

# Bayesian Nonlinear Blind Channel Equalizer based on Gaussian Weighted MFCM

Soowhan Han<sup>†</sup>, Sungdae Park<sup>\*\*</sup>, Jongkeuk Lee<sup>\*\*\*</sup>

## ABSTRACT

In this study, a modified Fuzzy *C*-Means algorithm with Gaussian weights (MFCM\_GW) is presented for the problem of nonlinear blind channel equalization. The proposed algorithm searches for the optimal channel output states of a nonlinear channel based on received symbols. In contrast to conventional Euclidean distance in Fuzzy *C*-Means (FCM), the use of the Bayesian likelihood fitness function and the Gaussian weighted partition matrix is exploited in this method. In the search procedure, all possible sets of desired channel states are constructed by considering the combinations of estimated channel output states. The set of desired states characterized by the maximal value of the Bayesian fitness is selected and updated by using the Gaussian weights. After this procedure, the Bayesian equalizer with the final desired states is implemented to reconstruct transmitted symbols. The performance of the proposed method is compared with those of a simplex genetic algorithm (GA), a hybrid genetic algorithm (GA merged with simulated annealing (SA):GASA), and a previously developed version of MFCM. In particular, a relatively high accuracy and a fast search speed have been observed.

**Key words:** Modified Fuzzy *C*-Means, Gaussian Weighted Partition Matrix, Bayesian Likelihood, Desired Channel States, Nonlinear Blind Channel

## 1. INTRODUCTION

In digital communication systems, data symbols are transmitted at regular intervals. Time dispersion caused by non-ideal channel frequency response characteristics, or by multipath transmission, may create inter-symbol interference (ISI). This has become a limiting factor in many communication environments. Furthermore, the nonlinear character of ISI that often arises in high speed communication channels degrades the performance of the overall communication system [1].

To overcome the detrimental ISI effects and to achieve high-speed and reliable communication, we have to resort ourselves to nonlinear channel equalization.

The conventional approach to linear or nonlinear channel equalization requires an initial training period, with a known data sequence, to learn the channel characteristics. In contrast to standard equalization methods, the so-called blind (or self-recovering) equalization methods operate without a training sequence [2]. Given its superiority, the blind equalization method has gained practical interest during the last few years. Most of the studies carried out so far are focused on linear channel equalization because of its inherent simplicity [3-5].

Relatively a small number of papers have dealt explicitly with nonlinear channel models. The blind estimation of Volterra kernels, which characterize nonlinear channels, was presented in [6] while a maximum likelihood (ML) method implemented via

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expectation-maximization (EM) was introduced in [7]. In spite of the advantages of these approaches, these methods are not free from limitations. The Volterra approach suffers from enormous computational complexity. Furthermore the ML approach requires some prior knowledge of the nonlinear channel structure to estimate the channel parameters. The approach with a nonlinear structure such as multilayer perceptrons, being trained to minimize some cost function, have been investigated in [8]. However, in this method, the structure and complexity of the nonlinear equalizer must be specified in advance. The support vector (SV) equalizer proposed by Santamaria et al. [9] can be a possible solution for both of linear and nonlinear blind channel equalization at the same time, but it still suffers from the high computational cost of its iterative reweighted quadratic programming procedure. The deterministic approach provided by Raz et al. [10] offers a linearizing Volterra filter equalizer and the linearized channel that results from the cascade connection of the blind nonlinear channel with Volterra filter equalizer. However, this algorithm uses the oversampling technique that translates a single input single output (SISO) system to a single input multi output (SIMO) system. Furthermore the resulting oversampled channel matrix must be invertible over the transmitted symbols. For this method, the sampling rate for the received signal has to be higher than the baud rate, otherwise a multi-sensor array must be utilized. In addition, the signal to noise ratio (SNR) should be kept relatively high. A unique approach to nonlinear channel blind equalization was offered by Lin et al. [11]. In their study they used the simplex GA method to estimate the optimal channel output states instead of estimating the channel parameters in a direct manner. The desired channel states were constructed from these estimated channel output states, and placed at the center of their RBF equalizer. With this method, the complex modeling of the nonlinear channel can be avoided

and it works well under a simple SISO communication environment. Recently this approach has been implemented with a hybrid genetic algorithm (that is genetic algorithm, GA merged with simulated annealing (SA); GASA) [12] and a modified Fuzzy C-Means (MFCM) algorithm [13] instead of the simplex GA. The resulting better performance in terms of speed and accuracy have been reported. However, the estimation accuracy and convergence speed in search of the optimal channel output states needs further improvement for the heavy noise communication environments such as real-time use.

