Border Effect of Transmission Coverage in Mobile Wireless Communications

J. David Haughs and Dongsoo S. Kim

Abstract—In this paper, we demonstrate the boundary effect of a deployed regions on the effective coverage of a mobile node. A node coverage area is not uniform throughout the entire deployed region. Assuming a uniform coverage can result in significant error in calculations. In this study, we analyze the behavior of a node's coverage area as a function of its transmission range throughout the entire deployed region. Using this analysis, a mathematical model for effective coverage in mobile wireless communications is created. The mathematical model considers the effect of the deployed regions boundaries on the coverage area of a mobile node. Lastly, we present simulation results to verify the analytical model and to compare this model with that of a uniform coverage.

Index Terms—Wireless communication, mobility model, ad-hoc networks,

1 INTRODUCTION

AD hoc wireless networks are formed by a group of wireless mobile nodes. The wireless nodes can be any sort of microprocessor device with the ability for wireless communication. By nature, a wireless ad hoc network lacks any fixed network infrastructure. Users are provided connectivity with unrestricted mobility due to the self-organizing, rapidly deployable architecture of wireless ad hoc networks.

Because a node in a wireless ad hoc network is connected with unrestricted mobility, the topology of the network is dynamic. Realistic mobility modeling becomes very critical for analyzing node behavior and network performance. Common mobility models are the random walk and random waypoint mobility models [1], [4], [5], [10].

The random walk mobility model was derived from the Brownian motion, which is a stochastic process for modeling random continuous motion [9]. In this model, a mobile node moves from its current location with a randomly selected speed in a randomly selected direction. The new speed and direction are both chosen from pre-defined ranges [5]. The new speed and direction are maintained for an arbitrary length of time. At the end of the chosen time the node makes a memoryless decision of a new random speed and direction.

In the random waypoint mobility model, a mobile node chooses a random destination within the deployed area. With the destination chosen, the node randomly chooses a speed at which to travel arbitrarily from a pre-defined range. Upon reaching the destination, the node pauses for a random time before determining a new destination and speed. In the random waypoint model, nodes have a tendency to concentrate to the middle of the deployment region [4], [14], [16], indicating that this model does not present a truly uniform node distribution.

Both mobility models presented above are used regularly in the simulation of wireless mobile nodes [8], [15], [7], [12], [16], [17]. In addition to choosing the appropriate mobility model, one must also understand the coverage of a MN under a given situation. It is common to assume a node has a uniform coverage area independent of its location whithin the deployment area without full understanding of the

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impact of the transmission range in that deployment area [3], [12]. This paper will present the dependency of location and transmission power on the coverage of a mobile node and provide an analytical method for describing this behavior and analyzing connectivity of mobile nodes.

2 Effective Node Coverage

The effective coverage of a mobile node describes the number of mobile nodes that can communicate with a given node in the deployment region. Effective coverage is dependent on the location of the node and its transmission power in relation to the size of the deployment region and nodal density. For simplicity, it has been commonly assumed that a mobile node has a uniform circular transmission area that directly corresponds to its transmission power. The transmission power of a node has a range of length and can therefore be represented as the radius of the circular transmission area centered at the node. As a MN moves about the deployment region, however, the transmission area could be affected by the boundary. The boundary effect occurs when a node is at a distance less than its transmission range from one or more boundaries of the deployment region. At this distance a portion of its total transmission area extends beyond the boundary of the deployment region. This portion of transmission area should be considered unusable as there cannot exist nodes to communicate with beyond the boundary. Therefore, the effective coverage area of the affected mobile node is less than the circular range.

Figure 1 illustrates this behavior. The figure shows two identical nodes near one corner of a deployment region. The solid dot in the middle of the shaded circle is a mobile node, the shaded circle represents the transmission area of the node. The dashed line in the figure is located a distance of r from the boundary. This line represents a threshold into a region of cutoff for its effective coverage area. As can be seen in the figure, a node crossing this threshold line nearby the boundary has a portion of its transmission area extending beyond the boundary of the deployment. This hatched area serves no benefit to the node.

