

Joint Scheduling and Rate Optimization in Multi-channel Multi-radio Wireless Networks with Contention-based MAC

Dang Quang Bui[†], Myeonggil Choi^{**}, Won-Joo Hwang^{***}

ABSTRACT

Currently, Wireless Networks have some nice characteristics such as multi-hop, multi-channel, multi-radio, etc but these kinds of resources are not fully used. The most difficulty to solve this issue is to solve mixed integer optimization. This paper proposes a method to solve a class of mixed integer optimization for wireless networks by using AMPL modeling language with CPLEX solver. The result of method is scheduling and congestion control in multi-channel multi-radio wireless networks.

Key words: Wireless Network, Congestion Control, Network optimization, Mixed integer optimization

1. INTRODUCTION

In network, rate and congestion control is often modeled as a network utility maximization issue

$$\begin{aligned} & \text{maximize } \sum_s U_s(x_s) \\ & \text{subject to } Rx \leq c \end{aligned} \quad (1)$$

where source rate vector x is the only optimization variables, and routing matrix R and link capacity vector c are both constants. Utility functions $U_s(x_s)$ are often assumed to be smooth, increasing, concave, and depends on local rate only, although recent investigations have removed some of these assumptions for applications there they are invalid.

* Corresponding Author :Won-Joo Hwang, Address : (621-749) Obangdong, Gimhae, Gyeongnam, Korea, TEL : +82-55-320-3847, FAX : +82-55-322-6275, E-mail : ichwang@inje.ac.kr

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[†] Dept. of Information and Communication Systems, Inje University, South Korea. (E-mail : quangbd@gmail.com)

^{**} Dept. of Business Administration, ChungAng University, South Korea. (E-mail : mgchoi@cau.ac.kr)

^{***} Dept. of Information and Communications Engineering, High Safety Vehicle Core Technology Research Center, Inje University, South Korea.

The result of Kelly [1] has been widely accepted and used in wired networks. But the assumption that link capacity vector is constant is not true in wireless networks. Different from wired networks, there are interferences between links in wireless networks. In this paper, we use contention-based MAC to control interference, i.e., when a link is used to transfer data, some others, which are in its reference region, should not be used. This constraint usually leads to mixed integer programming problem. This is one of difficulties that we need to overcome in wireless networks. Furthermore, wireless networks have more nice characteristics such as multi-channel, multi-radio. It is well-known that these nice characteristics improve network performance significantly. However, the question how to use all resources efficiently is still open. The objective of the paper is to deal with rate optimization in wireless networks which have the above features.

The main contribution of the paper is as follows:

- We formulate rate control for contention-based wireless networks as a mathematical model. In this model, wireless network features, mean multi-channel and multi-radio, are considered. Interference is concretized in-

to mathematical constraints by using contention graph. The detail is presented in section 2.

- The original problem is complex. Therefore, we introduce a new variable to simplify the problem.
- Section 3 investigates in numerical analysis with a simple example. The result can be extended in more complicate networks without any difficulty.

2. MATHEMATICAL MODEL

We consider a network with the following model. Let $G=(S,L)$ be network graph where S is the source set and L is the link set. Assume that M is available channel set. Each source $s \in S$ transfers at end-to-end data rate x_s and characterized by a network utility function $U_s(x_s)$. Normally, this function is strictly concave, differentiable, and increasing. Each channel $m \in M$ is characterized by a capacity c_m . Time axis is divided in to slots with length of τ . Assume that the network operate periodically with period $T\tau$ where T is an integer. An indicator variable is denoted as follows

$$y_{lm}^t = \begin{cases} 1 & \text{if link } l \text{ uses channel } m \text{ is used to transfer} \\ & \text{data at slot time } t \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

If $y_{lm}^t = 1$, let f_{lm}^t be the intermediate data flow. Because data flow cannot exceed the channel capacity, therefore

$$0 \leq f_{lm}^t \leq c_m \quad \forall t = \overline{1, T}, l \in L, m \in M \quad (3)$$

Wireless network is influenced by interference. That means in every clique, at an arbitrary slot time t , if a link uses a channel, others links cannot use that channel anymore. To formulate the constraints, we use contention graph to express interference. In this graph, each link in networks is presented as a node. If two links interfere each

other then in the graph, two appropriate nodes are connected. This contention graph is divided in maximal cliques. A clique is a sub-graph that all nodes are connected. That means all links (presented as nodes in cliques) interference each other. A maximal clique is a clique that we cannot add any link to it to make another clique. Denote CL be set of maximal cliques. At any time $t = \overline{1, T}$, no more than one link in a maximal clique $C \in CL$ can be active in a channel $m \in M$, that means

$$\sum_{l \in C} y_{lm}^t \leq 1 \quad \forall t, m, C \quad (4)$$

Aggregate flow of an arbitrary link $l \in L$ is $\frac{\sum_m \sum_t y_{lm}^t f_{lm}^t}{T}$. This aggregate flow is shared by a source set $\mathcal{S}(l) \subset S$, the set of sources that use link l to transmit data. Therefore,

$$\sum_{s \in \mathcal{S}(l)} x_s = \frac{\sum_m \sum_t y_{lm}^t f_{lm}^t}{T} \quad (5)$$

In (5), the left hand side is aggregate data rate of all sources that use link l to transfer data while the right hand side is data rate that the network can support.