In this study, we propose a new modified Fuzzy C-Means algorithm with Gaussian weights (MFCM\_GW) to determine the optimal output states of a nonlinear channel. In the proposed algorithm, the Gaussian weighted partition matrix is developed and applied to the previous version of MFCM [13] for the reduction of noise effect. Thus, even the received symbols are corrupted by a heavy noise, the MFCM\_GW can estimate the optimal output states with the relatively high accuracy and fast convergence speed. Its performance is compared with those of a simplex GA, a GASA and a MFCM. In the experiments, the optimal output states are estimated by each of four search algorithms. Using the estimated channel output states, the desired channel states are derived and then utilized to compute the decision probability of Bayesian equalizer for the reconstruction of transmitted symbols.

The organization of this paper is as follows. Section 2 includes a brief introduction to the equalization of nonlinear channel using the Bayesian equalizer; Section 3 shows the relation between the desired channel states and the channel output states. In section 4, the proposed MFCM\_GW is introduced. The simulation results including comparisons with three other algorithms are provided in section 5. Conclusions are presented in Section 6.

## 2. EQUALIZATION OF NONLINEAR CHANNEL USING BAYESIAN EQUALIZER

A nonlinear channel equalization system is shown in Fig. 1. A digital sequence  $s(k)$  is transmitted through the nonlinear channel, which is composed of a linear portion described by  $H(z)$  and a nonlinear component  $N(z)$ , governed by the following expressions,

$$\bar{y}(k) = \sum_{i=0}^p h(i)s(k-i) \quad (1)$$

$$\hat{y}(k) = D_1\bar{y}(k) + D_2\bar{y}(k)^2 + D_3\bar{y}(k)^3 + D_4\bar{y}(k)^4 \quad (2)$$

where  $p$  is the channel order and  $D_i$  stands for the coefficient of the  $i^{\text{th}}$  nonlinear term. The transmitted symbol sequence  $s(k)$  is assumed to constitute an equiprobable and independent binary sequence taking values from a two-valued set  $\{\pm 1\}$ . We assume that the channel output is corrupted by an additive white Gaussian noise  $e(k)$ . Given this, the channel observation  $y(k)$  can be written as

$$y(k) = \hat{y}(k) + e(k) \quad (3)$$

If  $q$  denotes the equalizer order (number of tap delay elements in the equalizer), then there exist  $M = 2^{p+q+1}$  different input sequences

$$\mathbf{s}(\mathbf{k}) = [s(k), s(k-1), \dots, s(k-p-q)] \quad (4)$$

that may be received (where each component is either equal to 1 or -1). For a specific channel order and equalizer order, the number of input patterns that influence the equalizer is equal to  $M$ , and

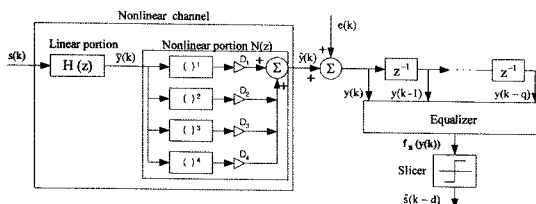


Fig. 1. An overall structure of a nonlinear channel equalization system

the input vector of equalizer without noise is

$$\hat{\mathbf{y}}(\mathbf{k}) = [\hat{y}(k), \hat{y}(k-1), \dots, \hat{y}(k-q)] \quad (5)$$

The noise-free observation vector  $\hat{\mathbf{y}}(\mathbf{k})$  is referred to as the desired channel states, and can be partitioned into two sets,  $\mathbf{Y}_{q,d}^{+1}$  and  $\mathbf{Y}_{q,d}^{-1}$ , as shown in (6) and (7), depending on the value of  $s(k-d)$ , where  $d$  is the desired time delay.

$$\mathbf{Y}_{q,d}^{+1} = \{ \hat{\mathbf{y}}(\mathbf{k}) | s(k-d) = +1 \} \quad (6)$$

$$\mathbf{Y}_{q,d}^{-1} = \{ \hat{\mathbf{y}}(\mathbf{k}) | s(k-d) = -1 \} \quad (7)$$