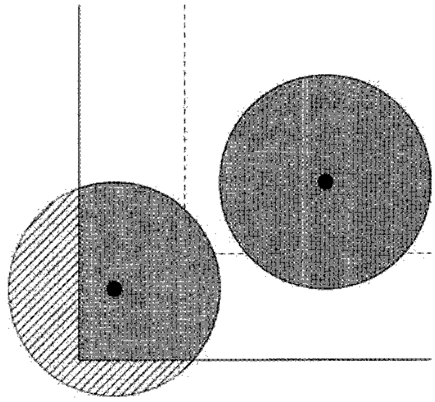


Fig. 1. Illistration of boundary effect using two identical nodes. One node's coverage extends beyond the boundary of the deplyment region.

The boundary effect can have a major impact on the coverage area of a mobile node. When analyzing node connectivity in wireless ad hoc networks it is important to take into account the boundary effect to prevent an overestimation of coverage area. Total transmission area data can be skewed by the cutoff areas at the boundaries.

3 Effective Connectivity Model

The goal of this paper is to predict node connectivity based on transmission range and to model the behavior of nodes under different simulations. An average effective connectivity area in two dimensional space, $\mu(r)$, is given as

$$\mu(r) = \iint S(r, x, y) \times p(x, y)\, dx\, dy, \qquad (1)$$

where x and y represent the location of the node, $p(x, y)$ is the probability that a node is located at the point (x, y) and $S(r, x, y)$ denotes the effective coverage of the node with the transmission range r and at the location (x, y).

Our analytical model considers a node's effective coverage area in different sections, or regions, in a deployment area. Without loss of generality, consider a square delolyment area of $L \times L$ and the transmission range r is not larger than $L/2$. For a detailed analysis for $r > L/2$, readers can refer [6]. The regions are created based on the transmission range, r, of a mobile node. As r nears 0, region inside of the threshold dominates the deployment, conversely as r nears $\frac{L}{2}$, regions either around the border or at the corner will dominate.

Nodes lying within the threshold, says region A, do not have coverage areas intersecting any boundary and therefore, are able to utilize their entire circular coverage area given as $S_A(r,x,y) = \pi r^2$.

A node beyond one and only threshold, says region B, yields the effective area as a circular segment. At the position of $x \le r$ and $r < y \le L - r$, the area of the circular segment is given as

$$S_B(r,x,y) = \pi r^2 + x\sqrt{r^2 - x^2} - r^2 \arccos(x/r).$$

A node beyond thresholds in both dimensions has the coverage area intersecting both boundaries. There are two cases to take into consideration for a node located in this region. The first case(C) occurs when a node is located within a distance of r from the very corner. The second case(D) occurs when a node is located out of the distance r from the corner but still within the bounds of the thresholds.

For a node located at $x \le r$ and $y \le r$, the coverage areas of C_1 and C_2 are given as

$$\begin{aligned} S_C(r,x,y) = {} & \frac{3}{4}\pi r^2 + xy + \frac{1}{2}x\sqrt{r^2 - x^2} \\ & + \frac{1}{2}y\sqrt{r^2 - y^2} \\ & - \frac{1}{2}r^2(\arccos(x/r) + \arccos(y/r)), \end{aligned}$$

and

$$\begin{aligned} S_C(r,x,y) = {} & \pi r^2 + x\sqrt{r^2 - x^2} + y\sqrt{r^2 - y^2} \\ & - r^2(\arccos(x/r) + \arccos(y/r)). \end{aligned}$$

Figure 2 demonstrates the boundary effect by representing the recently described $S(r,x,y)$ as a ratio out of a complete circular transmission area. The x and y axes of the figure represent the node's location in one quarter of a deployment map. The figure shows that as the node moves away from a corner or a map edge, the number of nodes connected directly to the node increases until it reaches a constant.

