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주문량 조정이 가능한 커스텀 부품 공급계약의 가치분석

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Analysis of Quantity Adjustment Flexibility in Custom Component Supply Contracts

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완제품 생산을 위하여 특별한 사양의 커스텀 부품이 필요한 경우 부품 공급자의 공급능력은 완제품 생산자의 생산능력을 제한하는 요소로 작용하여 궁극적으로 완제품 고객 서비스 수준에 큰 영향을 미친다. 완제품 수요가 불확실한 경우 수요 변동에 따른 부품 공급량 증감을 허용하는 커스텀 부품 공급자의 유연성은 완제품 생산자에게 목표 고객 서비스 수준을 만족하기 위한 부품 조달 비용을 절감할 수 있도록 하는 등의 이익을 가져온다. 본 논문에서는 이러한 커스텀 부품 공급 계약의 유연성이 갖는 가치를 완제품 생산자 측면에서 분석하는 방안을 제시한다.

Keywords : Supplier Flexibility, Quantity Adjustment, Service Level

1. Introduction

In highly-competitive industries such as the automobile and consumer electronics industries, assemblers introduce new product models every few months. With short product life cycles, it is risky for an assembler to keep excessive inventory of finished-products or components for specific products. Quick response of suppliers is emphasized more than ever to make sure that the components will be available when needed and as much as needed. In today's decentralized environment, a substantial proportion of components are purchased from external suppliers while some components are manufactured at the assembler site. As component procurement constitutes a large part of the assembler's business processes, the assembler seeks to improve efficiency and save costs in component supply.

There are mainly two types of components : custom and standard (Ulrich and Eppinger [17]). Custom components are designed and produced for specific products, while standard components are used for diverse products by different assemblers. In many cases, assemblers are involved in the early stage of developing custom components and sometimes it is an assembler who provides the design and specification of components. Engine parts, body panels and seats are examples of custom components in the automobile assembly. On the other hand, there exist established specifications for standard components. Multiple suppliers share the market of a standard component and the price is driven by competition between the suppliers. Examples of standard components for car assembly include tires, batteries, belts and so on (Dyer et al. [3]).

Supply of custom and standard components may have dif-

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ferent impacts on the finished-product assembly management. In order to maintain smooth assembly operations, an assembler needs to prepare for uncertainty in component supply. The common methods for protecting assembly operations from component shortage include utilizing multiple supply sources and keeping safety stocks of standard components. However, unlike standard component supply, it may not be possible nor cost effective to procure custom component from multiple sources. In order to keep a steady supply of custom components, it is essential to maintain a close relationship with the supplier. Thus, component supply management requires different considerations, depending on component types.

Another reason for the necessity of distinguishing procurement management of custom and standard components is the distinct relationship between assemblers and component suppliers. Since assemblers can hardly procure custom components from other sources, flexibility is one of the most important factors when an assembler signs supply contracts with a custom component supplier. In the production of a custom component with several configurable options, a part of the customizing process may take place only after customer demand is realized. To serve the customer demand in a competitive manner, the assembler must have a business partnership with a highly flexible supplier who can deliver unusually large orders, even on short notice.

While the flexibility of the custom component supplier has a substantial influence on the performance of assembly operations, it is not straightforward for the assembler to assess how much to pay for a certain level of supplier flexibility. The cost associated with a certain level of flexibility is closely related to the supplier's operation properties, which are in most cases not known to the assembler. On the other hand, the assembler can control the influence of standard component supply on assembly operations by keeping safety stocks or utilizing multiple sources. Usually the assembler has the information about how much it costs to keep standard component inventory or to procure from other suppliers. From the assembler's viewpoint, it is easy to analyze the cost of managing standard component supply. This observation provides a basic idea of our approach to suggest guidelines for the assembler's negotiation with the custom component supplier. We suggest a framework that evaluates the influence of the custom component supplier flexibility by the cost of standard component supply, which can be more easily assessed.

In our research, we focus on a particular supply flexibility model that is commonly used in the industry. Assume projected orders are placed one period ahead of time (in the amount of $\widetilde{O}_t \geq 0$). By the time of delivery, the assembler may adjust the actual order O_t to other quantity as long as it is between $\widetilde{O}_t - l_t$ and $\widetilde{O}_t + u_t$ ($l_t \geq 0, u_t \geq 0$). Usually, higher flexibility (larger l_t and/or u_t) imposes more cost to the supplier. Thus, the unit price of custom components may vary according to the flexibility level that the supplier provides. It is an important question for some industries to find a nominal unit price for custom component corresponding to a particular flexibility contract. The additional unit price that the assembler is willing to pay for an improved flexibility is referred to as the value of flexibility in our study.

