_n R_o) J_0(\lambda_n R)}{2\lambda_n g_1(\lambda_n)\{J_0^2(\lambda_n R_o) + J_1^2(\lambda_n R_o)\}} \tag{6}$$

where,

$$g_1(\lambda_n) = \{1 - \lambda_n L_b g_2(\lambda_n)\}\cosh(\lambda_n L_b) + \{g_2(\lambda_n) - \lambda_n L_b\}\sinh(\lambda_n L_b) \tag{7}$$

$$g_2(\lambda_n) = -\frac{\lambda_n \sinh(\lambda_n L_e) + M_e \cosh(\lambda_n L_e)}{\lambda_n \cosh(\lambda_n L_e) + M_e \sinh(\lambda_n L_e)} \tag{8}$$

The eigenvalues λ_n can be calculated using Eq. (9) which comes from Eq. (5).

$$M = \frac{\lambda_n J_1(\lambda_n R_o)}{J_0(\lambda_n R_o)} \tag{9}$$

2.2. **Heat loss.** The heat loss conducted into the pin fin through the fin base is calculated by Eq. (10).

$$q = \int_0^{r_o} -k\left.\frac{\partial T}{\partial x}\right|_{x=l_b} 2\pi r\,dr \tag{10}$$

Dimensionless heat loss from the pin fin is given as Eq. (11).

$$Q = q/(kl_c\phi_i) = -\sum_{n=1}^{\infty} \frac{4\pi\{\sinh(\lambda_n L_b) + g_2(\lambda_n)\cosh(\lambda_n L_b)\} J_1^2(\lambda_n R_o)}{\lambda_n g_1(\lambda_n)\{J_0^2(\lambda_n R_o) + J_1^2(\lambda_n R_o)\}} \tag{11}$$

2.3. **Thermal resistance.** The thermal resistance for a fin is defined by the ratio of difference of fin base temperature and ambient temperature to heat loss from the fin and is expressed by Eq. (12).

$$r_t = \frac{T|_{x=l_b} - T_\infty}{q} \tag{12}$$

The dimensionless thermal resistance form is given by Eq. (13).

$$R_t = kl_c r_t = \frac{\theta|_{X=L_b}}{Q} \tag{13}$$

3. RESLUTS AND DISCUSSION

For fixed fin outer radius, the variation of heat loss as a function of the fin length for different values of R_o and M is presented in Fig. 2. It shows that the heat loss increases and revels off and then approaches the maximum value as the fin length increases. It can be known that the heat loss reaches the maximum value more rapidly as R_o decreases and/or M increases.

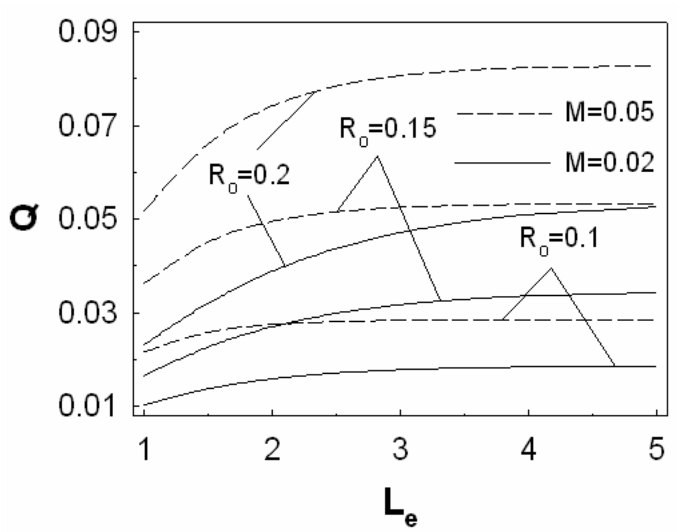

FIGURE 2. Heat loss versus the fin length (L_b=0.1, β=1)

Table 1 lists the increasing rate of heat loss with variation of the fin length [i.e. $\{Q(L_e+0.1)-Q(L_e)\}/Q(L_e)$]. For $0.9 \le Q/Q_{max} \le 0.98$, the increasing rate is between 1.0434% and 0.1987% in the case of M=0.02 whereas that is between 1.6069% and 0.3050% in the case of M=0.05. In this study, the optimum heat loss is somewhat arbitrarily taken as the heat loss for which the increasing rate (i.e. $\{Q(L_e+0.1)-Q(L_e)\}/Q(L_e)$) is equal to 0.5%. Also, the optimum fin length and thermal resistance are referred to the values when the heat loss becomes the optimum heat loss.

Figure 3 shows the variation of the optimum values as a function of the fin base thickness

TABLE 1. Increasing rate of heat loss (R_o=0.15, L_b=0.1, β=1).

