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Minimum Transmit Power Strategy for Poisson Distributed Wireless Ad-hoc Relay Networks in Rayleigh Fading Channels

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Abstract

In this paper, the transmit power minimization for Poisson distributed wireless ad-hoc relay networks in Rayleigh fading channels is considered. We investigate two power allocation methods one is a minimum power allocation (MPA) strategy and the other is an equal outage power allocation (EOPA) strategy. We analyze the total transmit power of two allocation methods under the given end-to-end outage probabilityconstraint. Our results show that the MPA achieves more power saving than EOPA, and the power saving is more significant as the number of relay nodes increases.

Key Words : *component; relay power allocation fading ad-hoc network*

I. Introduction

Recently, wireless ad-hoc networks that can self-configure to form a network topology have been studied for many wireless applications. Inherently the multi-hop routing in wireless ad-hoc networks is used to deliver messages from a source to a destination. Since most nodes of wireless ad-hoc networks are power-limited, the power consumption becomes a critical issue in modern ad-hoc network design [1], [2]. In [3], a power optimized transmission to minimize the end-to-end outage probability of a relay network is considered in Rayleigh fading channels. The total power minimization in a wireless relay system is investigated in [4] subject to a given end-to-end outage probability in binary symmetric channels.

Most of previous works for transmit power optimization in wireless relay networks are based on the deterministic nodes [3]-[6]. In this paper, however, we assume the nodes of ad-hoc networks are randomly distributed. Then, we obtain the average distance between nodes from the probability density function (pdf) of the distance. We consider a minimum power allocation (MPA) strategy and an equal outage

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power allocation (EOPA) strategy under the constraint of predetermined outage probability of an ad-hoc relay network in Rayleigh fading channels. We analyze the performance of two power allocation methods and show that the MPA attains significant power saving compared with the EOPA.

The rest of this paper is organized as follows. Section II and III describe the system model of the multi-hop relay and the dual hop relay system, respectively. In Section IV, the MPA and the EOPA are investigated, and the power consumption of two power allocation strategies is compared. Extension to multi-hop relay system is considered in Section V. In Section VI, the numerical resultsare given, and finally we conclude our work in Section VII.

II. System model

We assume that nodes are distributed uniformly with a density λ in a network area. Then, the probability that there are N nodes in an area A is given by Poisson distribution [7]

$$\Pr[N\ nodes\ \text{in}\ A] = e^{-\lambda A}\frac{(\lambda A)^N}{N!}\ . \qquad (1)$$

In a random network with uniform distribution and unit density ($\lambda = 1$), the probability density function of the distance R_n to the nth nearest neighbor in a sector ϕ of source-destination (SD) axis in Fig.1 is given by [7]

$$f_{R_n}(r) = r^{2n-1}\left(\frac{\phi}{2}\right)^n \frac{2}{(n-1)} \qquad (2)$$

And the mean distance between the source node to the nth nearest neighbor node that lie within a sector ϕ of source-destination axis in Fig.1 are given by [8]

$$E(R_n) = \sqrt{2/\phi}\ \frac{\Gamma(n+1/2)}{\Gamma(n)} \qquad (3)$$

where $\Gamma(\cdot)$ is the Gamma function.

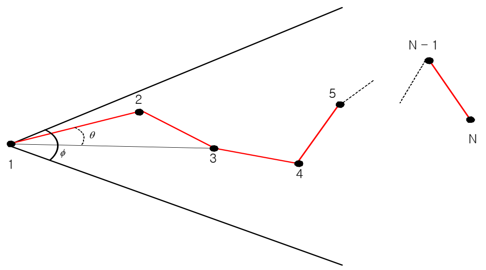

Fig.1 The system model of a multi-hop relay

III. Dual hop relay system

In this section, we consider a dual-hop relay system which consists of three nodes. For convenience, we represent source node, relay node, and destination node instead of node1, node2, and node3 in Fig.1, respectively. Outage is declared if the received signal-to-noise (SNR) ratio is below the predetermined threshold SNR. Hence, outage probability of the dual-hop regenerative system at the destination node is given by [9]

$$P_o = 1 - \exp[-\gamma_{th}(\frac{1}{\overline{\gamma}_R} + \frac{1}{\overline{\gamma}_D})] \qquad (4)$$

where γ_{th} denotes the threshold SNR. $\overline{\gamma}_R$ and $\overline{\gamma}_D$ are the average SNRs of the relay node and destination node, respectively.