4 Random Walk Analytical Model

Mobile nodes under the random walk mobility model remain distributed uniformly during the entire simulation [5], [14]. Therefore, the probability of a node located at a given (x,y), $p(x,y)$, is identical for every point within a region. Therefore, it is equivalent to the ratio of the area of a region to the area of the entire deployed region. The probabilities of a node located in each region are given as

$$\begin{aligned} p_A(x,y) &= (1 - 2\frac{r}{L})^2 \\ p_B(x,y) &= 4(1 - 2\frac{r}{L})\frac{r}{L} \\ p_C(x,y) &= \pi(\frac{r}{L})^2 \\ p_D(x,y) &= 4(\frac{r}{L})^2 - \pi(\frac{r}{L})^2 \end{aligned}$$

These probabilities are then used along with the coverage functions, $S(r,x,y)$, into Equation 1 to determine the average effective connectivity, $\mu(r)$, as follows:

$$\begin{aligned} \mu(r) = {} & \frac{\pi r^2(L - 2r)^2}{L^2} + \frac{4r^3(3\pi - 2)(L - 2r)}{3L^2} \\ & + \frac{r^4(9\pi - 5)}{3L^2} \end{aligned} \tag{2}$$

To find a unit-less connectivity, we can convert Equation 2 into a relative average effective connectivity by dividing it by the deployed area and introduce a relative transmission ρ as r/L. The relative average connectivity,$\mu(\rho)$, can

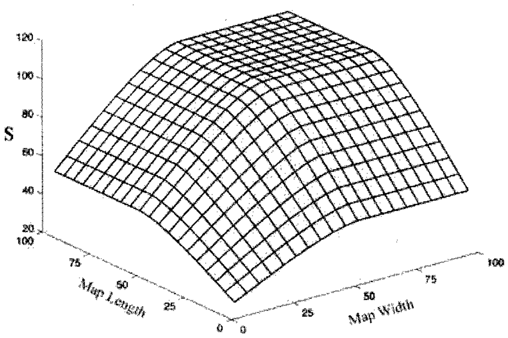


Fig. 2. Effective Transmission area due to border effect

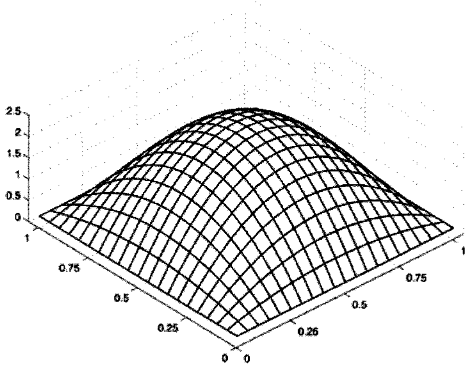


Fig. 3. Spatial Distribution of Random Waypoint

be given as

$$\mu(\rho) = \pi\rho^2 - \frac{8}{3}\rho^3 + (\frac{11}{3} - \pi)\rho^4$$

5 Random Waypoint Analytical Model

In a similar manner, the effective connectivity of the random waypoint mobility model can be obtained. However, the random waypoint mobility model lacks the uniformness in spatial distribution. Many researchers have worked to derive the random waypoint spatial density [2], [4], [13], [14]. Because of the dynamics of spatial distribution of the random waypoint [16], it is not easy to derive its average connectivity. For simplicity, we research uses the estimated spatial distribution presented in [14] as shown in Figure 3.