In case of a linear channel ( $D_1=1, D_2=0, D_3=0$  and  $D_4=0$ ),  $\hat{\mathbf{y}}(\mathbf{k})$  in (3), (5), (6) and (7) is replaced with  $\bar{\mathbf{y}}(\mathbf{k})$  in (1). The task of the equalizer is to recover the transmitted symbols  $s(k-d)$  based on the observation vector  $\mathbf{y}(\mathbf{k})$ . Because of the additive white Gaussian noise, the observation vector  $\mathbf{y}(\mathbf{k})$  is a random process having conditional Gaussian density functions centered at each of the desired channel states. The determination of the value of  $s(k-d)$  becomes a decision problem. Bayes decision theory [14] provides the optimal solution to the general decision problem, and thus can be applied here to derive the optimal solution for the equalizer. The solution forming the optimal Bayesian equalizer for the equiprobable transmitted symbols is given as follows [15,16]

$$f_B(\mathbf{y}(\mathbf{k})) = \sum_{i=1}^{n_s^{+1}} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{+1}\|^2 / 2\sigma_e^2) - \sum_{i=1}^{n_s^{-1}} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{-1}\|^2 / 2\sigma_e^2) \quad (8)$$

$$\hat{s}(k-d) = \text{sgn}(f_B(\mathbf{y}(\mathbf{k}))) = \begin{cases} +1, & f_B(\mathbf{y}(\mathbf{k})) \geq 0 \\ -1, & f_B(\mathbf{y}(\mathbf{k})) < 0 \end{cases} \quad (9)$$

where  $\mathbf{y}_i^{+1}$  and  $\mathbf{y}_i^{-1}$  are the desired channel states belonging to sets  $\mathbf{Y}_{q,d}^{+1}$  and  $\mathbf{Y}_{q,d}^{-1}$ , respectively, and their number of elements in these sets are denoted by  $n_s^{+1}$  and  $n_s^{-1}$ . Furthermore  $\sigma_e^2$  is the noise variance. The optimal equalizer solution in (8) depends on the desired channel states. In other words, the solution of nonlinear blind channel equalization crucially depends on how to find the desired channel

states,  $\mathbf{y}_i^{+1}$  and  $\mathbf{y}_i^{-1}$ , only from the observation vector  $\mathbf{y}(\mathbf{k})$ . In this study, the proposed MFCCM\_GW algorithm is investigated in search of the optimal output states of a nonlinear blind channel with the high accuracy and fast convergence speed. The desired channel states can be derived by considering their relationship with the searched channel output states. It will be explained in the next section. The optimal Bayesian decision probability in (8) is used to construct the fitness function of proposed algorithm, and it is also utilized as an equalizer, along with (9), for the reconstruction of the transmitted symbols.

### 3. DESIRED CHANNEL STATES AND CHANNEL OUTPUT STATES

The desired channel states,  $\mathbf{y}_i^{+1}$  and  $\mathbf{y}_i^{-1}$ , must be known for the Bayesian equalizer (8) - (9) in order to reconstruct the transmitted symbols. If the channel order is taken as  $p=1$  with  $H(z)=0.5+1.0z^{-1}$ , the equalizer order  $q$  is equal to 1, the time delay  $d$  is also set to 1, and the nonlinear portion is described by  $D_1=1, D_2=0.1, D_3=0.05, D_4=0.0$  (see Fig. 1), then the eight different channel states ( $2^{p+q+1}=8$ ) may be observed at the receiver in the noise-free case. Here the output of the equalizer should be  $\hat{s}(k-1)$ , as shown in Table 1. From this table, it can be seen that the desired channel states  $[\hat{y}(k), \hat{y}(k-1)]$  can be constructed from the elements of the dataset, called "channel output states",  $\{a_1, a_2, a_3, a_4\}$ , where for this particular channel we have  $a_1=1.89375, a_2=-0.48125, a_3=0.53125, a_4=-1.44375$ . The length of dataset,  $\tilde{n}$ , is determined by the channel order,  $p$ , such as  $2^{p+1}=4$ . In general, if  $q=1$  and  $d=1$ , the desired channel states for  $\mathbf{Y}_{i,1}^{+1}$  and  $\mathbf{Y}_{i,1}^{-1}$  are  $(a_1, a_1), (a_1, a_2), (a_3, a_1), (a_3, a_2)$ , and  $(a_2, a_3), (a_2, a_4), (a_4, a_3), (a_4, a_4)$ , respectively. In the case of  $d=0$ , the channel states,  $(a_{1,1} a_1), (a_{1,2} a_2), (a_{2,3} a_3), (a_{2,4} a_4)$ , belong to  $\mathbf{Y}_{i,1}^{+1}$ , and  $(a_{3,1} a_1), (a_{3,2} a_2), (a_{4,3} a_3), (a_{4,4} a_4)$  belong to  $\mathbf{Y}_{i,1}^{-1}$ . This relation is valid for the channel that has