Several researchers have considered supply chain models where the buyer is allowed to change order quantities. Moinzadeh and Nahmias [9] consider an inventory model in which the buyer has the option of increasing the delivery quantity at additional cost just prior to the periodic delivery of a fixed quantity. They suggest an approximate policy to reduce cost. Eppen and Iyer [4] consider "backup agreements" in which the buyer has options of purchasing a certain quantity at no additional cost but pays penalty for any quantity not purchased. Barnes-Schuster et al. [1] examine a supply contract with options in a two-period model. In addition to committed orders, the buyer may purchase and exercise options of increasing orders. They evaluate the option exercise price numerically and study the channel coordination. Huang et al. [6] also consider a two-period contract with volume flexibility. The buyer may adjust the initial order both upward and downward at given costs. They find the buyer's optimal policy in some special cases and derive the contract exercise cost. Tsay [15] consider a "quantity flexibility" contract and studies the coordination of a manufacturer-retailer channel in a similar setting to the newsvendor problem. Sethi et al. [12] consider a contract in which the buyer has an option to increase his order at cost of paying a higher price and discuss the buyer's optimal decision to maximize his profit. Kim et al. [7] study a contract which allows the buyer to increase or cancel the order within a certain proportion of the initial order and suggest a procedure to find a final order quantity which minimize the buyer's total cost. Wu [18] analyzes a contract in which the retailer updates demand information using Bayesian procedure before making the final purchase and shows the impact of the minimum purchase commitment on the profit of the retailer and the manufacturer. While most researches regarding quantity flexibility contracts focus on a single-purchase problem, there are several studies which consider a multi-period problem where the buyer orders from the manufacturer in a repeated manner. Bassok and Anupindi [2] study "rolling horizon flexibility" contracts and propose several heuristics for the buyer's ordering policy in order to reduce cost. Tsay and Lovejoy [16] analyze a multi-stage supply chain with quantity flexibility contracts. They examine where to position flexibility through approximate analysis and extensive numerical experiments.

We consider a two-stage multi-period supply chain model in which the buyer is allowed to adjust order quantities. The buyer places advance orders to the supplier with forecast data on future customer demands. Due to forecast errors, the buyer may need more or fewer components to meet actual customer demands. If the supplier provides no flexibility, the buyer cannot increase or decrease the order quantities and may end up with high costs. With a more flexible supplier, the buyer can adjust order quantities close to customer demands and reduce costs. Apparently, the supplier flexibility has a large impact on the buyer's costs. However, it is difficult to analyze the flexibility value directly through buyer's cost changes in multi-period supply chain models. Bassok and Anupindi [2] and Tsay and Lovejoy [16] address the value of flexibility issue, but they only provide insights by way of numerical experiments. In this paper, we suggest a new approach to estimate the value of flexibility. Our approach assumes an assemble-to-order system where the assembler does not keep finished-product inventory and there is no capacity constraint on the assembly operation. The finished-product consists of two components, one custom and one standard. To avoid the risk of obsolete cost by keeping no finished-product inventories, the assembler must have well-established partnerships with its suppliers to provide a competitive level of service to its customers. In this paper, we use average backorders of customer demand as the assembler's performance metric. Average backorders can be used to estimate average waiting time until the customer demand is satisfied. It is also used to measure γ-type customer service level, which is defined as average customer backorders divided by average demand (Schneider [10]).

The assembler may want to control the average number of backorders to be lower than a target level, or the customer might specify a maximum allowable backorder level. To achieve the target backorder level, it is critical to have the component supply not to interrupt the assembly operation. When the backorder level is higher than the target and custom component supply causes backorders, the assembler can improve the performance of custom component supply by signing a contract for higher flexibility. The assembler may be able to achieve the same target backorder level by enhancing standard component supply, if a portion of backorders has been caused by standard component shortage. In our framework of estimating the value of flexibility, we assume component shortages in both types of component supply and first identify the trade-offs between custom and standard component supply performance to deliver a target customer service level. Then the value of the custom component supplier flexibility is quantified indirectly through the cost of enhancing standard component supply to achieve the same customer service level. This framework provides the assembler with a guideline for acceptable custom component prices for different levels of flexibility.