The effective connectivity of nodes is calculated with the inclusion of the spatial node density function from [14]. Due to this inclusion, $S(r, x, y)$ becomes not only a function of area, but also a function of density.

$$S(r, x, y) = f(A(r, x, y), p(x, y))$$

To simplify the calculations a zero pause time is assumed and no node is stationary. The resulting simplified spatial density function is,

$$f(x, y) = 36xy(x-1)(y-1)$$

This spatial distribution function is based on a unit square deployment region. Combining the density function and the effective coverage functions for each region, we obtain the average connectivity of the random waypoint as

$$\begin{aligned}\mu(\rho) &= 1.44\pi\rho^2 - 15.26\rho^5 + 5.4\pi\rho^6 - 5.27\rho^7 \\ &\quad + 3.79\rho^8 - 6.01\rho^9 + 3.56\rho^{10} - 0.92\rho^{11} \\ &\quad + 0.09\rho^{12} \\ &\backsimeq 1.44\pi\rho^2 - 15.26\rho^5 + 5.4\pi\rho^6 - 5.27\rho^7 \quad (3)\end{aligned}$$

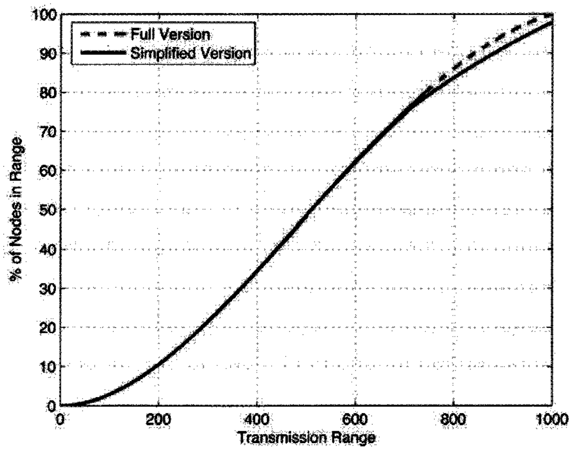


Fig. 4. Complete and Simplified

6 Analysis

6.1 Random Walk Analytical vs Simulated

Section 4 presented two versions of the random walk analytical model. The first version is a complete implementation of the effective coverage analytical model over the range of $0 < r \leq L$ made up of several parts. The second version is an expansion of the simpler beginning part of the lengthy first version. This function was originally intended to represent the effective coverage of the random walk mobility model from $0 < r \leq \frac{L}{2}$. However, once it was implemented the range was experimented with and it was found that its output is nearly identical to that of the lengthier version all the way to the extreme end of the range, L. Figure 4 shows both of these equations plotted together on the same graph. As one can see from the figure, the two functions are nearly identical all the way to about $r = 750$. At this point the simplified version produces a slightly lower estimation than the full version. Figure 5 shows both of these versions plotted with the results of a simulation. In this figure, it is clear that both models slightly deviate from the simulated data with the full version slightly over estimating and the abbreviated version slightly under estimating. The differences at this level of r are negligible as these levels of transmission range versus deployment region are not realistic. Therefore, the simplified version provides functional results.

To compare the model with the simulated data, multiple random walk simulations were run with the total number of simulated nodes ranging from 20 to 300 in increments of 20. Each of these simulations was conducted with a map size of 1000×1000 meters, a simulation time of 900 seconds, minimum node speed of 0.1 meters/second, maximum node speed of 10 meters/second and a uniformly distributed duration of [0,100] seconds. Data from every simulation was then analyzed, at transmission ranges from 25 meters to 1000 meters, for the average number of nodes within one node's transmission range. The data was plotted as the average percentage of nodes in the simulation that are within the transmission range of one node. As the transmission range increased, the curves of all simulations followed the same shape regardless of the number of nodes in the simulation.

The random walk simulation data was plotted with the random walk analytical model and can be seen by once again referring to Figure 5. From this figure one can see that the random walk simulation data and the random walk analytical model are nearly identical. The most noticeable difference in the analytical model as compared to the simulation data occurs at the upper bounds of the transmission range. This is attributed mainly to the approximation used in the analytical model. The analytical model is accurate up to approximately $r = \frac{3L}{4}$, this is where the simplified model deviates from the full model and the curve is slightly skewed.