Table 1. The relation between desired channel states and channel output states

| Nonlinear channel with $H(z)=0.5+1.0z^{-1}, D_1=1, D_2=0.1, D_3=0.05, D_4=0.0$ , and $d=1$ |          |          |                        |                |                     |
|--|----------|----------|------------------------|----------------|---------------------|
| Transmitted symbols  |          |          | Desired channel states |                | Output of equalizer |
| $s(k)$   | $s(k-1)$ | $s(k-2)$ | $\hat{y}(k)$           | $\hat{y}(k-1)$ | $\hat{s}(k-1)$      |
| By channel output states, $\{a_1, a_2, a_3, a_4\}$   |          |          |                        |                |                     |
| 1  | 1        | 1        | 1.89375                | 1.89375        | $(a_1, a_1)$        |
| 1  | 1        | -1       | 1.89375                | -0.48125       | $(a_1, a_2)$        |
| -1   | 1        | 1        | 0.53125                | 1.89375        | $(a_3, a_1)$        |
| -1   | 1        | -1       | 0.53125                | -0.48125       | $(a_3, a_2)$        |
| 1  | -1       | 1        | -0.48125               | 0.53125        | $(a_2, a_3)$        |
| 1  | -1       | -1       | -0.48125               | -1.44375       | $(a_2, a_4)$        |
| -1   | -1       | 1        | -1.44375               | 0.53125        | $(a_4, a_3)$        |
| -1   | -1       | -1       | -1.44375               | -1.44375       | $(a_4, a_4)$        |

a one-to-one mapping between the channel inputs and outputs [11]. Thus the desired channel states can be derived from the channel output states if we assume that  $p$  is known, and the main problem of blind equalization can be changed to focus on finding the optimal channel output states from the received patterns.

It is known that the Bayesian likelihood (BL), given by (10), is always maximized with respect to the desired channel states derived from the optimal channel output states [16].

$$BL = \prod_{k=0}^{L-1} \max(f_B^{+1}(k), f_B^{-1}(k)) \tag{10}$$

where  $f_B^{+1}(k) = \sum_{i=1}^{n_1} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{+1}\|^2 / 2\sigma_e^2)$ ,  $f_B^{-1}(k) = \sum_{i=1}^{n_2} \exp(-\|\mathbf{y}(\mathbf{k}) - \mathbf{y}_i^{-1}\|^2 / 2\sigma_e^2)$  and  $L$  is the length of received sequences. Therefore, the BL is utilized as the fitness function (FF) of the proposed algorithm to find the optimal channel output states. Being more specific, the fitness function is taken as the logarithm of the BL, that is

$$FF = \sum_{k=0}^{L-1} \log(\max(f_B^{+1}(k), f_B^{-1}(k))) \tag{11}$$

The optimal channel output states, which maximize the fitness function FF, cannot be obtained

with the use of the conventional gradient-based methods given the fact that the channel structure is not known in advance [11]. For carrying out search of these optimal channel output states, the proposed MFCM\_GW is utilized in this study, and its performance is compared with those of three other search algorithms introduced in [11-13].

#### 4. A MODIFIED FUZZY C-MEANS WITH GAUSSIAN WEIGHTS (MFCM\_GW)

Before the introduction of the proposed MFCM\_GW to solve the problem of nonlinear blind channel equalization, the previously developed version of MFCM presented in [13] should be investigated at first because both algorithms have the same structure.

The MFCM comes with two additional stages in comparison with the standard version of the Fuzzy C-Means (FCM) [13,17]. One of them concerns the construction stage of possible data set of desired channel states with the derived elements of channel output states. The other is the selection stage for the optimal desired channel states among them based on the Bayesian likelihood fitness function. For the channel shown in Table 1, the four elements of channel output states ( $2^{n+1} = 4$ ) are required to construct the optimal desired channel states. If the candidates,  $\{c_1, c_2, c_3, c_4\}$ , for the elements of optimal channel output states  $\{a_1, a_2, a_3, a_4\}$ , are extracted from the centers of a conventional FCM algorithm (or randomly initialized at first), twelve (4!/2) different possible data sets of desired channel states can be constructed by completing matching between  $\{c_1, c_2, c_3, c_4\}$  and  $\{a_1, a_2, a_3, a_4\}$ . To facilitate fast matching, the arrangements of  $\{c_1, c_2, c_3, c_4\}$  are saved to a certain mapping set  $C$  such as  $C(1)=1,2,3,4$ ,  $C(2)=1,2,4,3$ , ...,  $C(12)=3,2,1,4$  before the search process starts. For example,  $C(2)=1,2,4,3$  means that the set of desired channel states is constructed with  $c_1$  for  $a_1$ ,  $c_2$  for  $a_2$ ,  $c_4$  for