Q/Q_{max}	M	L_e for Q/Q_{max}	$\{Q(L_e+0.1)-Q(L_e)\}/Q(L_e)$ (%)
0.9	0.02	2.8305	1.0434
	0.05	1.7841	1.6069
0.95	0.02	3.5255	0.5054
	0.05	2.2233	0.7771
0.98	0.02	4.4264	0.1987
	0.05	2.7934	0.3050

when dimensionless fin outer radius is fixed as 0.15. Both the optimum heat loss and fin length decrease linearly with increase of the fin base thickness. It can be noted that the corresponding optimum thermal resistance increases very slightly as the fin base thickness increases. Physically it means that decreasing rate of the fin base temperature is almost the same as that of heat loss with increases of the fin base thickness.

The variation of the optimum values as a function of fin outer radius is presented in Fig. 4. The optimum heat loss increases slowly first and then increases rapidly whereas the optimum

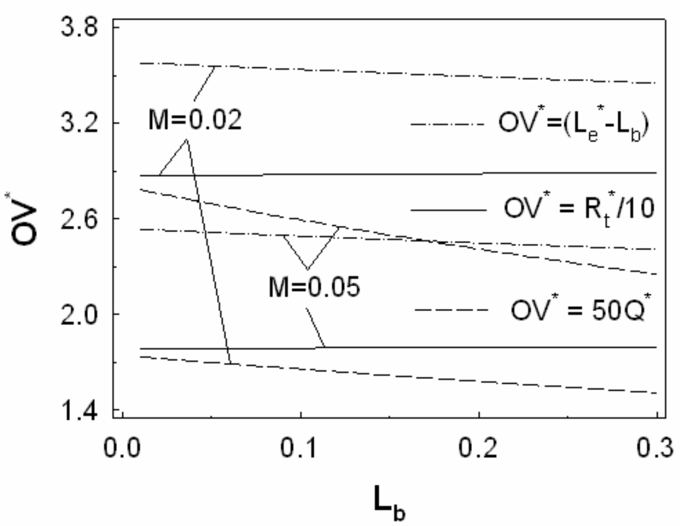

FIGURE 3. Optimum values versus the fin base thickness (R_o=0.15, β=1)

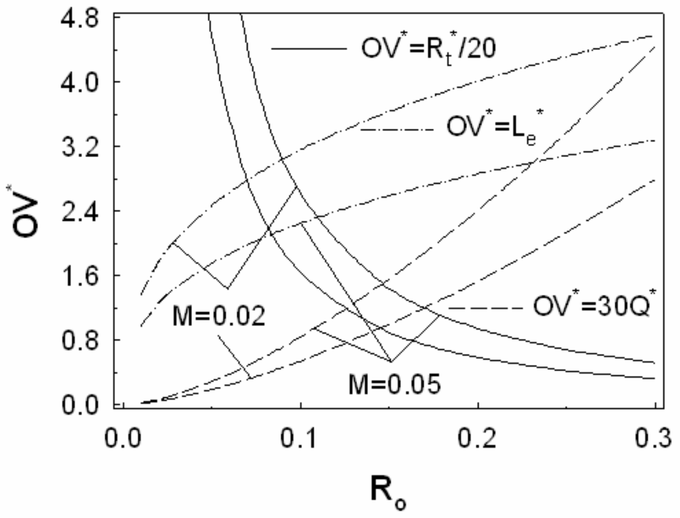

FIGURE 4. Optimum values versus the fin outer radius (L_b=0.1, β=1)

fin length increases rapidly first and then revels off with the increase of fin outer radius. The optimum thermal resistance increases very rapidly as the fin outer radius approaches fin center line (i.e. $R_o \rightarrow 0$) because heat loss becomes very small value whereas the variation of fin base temperature is not so much.

Figure 5 presents the effect of convection characteristic numbers ratio on the optimum values. It can be known that both the optimum heat loss and thermal resistance independent on the variation of the convection characteristic numbers ratio. The optimum fin length

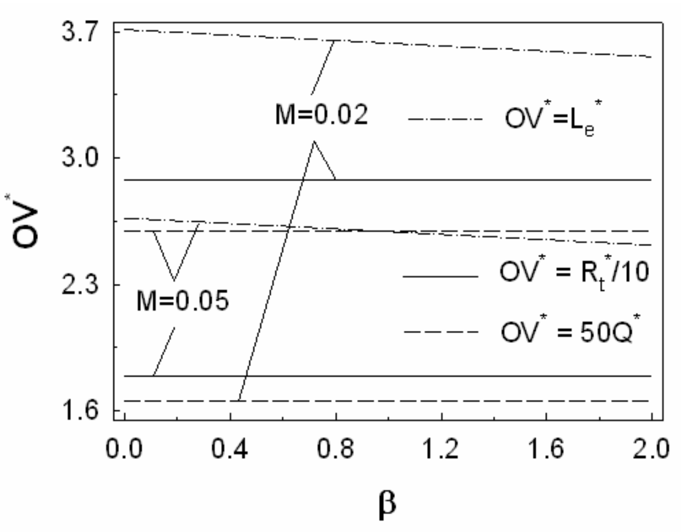

FIGURE 5. Optimum values versus convection characteristic numbers ratio (L_b=0.1, R_o=0.15)