The forth curve in Figure 5 represents the behavior of node coverage if no cutoff is taken into consideration, that is, a uniform coverage area is assumed. For a relatively small transmission range this assumption could be acceptable, but as the transmission range increases the assumed and actual node coverage differ greatly.

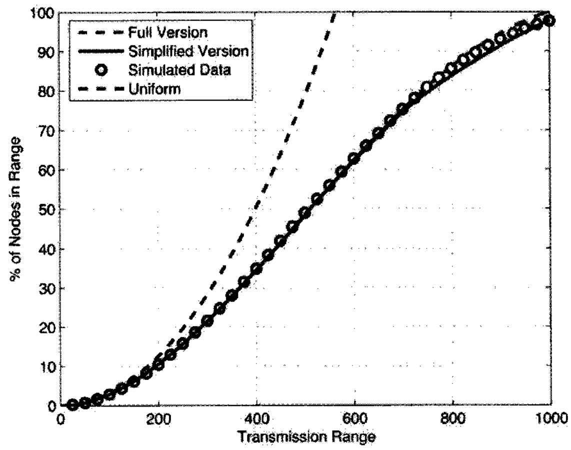


Fig. 5. Complete, Simplified and Simulated

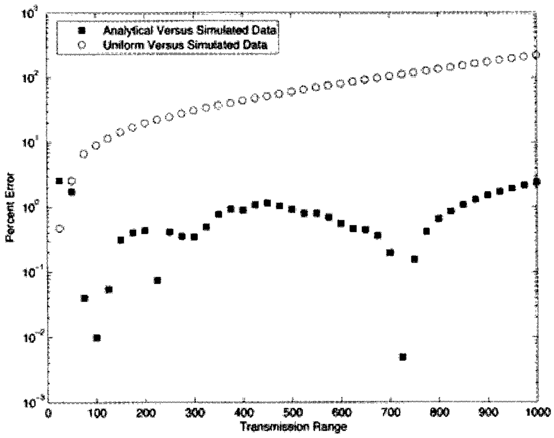


Fig. 6. Error of Random Walk

The analytical model and the uniform model are compared with the random walk simulation data and the difference is plotted in Figure 6. From this figure, it can be seen that the level of error increases at an exponential rate for the uniform distribution as r is increased, However, the analytical model retains a very small level of error throughout all values of r. From this data, it can be concluded that the uniform distribution can be used with a 95% confidence level if $r \leq 75$ and with a 90% confidence level if $r \leq 125$. These confidence levels decrease at a rapid rate beyond $r = 125$. However, using the analytical model the confidence level never drops below 95% and is actually better than

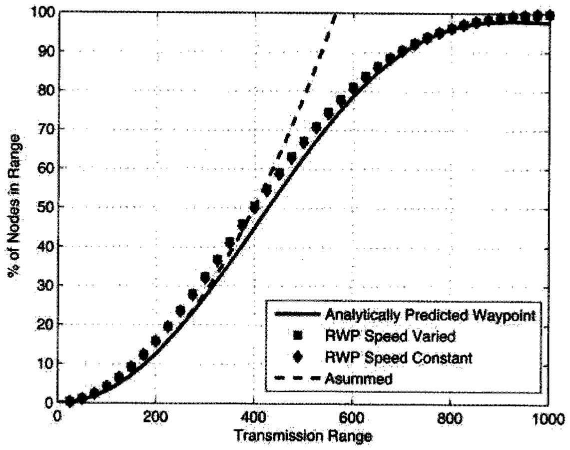


Fig. 7. Comparison of Random Waypoint

97.5% for $r \leq 900$.

6.2 Random Waypoint Analytical vs Simulated

As with the random walk simulations, multiple simulations where run for the random waypoint model. The number of nodes, simulation map size and simulation time was the same as in the random walk simulations. However, duplicate simulations where run with different speed ranges. The first simulation set was run with a minimum and maximum speed identical to that of the random walk simulations. The second simulation set was configured with a minimum and maximum speed of 5 meters/second. The maximum rest time for a node in both simulation sets was set to zero. Again data from each simulation was analyzed, at transmission ranges from 25 meters to 1000 meters, for the average number of nodes within one nodes transmission range. This data was also plotted and as was the case with the random walk model, all plots were the same irrespective of the total number of nodes in the simulation. The two speed variations in the simulations resulted in very similar plots as well.