$a_3$ , and  $c_3$  for  $a_4$  in Table 1. The desired channel states for this set are described as  $y_{i, C(2)}$  ( $y_{i, C(2)}^1$  and  $y_{i, C(2)}^{-1}$  for sets  $Y_{q,d}^{+1}$  and  $Y_{q,d}^{-1}$ , respectively), and its fitness function in (11) is presented by  $FF(2)$ . At the next stage, a data set of desired channel states, which has a maximum Bayesian fitness value, is selected as shown below

$$[index\_j, \max\_FF] = \max(FF(1), FF(2), \dots, FF(12)) \quad (12)$$

This data set ( $y_{i, C(index\_j)}$ ), the set of desired channel states configured by the selected  $C(index\_j)$ , is utilized as a center set in the conventional FCM algorithm. Subsequently the partition matrix  $U$  is updated and a new center set  $y_i$  is sequentially derived with the use of this updated matrix  $U$ . These are shown in (13) and (14).

$$U_{ik}^{(m+1)} = \frac{1}{\sum_{l=1}^{n_s} \left( \frac{\|y(k) - y_{i, C(index\_j)}^{(m)}\|}{\|y(k) - y_{l, C(index\_j)}^{(m)}\|} \right)^2} \quad (13)$$

$$y_i^{(m+1)} = \frac{\sum_{k=0}^{L-1} (U_{ik}^{(m+1)})^2 y(k)}{\sum_{k=0}^{L-1} (U_{ik}^{(m+1)})^2} \quad (14)$$

where  $y_i^{(m+1)}$  is the estimated center set at the  $(m+1)^{th}$  iteration and  $n_s$  is the total number of center vectors ( $n_s=8$  for the channel in Table 1). The new four candidates for the elements of optimal output states are extracted from this new center set,  $y_i^{(m+1)}$ , based on the relation presented in Table 1. The eight centers in the new center set are treated as the desired channel states constructed by the elements of channel output states,  $\{a_1, a_2, a_3, a_4\}$ , shown in Table 1, and thus each value of the new  $\{c_1, c_2, c_3, c_4\}$  is replaced with each one of the  $\{a_1, a_2, a_3, a_4\}$  in the new center set as in (15), respectively.

$$c_r^{(m+1)} = a_r \text{ in } y_i^{(m+1)} \text{ where } r=1,2,3,4 \quad (15)$$

These steps are repeated until the Bayesian likelihood fitness function has not been changed or the

maximum number of iteration has been reached. More details about MFCM can be found in [13].

However, the performance of MFCM is easily affected by a heavy noise, because its partition matrix  $U$  and center set  $y_i$  are updated based on Euclidean distance measure such as shown in equations (13) and (14) by the clustering process of a standard FCM algorithm. As mentioned in section 2, the received symbol  $y(k)$  is a random process having conditional Gaussian density functions centered at each of the desired channel states because of the additive white Gaussian noise. Thus to avoid this noise effect, the proposed MFCM\_GW utilizes the Gaussian density function to derive the membership matrix  $U$  and a new center set  $y_i$  by replacing the equations (13) and (14) in the MFCM with equations (16) and (17), respectively.

$$U_{ik}^{(m+1)} = \frac{\exp(-\|y(k) - y_{i,C(index..j)}^{(m)}\|^2 / 2\sigma_e^2)}{\sum_{l=1}^{n_c} \exp(-\|y(k) - y_{l,C(index..j)}^{(m)}\|^2 / 2\sigma_e^2)} \quad (16)$$

$$y_i^{(m+1)} = \sum_{k=0}^{L-1} U_{ik}^{(m+1)} y(k) \quad (17)$$

where  $\sigma_e^2$  is the noise variance. For the communication channels corrupted by the additive white Gaussian noise, it has a more robust characteristic to the noise than the MFCM does. It will be clearly shown in our experiments. The proposed MFCM\_GW algorithm can be concisely described as some pseudo-code along with its flowchart shown in Fig. 2.

*begin*

*save arrangements of candidates,  $\{c_1, c_2, c_3, c_4\}$ , to  $C$*   
*randomly initialize the candidates,  $\{c_1, c_2, c_3, c_4\}$*   
*while (new fitness function - old fitness function) < threshold value*  
*for  $j=1$  to  $C$  size*  
*map the arrangement of candidates,  $C[j]$ , to  $\{a_1, a_2, a_3, a_4\}$*   
*construct a set of desired channel states*

*based on the relation shown in table 1 calculate its fitness function (FF[j]) by equation (11)*

*end*

*find a data set which has a maximum FF in  $j=1..C$  size : equation (12)*

*update the membership matrix  $U$  by the data set*

*utilized as a center set : equation (16)*

*derive a new center set by the  $U$ : equation (17)*

*extract the candidates,  $\{c_1, c_2, c_3, c_4\}$ , from the new center set*

*based on the relation shown in table 1: equation (15)*

*end*

*end*

## 5. EXPERIMENTAL STUDIES

In this section, the nonlinear blind equalizations realized with the use of the simplex GA, GASA, MFCM and MFCM\_GW are taken into account to demonstrate the effectiveness of the proposed method. Four nonlinear channels in [11] and [18]