The problem is summarized as follows:

$$\begin{aligned} & \text{maximize} && \sum_{s \in S} U_s(x_s) \\ & \text{subject to} && y_{lm}^t \in \{0, 1\} \quad \forall t = \overline{1, T}, l \in L, m \in M \\ & && 0 \leq f_{lm}^t \leq c_m \quad \forall t = \overline{1, T}, l \in L, m \in M \\ & && \sum_{l \in C} y_{lm}^t \leq 1 \quad \forall t = \overline{1, T}, m \in M, C \in CL \\ & && \sum_{s \in \mathcal{S}(l)} x_s = \frac{\sum_m \sum_t y_{lm}^t f_{lm}^t}{T} \quad \forall l \in L \\ & && x_{s, \min} \leq x_s \leq x_{s, \max} \quad \forall s \in S \end{aligned} \quad (A)$$

Our objective is to choose data rate x , data flow f , and scheduling y that maximize aggregate network utility function $\sum_{s \in S} U_s(x_s)$ and at the same time all constraints are satisfied. The objective to find data rate x and data flow f is rate control problem while the objective to find y is scheduling problem. Look at the problem (A) we can see that objective function is concave. Unfortunately, con-

straints are not convex. They are mixed integer constraints. Thus, the above problem is non-convex.

To remove integer variables, we use the following assumptions:

- If a link uses a channel, it will use full capacity of the channel. That means $f_{lm}^t = c_m \forall t = \overline{1, T}, l \in L, m \in M$. This assumption is acceptable because the more data flow, the more obtained data rate, and therefore, the more utility we get.
- T is large enough. This assumption is to guarantee the continuity of the variable p which is defined later.

Denote p_{lm} be the probability that link l uses channel m during T period. We have

$$p_{lm} = \frac{\sum_t y_{lm}^t}{T} \tag{6}$$

Because T is large enough, therefore we can consider that p_{lm} is continuous. And from (4) and (6) we have

$$\begin{aligned} \sum_{l \in C} p_{lm} &= \frac{1}{T} \sum_{l \in C} \sum_t y_{lm}^t = \frac{1}{T} \sum_t \sum_{l \in C} y_{lm}^t \\ &= \frac{1}{T} \sum_t \left(\sum_{l \in C} y_{lm}^t \right) \leq \frac{1}{T} \sum_t 1 = 1 \end{aligned} \tag{7}$$

So the constraint (4) is replaced with (7). Next using the first assumption and (5) we induce

$$\begin{aligned} \sum_{s \in S(l)} x_s &= \frac{\sum_m \sum_t y_{lm}^t f_{lm}^t}{T} = \frac{\sum_m \sum_t y_{lm}^t c_m}{T} = \\ &= \frac{\sum_m c_m \left(\sum_t y_{lm}^t \right)}{T} = \sum_m p_{lm} c_m \quad \forall l \in L \end{aligned} \tag{8}$$

So the constraint (5) is replaced with (8). And finally, the problem (A) becomes

$$\begin{aligned} &\text{maximize} \quad \sum_{s \in S} U_s(x_s) \\ &\text{subject to} \quad \sum_{l \in C} p_{lm} \leq 1 \quad \forall m \in M, C \in CL \\ &\quad \quad \quad \sum_{s \in S(l)} x_s = \sum_m p_{lm} c_m \quad \forall l \in L \\ &\quad \quad \quad x_{s \min} \leq x_s \leq x_{s \max} \quad \forall s \in S \\ &\quad \quad \quad 0 \leq p_{lm} \leq 1 \quad \forall l \in L, m \in M \end{aligned} \tag{B}$$

Clearly the problem (B) is convex problem because all constraints are linear. We can use Lagrangian Relaxation and Gradient Projection Method to solve (B).

After finding x and p , we have to find y such that

$$\begin{aligned} &\text{Find} \quad y \\ &\text{subject to} \quad y_{lm}^t \in \{0, 1\} \quad \forall t = \overline{1, T}, l \in L, m \in M \\ &\quad \quad \quad \sum_{l \in C} y_{lm}^t \leq 1 \quad \forall t = \overline{1, T}, m \in M, C \in CL \\ &\quad \quad \quad \frac{\sum_t y_{lm}^t}{T} = p_{lm} \quad \forall l \in L, m \in M \end{aligned} \tag{C}$$

The problem (C) is an integer problem. In the next problem, we consider an example and how to solve that problem.