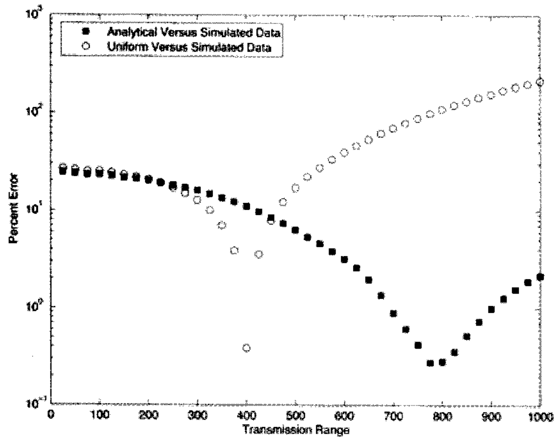
The data from the simulation was plotted with the analytical model and can be seen in Figure 7. The random waypoint simulation data and the analytical model are very similar in shape. There are a few slight variations that can be seen in the figure and these can be attributed to the approximation of the area under the spatial density used in the calculation of the analytical model. The random waypoint analytical model proves to be much more accurate than the uniform model in the higher regions of r. The two are very similar in the lower regions of r.

Figures 8(a) and 8(b) show the RWP model and uniform model error as compared to the simulated data. The random waypoint model has a confidence level of greater than 90% for $\frac{L}{2} < r$. The error is greater below $\frac{L}{2}$ and is attributed to the lower overall connectivity in these ranges. A slight deviation in the percent of nodes in range for a low value already equates to an unfairly large percentage of error. Simulations themselves can and do vary by a small amount due to the randomness of their behavior. Regardless of the unfair percentage of error analysis, this model is a very reasonable estimate of the random walk mobility model's effective coverage.

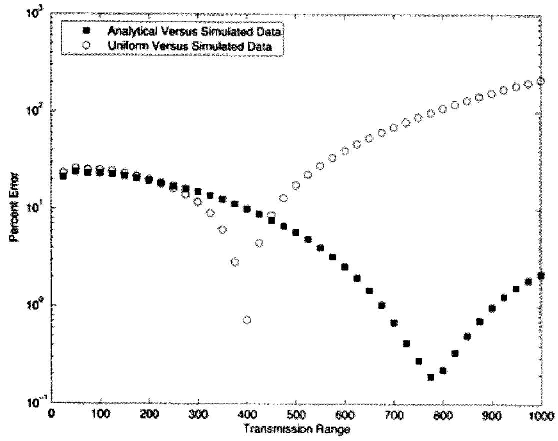
7 Conclusion

In this paper we have shown that the location and transmission range of a mobile node can have a considerable effect on expected node connectivity and density. We presented a mathematical model to predict the behavior of node coverage. The model was extended to the random walk and random waypoint mobility models and examined for accuracy in each. It was shown that as the transmission range, r, of a node is increased, the effective mean coverage also increases, but not in a uniform manner. Extensive simulations were conducted to verify these mathematical findings and to demonstrate the importance of the boundary effect.

Future work for effective coverage includes adaptations to additional mobility models, including group mobility models. In addition to adaptations, further refining the spatial node density of the random waypoint model could reduce the variations between the simulated and analytical results. Likewise, comparisons between the presented analytical models, simulated data and real world data could provide extremely useful to the research community.



(a) Error of Variable Speed



(b) Error of Constant Speed

Fig. 8. Random Waypoint Error

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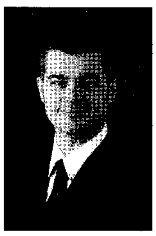

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