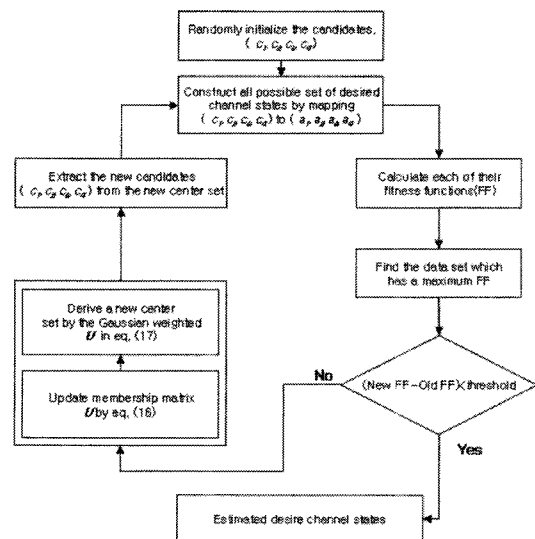


Fig. 2. The flowchart of the MFCM\_GW.

with different channel orders are discussed. Channel 1 is shown in Table 1 while Channel 2, 3 and 4 are described as follows.

Channel 2:  $H(z) = 0.5 + 1.0z^{-1}$ ,  $D_1 = 1, D_2 = 0.1, D_3 = -0.2$ ,  
 $D_4 = 0.0$ , and  $d=1$

Channel 3:  $H(z) = 0.5 + 1.0z^{-1}$ ,  $D_1 = 1, D_2 = 0.0, D_3 = -0.9$ ,  
 $D_4 = 0.0$ , and  $d=1$

Channel 4:  $H(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$ ,  $D_1 = 1$ ,  
 $D_2 = 0.2, D_3 = 0.0, D_4 = 0.0$ ,

and  $d=1$

In channel 1, 2 and 3, the channel order  $p$ , the equalizer order  $q$ , and the time delay  $d$  are 1, 1, 1, respectively. Thus for each of them, the output of the equalizer should be  $\hat{s}(k-1)$ , and the eight desired channel states ( $2^{p+q} = 8$ ) composed of the four channel output states ( $2^{p+1} = 4$ ,  $a_1, a_2, a_3, a_4$ ) as shown in Table 1 will be observed at the receiver in the noise-free case. For channel 4, the sixteen desired channel states with the eight channel output states ( $2^{p+1} = 8$ ,  $a_1, a_2, a_3, \dots, a_8$ ) exist because the channel order  $p$  is 2. The desired channel states,  $(a_1, a_1), (a_1, a_2), (a_2, a_3), (a_2, a_4), (a_5, a_1), (a_5, a_2), (a_6, a_3), (a_6, a_4)$ , belong to  $\mathbf{Y}_{1,1}^{+1}$ , and  $(a_3, a_5), (a_3, a_6), (a_4, a_7), (a_4, a_8), (a_7, a_5), (a_7, a_6), (a_8, a_7), (a_8, a_8)$  belong to  $\mathbf{Y}_{1,1}^{-1}$ , where  $a_1, a_2, a_3, \dots, a_8$  are 2.0578, 1.0219, -0.1679, -0.7189, 1.0219, 0.1801, -0.7189 and -1.0758, respectively. These sixteen desired channel states for channel 4 are summarized in [13].

In the experiments, 10 independent simulations for each of four channels with six different noise levels (SNR=0,5,10,15,20 and 25db) are performed with 1,000 randomly generated transmitted symbols. Afterwards the obtained results are averaged. The four search algorithms, simplex GA, GASA, MFCM and MFCM\_GW, have been implemented in a batch mode to facilitate comparative analysis. With this regard, we determine the normalized root mean squared errors (NRMSE)

$$\text{NRMSE} = \frac{1}{\|\mathbf{a}\|} \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{a} - \hat{\mathbf{a}}_i\|^2} \quad (18)$$

where  $\mathbf{a}$  is the dataset of optimal channel output states,  $\hat{\mathbf{a}}_i$  is the dataset of estimated channel output states in the  $i^{\text{th}}$  simulation, and  $N$  is the total number of independent simulations ( $N=10$ ). As shown in Fig. 3, the proposed MFCM\_GW comes with the lowest NRMSE for all four channels, and the performance differences are more severe under the high noise levels such as SNR=0,5,10db. It is caused by the fact that the MFCM\_GW uses the Gaussian weights shown in equations (16) and (17) to reduce the noise interference as mentioned in section 4. A sample of 1,000 received symbols under 5db SNR for channel 4 and its desired channel states constructed from the estimated channel output states by each of four search algorithms are shown in Fig. 4.