3. NUMERICAL ANALYSIS

Assume that we have a network as shown in Figure 1. The network is composed of five nodes and four links. Two flows x_1 and x_2 share common links. Assume that there are two channels with capacities of $c_1 = 1MBps$ and $c_2 = 3MBps$. The application requires that data flows are in the range $[1, 4]MBps$. And we consider an assumption that interference range is two hops. Therefore, we have

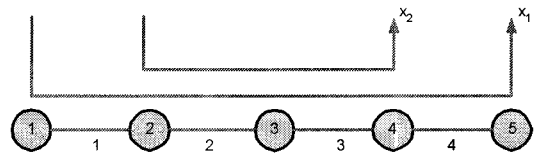


Fig. 1. Network architecture

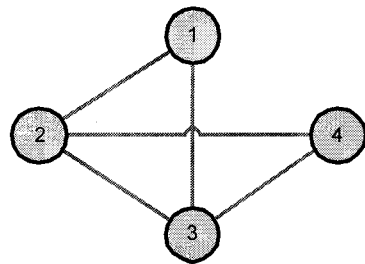


Fig. 2. Contention graph

the contention graph as shown in Figure 2. In this graph, each node represents a link, a link represents interference.

From the contention graph, we see that there are two cliques in graph. The first one includes links {1,2,3} while the second one composes of links {2,3,4}. Therefore, the problem (B) becomes

$$\begin{aligned}
 &\text{maximize } U_1(x_1) + U_2(x_2) \\
 &\text{subject to } p_{11} + p_{21} + p_{31} \leq 1 \\
 &\quad p_{21} + p_{31} + p_{41} \leq 1 \\
 &\quad p_{12} + p_{22} + p_{32} \leq 1 \\
 &\quad p_{22} + p_{32} + p_{42} \leq 1 \\
 &\quad x_1 = p_{11}c_1 + p_{12}c_2 \\
 &\quad x_1 + x_2 = p_{21}c_1 + p_{22}c_2 \\
 &\quad x_1 + x_2 = p_{31}c_1 + p_{32}c_2 \\
 &\quad x_1 = p_{41}c_1 + p_{42}c_2 \\
 &\quad 1 \leq x_1, x_2 \leq 4 \tag{D} \\
 &\quad 0 \leq p_{ij} \leq 1 \quad i = \overline{1,4}, j = \overline{1,2}
 \end{aligned}$$

To solve the problem (D), we use AMPL modeling language [2] with CPLEX solver [3]. The obtained result is shown in Table 1. This result shows that interference influences on throughput so much. We can see that total bandwidth is $c_1 + c_2 = 4$ but aggregate throughput is only $x_1 + x_2 = 1.5$, it is less than 40% bandwidth.

So we solved the congestion control problem.

Table 1. Optimal congestion control solution

Parameters	Solutions
x_1	1
x_2	0.5
$x_3 \equiv p_{11}$	1
$x_4 \equiv p_{12}$	0
$x_5 \equiv p_{21}$	0
$x_6 \equiv p_{22}$	0.5
$x_7 \equiv p_{31}$	0
$x_8 \equiv p_{32}$	0.5
$x_9 \equiv p_{41}$	1
$x_{10} \equiv p_{42}$	0

Next, we have to solve the scheduling problem. The scheduling problem is formulated as (C). Assume that $T=2$, and (C) becomes

$$\begin{aligned}
 &\text{Find } y \\
 &\text{subject to } y_{lm}^i \in \{0,1\} \quad \forall t = \overline{1,2}, l = \overline{1,4}, m = \overline{1,2} \\
 &\quad y_{11}^1 + y_{21}^1 + y_{31}^1 \leq 1 \\
 &\quad y_{21}^1 + y_{31}^1 + y_{41}^1 \leq 1 \\
 &\quad y_{12}^1 + y_{22}^1 + y_{32}^1 \leq 1 \\
 &\quad y_{22}^1 + y_{32}^1 + y_{42}^1 \leq 1 \\
 &\quad y_{11}^2 + y_{21}^2 + y_{31}^2 \leq 1 \\
 &\quad y_{21}^2 + y_{31}^2 + y_{41}^2 \leq 1 \\
 &\quad y_{12}^2 + y_{22}^2 + y_{32}^2 \leq 1 \\
 &\quad y_{22}^2 + y_{32}^2 + y_{42}^2 \leq 1 \\
 &\quad y_{11}^1 + y_{11}^2 = 2 \\
 &\quad y_{12}^1 + y_{12}^2 = 0 \\
 &\quad y_{21}^1 + y_{21}^2 = 0 \\
 &\quad y_{22}^1 + y_{22}^2 = 1 \\
 &\quad y_{31}^1 + y_{31}^2 = 0 \\
 &\quad y_{32}^1 + y_{32}^2 = 1 \\
 &\quad y_{41}^1 + y_{41}^2 = 2 \\
 &\quad y_{42}^1 + y_{42}^2 = 0 \tag{E}
 \end{aligned}$$