We assume the propagation loss between a transmitter and a receiver is proportional to the inverse of the power of the path loss coefficients of the distance. Then the average SNR at relay node is written by

$$\overline{\gamma}_R = \frac{p_s}{N_R} R_{SR}^{-\alpha} = \frac{p_s}{N_R} G_{SR} \qquad (5)$$

where p_s is the transmit power at the source node, N_R is the noise power at the relay node, R_{SR} is the distances between the source nodeand the relay node, and G_{SR} is the path gain between the source nodeand the relay node. The average SNR at the destination node $\overline{\gamma}_D$ and the threshold SNR γ_{th} can be represented by similar manner: $\overline{\gamma}_D = (p_R / N_D) G_{RD}$, $\gamma_{th} = p_{th} / N_{th}$. We assume that the noise power at each node is the same.

There are following relations among distances,

$$R_{RD}^2 = R_{SR}^2 + R_{SD}^2 - 2 R_{SR} R_{SD} \cos\theta \qquad (6)$$

where R_{SD} is the distance between the source and the destination nodes, θ is the angle between the source-relay(SR) link and the source-destination(SD) link. The mean distances between nodes can be obtained by combining (3) and (6).

IV. Power optimization

In this section, we consider the minimization of the total transmit power subject to the predetermined end-to-end outage probability in ad-hoc networks.

A. Minimum Power Allocation (MPA) Method

We want to find a power allocation method to minimize the total transmit power of a dual-hop relay network. This problem is formulated as:

$$\text{Minimize} \qquad p_S + p_R = p_T\,, \qquad (7)$$

subject to

$$P_o = 1 - \exp[-\gamma_{th}(\frac{1}{\overline{\gamma}_R} + \frac{1}{\overline{\gamma}_D})]$$
$$p_S > 0, \quad p_R > 0 \qquad (8)$$

It is noted that (7) is a linear function and (8) is convex function. Therefore, we can obtain a unique optimal solution for the minimization problem [10]. Using the *Lagrange* method, the object function can be written as

$$J = p_S + p_R + \lambda\{1 - \exp[-\gamma_{th}(\frac{1}{\overline{\gamma}_R} + \frac{1}{\overline{\gamma}_D}) - P_o]\}. \qquad (9)$$

To solve this equation, taking derivative and replacing average SNR of (5) yield

$$\frac{\partial J}{\partial p_S} = 1 - \lambda \exp\left(-\frac{p_{th}}{G_{SR} p_S}\right)\frac{p_{th}}{G_{SR} p_S^2} = 0$$
$$\frac{\partial J}{\partial p_R} = 1 - \lambda \exp\left(-\frac{p_{th}}{G_{RD} p_R}\right)\frac{p_{th}}{G_{RD} p_R^2} = 0$$
$$\frac{\partial J}{\partial \lambda} = 1 - \exp\left[-p_{th}\left(\frac{1}{G_{SR} p_S} + \frac{1}{G_{RD} p_R}\right)\right] - P_o = 0. \qquad (10)$$

From the first and the second equations of (10), λ is not equal to zero, and

$$G_{SR}p_S^2 = G_{RD}p_R^2 . \qquad (11)$$

Substituting (11) to (8), the optimal transmit power of the source node, p_S^*, and that of the relay node, p_R^*, can be obtained as

$$p_S^* = -\frac{p_{th}}{\ln(1-P_o)}\left(\frac{1}{G_{SR}}+\frac{1}{\sqrt{G_{SR}G_{RD}}}\right),$$

$$p_R^* = -\frac{p_{th}}{\ln(1-P_o)}\left(\frac{1}{G_{RD}}+\frac{1}{\sqrt{G_{SR}G_{RD}}}\right). \qquad (12)$$