In addition, we compared the search time of the algorithms. The search times for each of four algorithms are included in Table 2; notably, the MFCM and MFCM\_GW offer much higher search speed for all channels and this could be attributed to their simple structures. The basic architecture of MFCM\_GW is shared with the one of MFCM introduced in [13]. However, the search speed for the proposed MFCM\_GW is much faster where the noise level is going up (SNR=0,5,10db) as shown

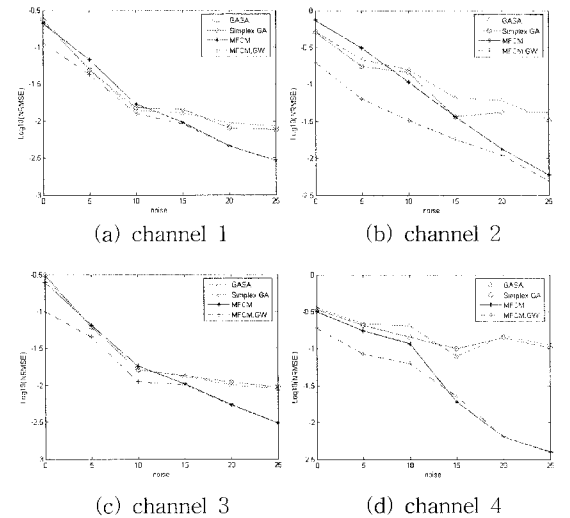


Fig. 3. NRMSE for channel 1, 2, 3, and 4.

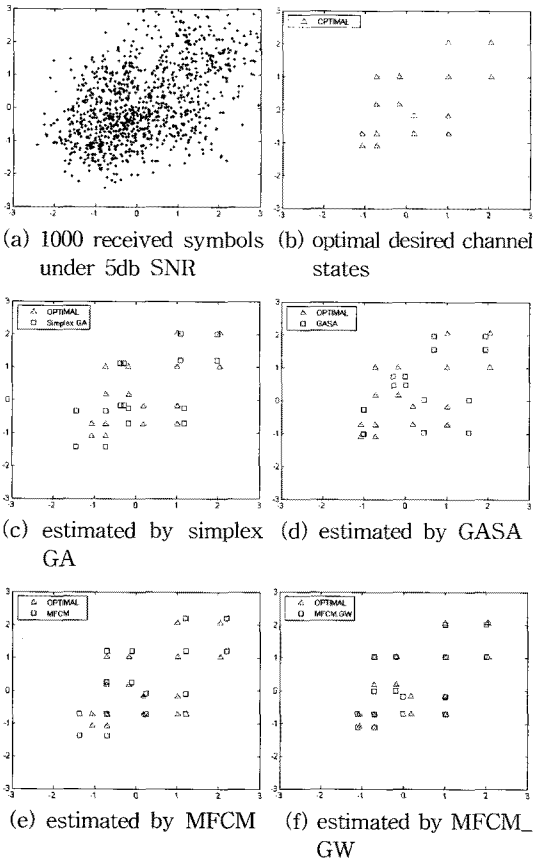


Fig. 4. A sample of received symbols for channel 4 and its desired channel states.

in the performance of NRMSE. Finally, we investigated the bit error rates (BER) when using the Bayesian equalizer; refer to Table 3. It becomes apparent that the BER with the estimated channel output states realized by the MFCM\_GW is almost the same as the one with the optimal output states for all four channels.

**6. CONCLUSIONS**

In this paper, we have introduced a new modified Fuzzy C-Means clustering algorithm with Gaussian weights and showed its application to nonlinear channel blind equalization. In this approach, the highly demanding modeling task of an unknown nonlinear channel becomes unnecessary

Table. 2. The averaged search time( in sec) for each of four algorithms.(Simulation environment : Pentium4 2.8Ghz, 2G Memory, code written in Matlab 7.1)