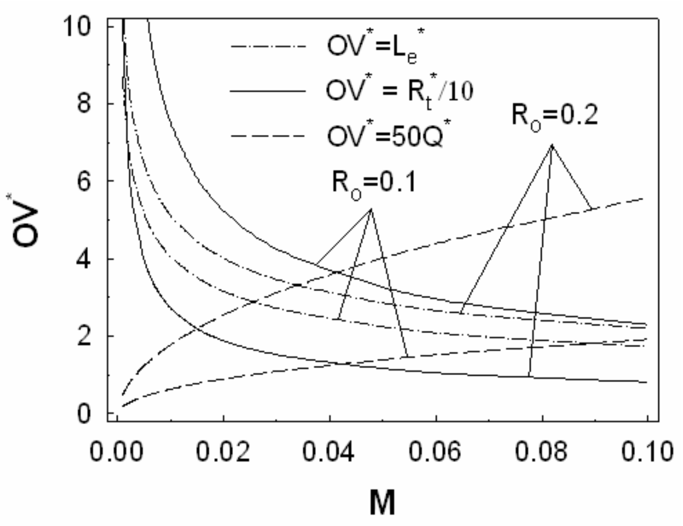

FIGURE 6. Optimum values versus ambient convection characteristic number (L_b=0.1, β=1)

decreases almost linearly as the convection characteristic numbers ratio increases.

Ambient convection characteristic number, M, is specified as 0.02 and 0.05 in the previous discussion. The variation of optimum values as a function of ambient convection characteristic number is depicted in Fig. 6. The optimum heat loss increases somewhat rapidly first and then revels off with the increase of ambient convection characteristic number. Both the optimum fin length and thermal resistance decrease very remarkably first and then decrease slowly as the ambient convection characteristic number increases.

4. CONCLUSIONS

From the optimization of a pin fin based on the increasing rate of heat loss by using a two-dimensional analysis, the following conclusions can be drawn:

1. Optimum heat loss, fin length and thermal resistance of a pin fin are presented as a function of fin base thickness, convection characteristic numbers ratio, fin outer radius and ambient convection characteristic number.
2. Both the optimum heat loss and fin length decrease linearly whereas the optimum thermal resistance increases very slightly with increase of the fin base thickness.
3. The effect of fin outer radius and ambient convection characteristic number on the optimum values is remarkable whereas the effect of fin base thickness and convection characteristic numbers ratio on that is relatively small.

NOMENCLUTURE

h	ambient heat transfer coefficient, W/m^2°C
h_e	heat transfer coefficient at the fin tip, W/m^2°C
J_0	Bessel function of the first kind with 0th order
J_1	Bessel function of the first kind with 1th order
k	thermal conductivity, W/m°C
l_b	fin base thickness, m
L_b	dimensionless fin base thickness, l_b/l_c
l_c	characteristic length, m
l_e	fin tip length, m
L_e	dimensionless fin tip length, l_e/l_c
M	ambient convection characteristic number, hl_c/k
M_e	convection characteristic number at the fin tip, $h_e l_c/k$
q	heat loss from a pin fin, W
Q	dimensionless heat loss from a pin fin, $q/(kl_c\phi_i)$
r	pin fin radius coordinate, m
R	dimensionless pin fin radius coordinate, r/l_c
r_o	outer radius of a pin fin, m
R_o	dimensionless outer radius of a pin fin, r_o/l_c
r_t	thermal resistance, °C /W
R_t	dimensionless thermal resistance, $kl_c r_t$
T	temperature, °C

T_i inner wall temperature, °C
T_∞ ambient temperature, °C
x coordinate along the fin length, m
X dimensionless coordinate along the fin length, x/l_c

Greek symbol
β convection characteristic numbers ratio, M_e/M
θ dimensionless temperature, $(T-T_\infty)/(T_i-T_\infty)$
ϕ_i adjusted fin base temperature, (T_i-T_∞)
λ_n eigenvalues, n=1, 2, 3, • • •

Subscripts
b fin base
c characteristic
e fin tip
i inner wall
o outer radius
t thermal
∞ ambient

Superscripts
*

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