Hence, the minimized total transmit power, p_T^*, of the dual-hop system subject to the predetermined end-to-end outage probability is

$$\begin{aligned} p_T^* &= p_S^* + p_R^* \\ &= -\frac{p_{th}}{\ln(1-P_o)}\left(\frac{1}{G_{SR}}+\frac{1}{G_{RD}}+\frac{2}{\sqrt{G_{SR}G_{RD}}}\right) . \end{aligned} \qquad (13)$$

B. Equal Outage Power Allocation (EOPA) Method

If we assume that each node has the same outage probability under the given end-to-end outage probability, then the end-to-end outage probability can be written by

$$\begin{aligned} P_o &= 1-(1-P_{o,SR})(1-P_{o,RD}) \\ &= 1-(1-P_{o,SR})^2 \end{aligned} \qquad (14)$$

where $P_{o,SR}$ and $P_{o,RD}$ are the outage probability of the source-relay link and the relay-destination link, respectively. In Rayleigh fading channels, the outage probability can be written by

$$\begin{aligned} P_{o,SR} &= 1-\exp(-\gamma_{th}/\bar{\gamma}_{SR}) \\ &= 1-\exp\{-p_{th}/(G_{SR}p_s)\} . \end{aligned} \qquad (15)$$

After rearrangement, we can obtain the transmit power of the source node and the relay node,

$$p_s = -\frac{p_{th}}{G_{SR}}\frac{1}{\ln\left(\sqrt{1-P_o}\right)}$$

$$p_R = -\frac{p_{th}}{G_{RD}}\frac{1}{\ln\left(\sqrt{1-P_o}\right)} \cdot \qquad (16)$$

Therefore, the total transmit power of EOPA, p_{eq}, can be obtained by

$$\begin{aligned} p_{eq} &= p_S + p_R \\ &= -\frac{p_{th}}{\ln\left(\sqrt{1-P_o}\right)}\left(\frac{1}{G_{SR}}+\frac{1}{G_{RD}}\right) \end{aligned} \qquad (17)$$

The power reduction ratioof the total transmit power achieved by the MPA relative to EOPA is defined as

$$\begin{aligned} \mathrm{K} &= \frac{p_T^*}{p_{eq}} \\ &= \frac{1}{2}\frac{\left(\frac{1}{G_{SR}}+\frac{1}{G_{RD}}+\frac{2}{\sqrt{G_{SR}G_{RD}}}\right)}{\left(\frac{1}{G_{SR}}+\frac{1}{G_{RD}}\right)} . \end{aligned} \qquad (18)$$

We can make two crucial observations from (18): first, the power reduction ratio is independent of the end-to-end outage probability second, there is no power reduction when the path gain G_{SR} and G_{RD} are the same which means the distances R_{SR} and R_{RD} are equal. This result is identical to [4], [6].

V. Multi-hop relay system

A. MPA Method

The end-to-end outage probability of multi-hop relay systems in Rayleigh fading can be written by

$$P_o = 1-\exp\left(-\gamma_{th}\sum_{i=1}^{N-1}\frac{1}{\bar{\gamma}_i}\right) \qquad (19)$$

where $\bar{\gamma}_i$ is the average SNR of the i^{th} node. Similarly, the optimal transmit power of each node meeting the given end-to-end outage probability can be obtained as

$$p_1^* = -\frac{p_{th}}{\ln(1-P_o)}\left(\frac{1}{G_{12}}+\sum_{i=2}^{N-1}\frac{1}{\sqrt{G_{12}G_{i(i+1)}}}\right)$$
$$p_i^* = \sqrt{\frac{G_{(i-1)i}}{G_{i(i+1)}}}\,p_{(i-1)}\ ,\ i=2,\ 3,\ \ldots,\ N-1. \qquad (20)$$

Accordingly, the minimum total transmit power, p_T^*, is the sum of the transmit power of each node

$$p_T^* = \sum_{i=1}^{N-1} p_i^* . \qquad (21)$$

B.. EOPA Method

In the multi-hop case, the end-to-end outage probability can be written by

$$P_o = 1-(1-P_{o,k(k+1)})^{N-1},\ \ k=1,\ 2,\ldots,\ N \qquad (22)$$

where $P_{o,k(k+1)}$ denotes the outage probability of the communication link between the k^{th} node and the $(k+1)^{th}$ node. The transmit power of the i^{th} node is given by

$$p_i = -\frac{p_{th}}{G_{i(i+1)}}\frac{N-1}{\ln(1-P_o)}\ ,\quad i=1,\ 2,\ldots,\ N-1 \qquad (23)$$

Then, the total transmit power of the network is the sum of transmit power of each nodes.