| Channel   | SNR  | Simplex GA | GASA    | MFCM   | MFCM_GW |
|-----------|------|------------|---------|--------|---------|
| Channel 1 | 0db  | 41.6844    | 41.6735 | 0.4438 | 0.3766  |
|           | 5db  | 42.2844    | 42.5594 | 0.2266 | 0.2219  |
|           | 10db | 42.6500    | 42.2641 | 0.1469 | 0.2203  |
|           | 15db | 42.4703    | 42.4078 | 0.1250 | 0.1188  |
|           | 20db | 41.9703    | 42.1688 | 0.1078 | 0.1469  |
| Channel 2 | 0db  | 40.8609    | 41.2141 | 0.5391 | 0.3188  |
|           | 5db  | 41.9953    | 42.2703 | 0.3016 | 0.2594  |
|           | 10db | 42.2578    | 42.1203 | 0.1750 | 0.1688  |
|           | 15db | 42.0000    | 42.1953 | 0.1532 | 0.1609  |
|           | 20db | 42.3781    | 42.3860 | 0.1516 | 0.1531  |
| Channel 3 | 0db  | 40.5922    | 40.8000 | 0.3578 | 0.3312  |
|           | 5db  | 42.7297    | 42.1750 | 0.2109 | 0.1969  |
|           | 10db | 42.5063    | 42.0656 | 0.1547 | 0.1656  |
|           | 15db | 42.2813    | 42.1969 | 0.1406 | 0.1234  |
|           | 20db | 42.2156    | 42.4906 | 0.1109 | 0.1266  |
| Channel 4 | 0db  | 56.8953    | 56.7391 | 2.9844 | 2.6000  |
|           | 5db  | 59.2610    | 59.6953 | 3.3172 | 1.3187  |
|           | 10db | 59.9703    | 59.7000 | 2.0844 | 0.9750  |
|           | 15db | 60.6172    | 59.8954 | 1.6891 | 0.7063  |
|           | 20db | 60.2391    | 60.0985 | 3.4547 | 0.8922  |
|           | 25db | 58.3188    | 58.0703 | 1.6969 | 1.4969  |

as the construction of the desired channel states is accomplished directly on a basis of the estimated channel output states. It has been shown that the proposed MFCM\_GW with the Bayesian likelihood treated as the fitness function offers better performance in comparison to the solutions provided by the simplex GA, GASA, and the previous version of MFCM approach. In particular, it successively estimates the channel output states with relatively high speed and substantial accuracy even when the received symbols are significantly corrupted by a heavy noise. Therefore, the Bayesian equalizer based on MFCM\_GW can constitute a viable solution for various problems of nonlinear blind channel equalization. Our future research pursuits are oriented towards the use of the MFCM\_GW under more complex optimization en-



Table 3. Averaged BER%(no. of errors/no. of transmitted symbols)

| Channel \ SNR |      | with optimal states | Simplex GA | GASA  | MFCM  | MFCM_GW |
|---------------|------|---------------------|------------|-------|-------|---------|
|               |      |                     |            |       |       |         |
| Channel 1     | 0db  | 17.26               | 17.73      | 18.04 | 17.66 | 17.39   |
|               | 5db  | 07.87               | 07.85      | 08.00 | 08.07 | 07.96   |
|               | 10db | 01.15               | 01.16      | 01.15 | 01.18 | 01.17   |
|               | 15db | 00.01               | 00.01      | 00.01 | 00.01 | 00.01   |
|               | 20db | 00.00               | 00.00      | 00.00 | 00.00 | 00.00   |
|               | 25db | 00.00               | 00.00      | 00.00 | 00.00 | 00.00   |
| Channel 2     | 0db  | 27.94               | 28.10      | 28.01 | 28.50 | 28.19   |
|               | 5db  | 15.18               | 15.21      | 15.39 | 15.70 | 15.21   |
|               | 10db | 4.52                | 4.98       | 4.71  | 4.69  | 4.49    |
|               | 15db | 0.20                | 0.24       | 0.25  | 0.23  | 0.23    |
|               | 20db | 0.00                | 0.00       | 0.00  | 0.00  | 0.00    |
|               | 25db | 0.00                | 0.00       | 0.00  | 0.00  | 0.00    |
| Channel 3     | 0db  | 19.64               | 21.53      | 21.49 | 19.91 | 19.87   |
|               | 5db  | 10.44               | 10.52      | 10.53 | 10.59 | 10.53   |
|               | 10db | 02.64               | 02.66      | 02.68 | 02.68 | 02.66   |
|               | 15db | 00.09               | 00.09      | 00.09 | 00.09 | 00.09   |
|               | 20db | 00.00               | 00.00      | 00.00 | 00.00 | 00.00   |
|               | 25db | 00.00               | 00.00      | 00.00 | 00.00 | 00.00   |
| Channel 4     | 0db  | 21.03               | 21.19      | 21.65 | 21.51 | 21.49   |
|               | 5db  | 11.93               | 12.24      | 12.49 | 11.98 | 11.91   |
|               | 10db | 4.11                | 4.68       | 4.87  | 4.65  | 4.86    |
|               | 15db | 1.01                | 1.57       | 1.34  | 1.00  | 1.00    |
|               | 20db | 0.09                | 2.42       | 2.28  | 0.09  | 0.09    |
|               | 25db | 0.00                | 1.22       | 1.13  | 0.00  | 0.00    |

vironments, such as those encountered when dealing with channels of high dimensionality and equalizers of higher order.

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