Solve the problem (E); we obtain the result in Table 2 and Figure 3. This result shows that in the first slot time, links 1 and 4 are active in channel 1 and link 2 is active in channel 2. Because interference range is two hops, therefore, there is not interference between links in the first slot time. Similarly, in the second slot time, links 1 and 4 use channel 1 and link 3 uses channel 2.

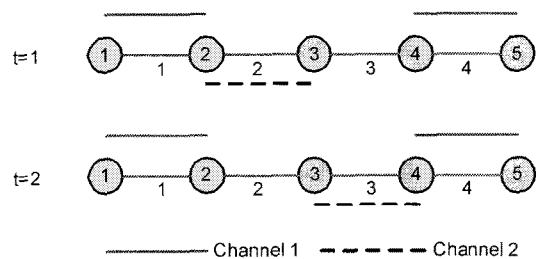


Fig. 3. Optimal scheduling solution.

Table 2. Optimal scheduling solution

Parameters	Solutions
y_{11}^1	1
y_{11}^2	1
y_{12}^1	0
y_{12}^2	0
y_{21}^1	0
y_{21}^2	0
y_{22}^1	1
y_{22}^2	0
y_{31}^1	0
y_{31}^2	0
y_{32}^1	0
y_{32}^2	1
y_{41}^1	1
y_{41}^2	1
y_{42}^1	0
y_{42}^2	0

4. RELATED WORKS

Currently, motivated by Kelly's works many researchers adapt it in wireless networks. The hot-test trend to solve this problem is to cooperate between layers in networks to achieve global optimal network utility. Network protocols may instead be holistically analyzed and systematically designed as distributed solutions to some global optimization problems in the form of generalized Network Utility Maximization (NUM) [4-9]. Some co-operation schemes are presented, e.g., physical layer and MAC layer [5,6], MAC layer and transport layer [5,7], physical layer and transport layer [8], network layer and transport layer [9].

In most cases, authors use iterative methods to solve optimization problems [4-13]. To use these methods, the problem should be convex. Unfortunately, this assumption is not always true. Some authors are thinking solutions for non-convex problem [6,10-12].

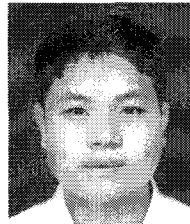
5. CONCLUSION

This paper solved rate optimization problem in multi-channel and multi-radio wireless networks with contention-based MAC. This problem is formulated as an optimization in which the mathematical problem is mixed integer optimization; therefore, we introduce a new variable to convert it into two simpler problems, one is for rate control and the other for scheduling. The solution is obtained with an optimization tool named AMPL modeling language with CPLEX solver. By using the above method, we can solve the problem although it is mixed integer optimization. Until now, we only solved the problem mathematically. The future work is to propose an algorithm to solve the problem and implemented in a distributed way.

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Bui Dang Quang

He received the B.S. degree in Information and Communication Systems from Hanoi University of Technology, in 2004 and the M.S. degree in Department of Electronic Telecommunication Engineering from Inje University. He is currently working towards his Ph.D. at Department of Information and Communication Systems, Inje University. His research interests are in Network Optimization and Robust Design.



Myeonggil Choi

He is an assistant professor at ChungAng University in Korea. He received the B.S. degree from Pusan National University and Ph.D. degree in Management Information Systems from Korea Advanced Institute of Science and Technology (KAIST) in 2004. From 1995 to 2005, he worked at Agency for Defense Department (ADD) as researcher and has worked for National Security Research Institute, Electronics and Telecommunications Research Institute (ETRI) in Korea. From 2005 to 2007, he served as an assistant professor at Inje University. His recent research issues include Information System Security Evaluation, E-Commerce Security and Information Security Management, and Industrial Security.



Won-Joo Hwang

He received the Ph.D Degree from Osaka University Japan in 2002. He received his bachelor's degree and M.S. degree in Computer Engineering from Pusan National University, Pusan, Republic of Korea, in 1998 and 2000. Since September 2002, he has been an assistance professor at Inje University, Republic of Korea. His research interests are in Network Optimization and Ubiquitous Sensor Networks.