VI. Numerical examples

Fig.2 shows the normalized mean distances to the distance between the source node and the relay node (1st neighbor), R_{SR}. It is noticed that the distance, R_{SD}, is increasing with the angle θ between SR and SD link.

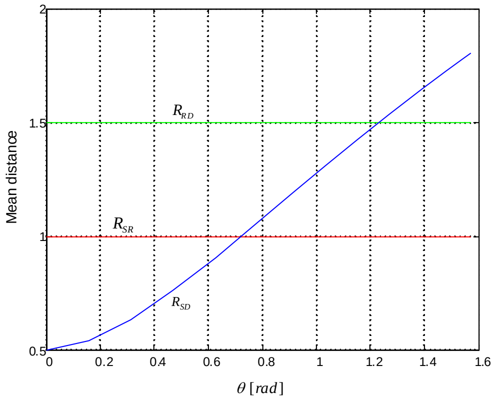

Fig. 2 Mean distance vs. angle θ between SR and SD link.

Fig. 3 is a plot of the transmit power of each node to meet the predetermined outage probability of 1×10^{-3} for N equal to 3. If the angle θ between SR and SD link is equal to zero, p_S^*, p_R^*, and p_T^* is 31.3 dB, 26.8 dB, and 32.6 dB, respectively. The distance between the relay node and the destination node increases with θ. The total transmit power of MPA, p_T^*, becomes 40.7 dB if θ is equal to $\pi/2$. In the case of EOPA, however, the total transmit power p_{eq} is greater than that of MPA the total transmit power becomes 33.5 dB and 41.4 dB if θ is zero and $\pi/2$, respectively. Consequently, MPA has a power gain of 0.9 dB and 0.7 dB when θ is zero and $\pi/2$, respectively, over EOPA. It is also noted that p_T^* is identical to p_{eq} if G_{SR} is equal to G_{RD} (implying R_{SR} is equal to R_{RD}).

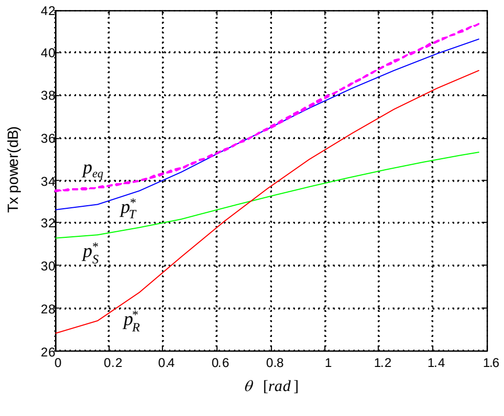

Fig.3 Transmit power vs. angle θ between SR and SD link ($P_o = 1\times10^{-3}, \alpha = 3, \phi = \pi/2, p_{th} = 1$).

Total transmit powers with $\theta = 0$ for several power allocation strategies are shown in Fig. 4. On the outage probability of 1×10^{-3}, the total transmit power of MPA, EOPA, and the direct transmission are 32.6 dB, 33.5 dB, and 35.3 dB, respectively. The total transmit power of the direct transmission indicates the required transmit power to satisfy the given end-to-end outage probability where the communication link from the source node to the destination node is a direct path. We noticed that the MPA obtains power gains of about 0.9 dB and 2.7 dB over EOPA and the direct power allocation, respectively.

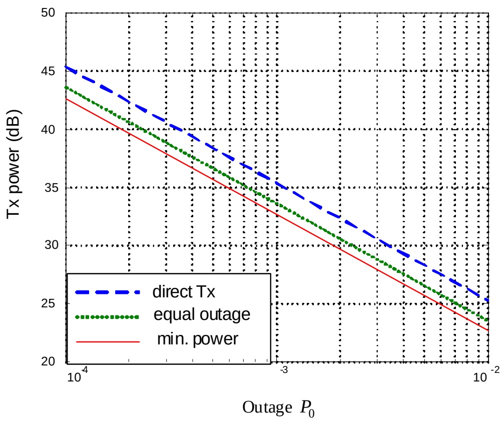

Fig.4 Total transmit powers for each power allocation strategies ($\alpha = 3, \phi = \pi/2, \theta = 0, p_{th} = 1$).

Fig.5 depicts the power reduction ratio as a function of θ when $P_o = 1\times10^{-3}, \phi = \pi/2, p_{th} = 10$. It is noted that the power reduction ratio increases with the path loss coefficient α. The power reduction ratio is 18 % and 33 % when α is 3 and 5, respectively, when $\theta = 0$. It is found that the power reduction ratio increases to unity and then decreases with θ, because the increasing ratio of p_{eq} and p_T^* with respect to θ is not the same as shown in Fig.2. When θ is about 0.72 [rad], especially, R_{SR} is equal to R_{RD}, the power reduction ratio becomes unity, which indicates no power reduction can be obtained.

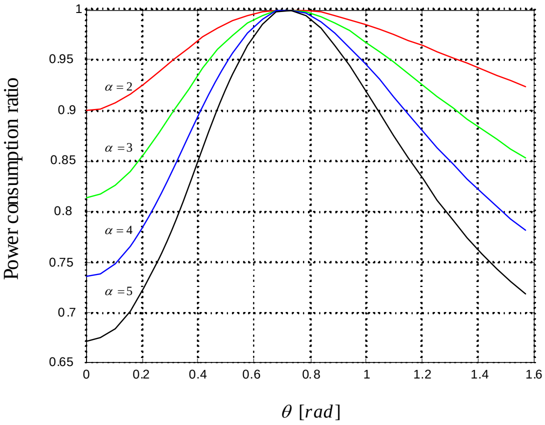

Fig. 5 Power reduction ratio vs. angle θ between SR and SD link
($P_o=1\times10^{-3}$, $\phi=\pi/2$, $p_{th}=10$).

For multi-hop relay routing, the total transmit powers as a function of the number of nodes is plotted Fig.6. It is shown that more total transmit power is required as the number of nodes increases. In multi-hop relay systems, the MPA can save more transmit power compared with the EPA or the direct transmission, and the power savings through the MPA method is significant with the number of nodes. It is seen that MPA reduces the power consumption of 2.60 dB and 6.86 dB over the EOPA and the direct transmission, respectively, of the multi-hop relay system with 7 nodes.

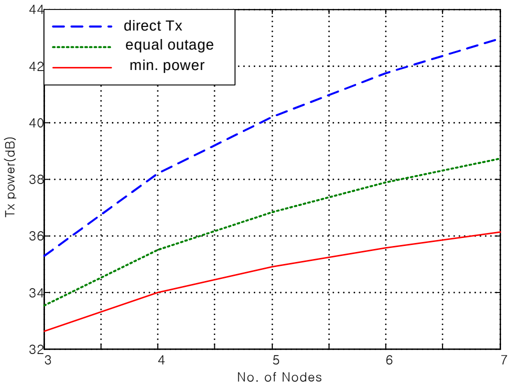

Fig. 6 Total transmit powers of Multi-hop relay routing system.
($\alpha=3$, $\phi=\pi/2$, $\theta=0$, $p_{th}=1$, $P_o=1\times10^{-3}$)

Ⅶ. Conclusions

In this paper, we have modeled the wireless multi-hop relay system as a Poisson point process in Rayleigh fading channels. We have investigated two power allocation methods, the minimum power allocation (MPA) and equal outage power allocation (EOPA). We have analyzed the total transmit power of two power allocation methodsunder the given end-to-end outage probability. We have shown that the achievable power saving through the MPA method is significant over EOPA method, and the power saving increases with the number of nodes.

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