The remainder of the paper is organized as follows. In the next section, we describe our supply chain model. In Section 3, we provide the background analysis for the framework of estimating value of flexibility. Section 4 illustrates how the framework is used to guide practice through examples. We end the paper with concluding remarks.

2. The Model

We consider a discrete-time model of a system assembling two components (one custom and one standard) into a finished-product. The assembler receives customer demand D_t for finished-products at the beginning of period t. One unit of the finished-product consists of one unit of each component. After observing new customer demand, the assembler places orders for both types of components. Although the assembler tries to order custom component as much as necessary, sometimes it is inevitable to carry custom component inventory due to demand forecast error.

When the assembler agrees on quantity adjustment flexibility, its order quantity O_t of custom component should satisfy the following constraint;

$$\widetilde{O}_t - l_t \le O_t \le \widetilde{O}_t + u_t \tag{1}$$

where $\widetilde{O}_t$ denotes a projected order quantity given to the supplier in the previous period. The downward and upward adjustment parameters measure the flexibility level that the custom component supplier provides.

Unlike the custom component, there is no boundary constraints such as (1) on the order quantities on the standard component; the standard component order is assumed to be as much as customer demand. If the supplier cannot satisfy a standard component order immediately, the unsatisfied standard component orders are backlogged and delivered later. The procurement of standard component is independent of that of custom component in the sense that the customer component supplier's flexibility does not affect the standard component oder quantities.

After component delivery, the assembler assembles as many products as possible to satisfy the current customer demand D_t and backorders B_t. We assume that there is no assembly capacity limit as long as the components are available. At the end of the period the assembler delivers the customer order (unsatisfied customer demand is backordered) and informs the custom component supplier of the projected order quantity $\widetilde{O_{t+1}}$ for the next period, which is decided based on demand forecast $\widetilde{D_{t+1}} \geq 0$.

3. Upper Bounds on Customer Backorders

Identifying an upper bound on the average customer backorders is the first step of our framework of estimating the supplier flexibility. Using the upper bound, we obtain the combination of custom and standard component supply performances that guarantee a target backorder level. In this section, we provide general procedures to get an upper bounds on customer backorders.

The following notation is used throughout the paper.

- D_t : customer demand quantity of finished- product for period t,
- $\widetilde{D}_t$: customer demand forecast for period t,
- ε_t : customer demand forecast error (i.e. $\varepsilon_t := D_t - \widetilde{D}_t$),
- B_t : backorders of finished-product at the beginning of period t, $B_1 = 0$,
- $\widetilde{O}_t$: projected order quantity for period t given to the custom component supplier in the previous period,
- O_t : actual order quantity of custom component for period t,
- I_t^c : inventory level of custom component at the beginning of period t,
- I_t^s : inventory level of standard component at the beginning of period t,
- Q_t^s : backorders of standard component at the beginning of period t, $Q_1^s = 0$.

With quantity adjustment flexibility, we find that the following inequality holds between customer backorders and component supply performance metrics. The proof can be found in <Appendix>.

Theorem 1 : If the assembler places a projected order from the custom component supplier every period using the expected component requirements in the next period;

$$\widetilde{O}_t = \widetilde{D}_t + B_t - I_t^c,$$

then

$$\overline{B} \leq \overline{\lim_{T\to\infty}} \frac{1}{T} \sum_{t=1}^{T} (\varepsilon_t - u_t)^+ + \overline{Q^s} \tag{2}$$

where $(x)^+ = \max\{x, 0\}$,

$$\overline{B} := \overline{\lim_{T\to\infty}} \frac{1}{T} \sum_{t=1}^{T} B_t,$$

and

$$\overline{Q^s} := \overline{\lim_{T\to\infty}} \frac{1}{T} \sum_{t=1}^{T} Q_t^s.$$

Theorem 1 gives an upper bound on the average customer backorders $\overline{B}$ under quantity adjustment flexibility. The first term of the upper bound in (2) includes the upward flexibility parameter and demand forecast error. Note that Theorem 1 does not require any assumptions on demand process. When demand forecast errors follow an identical distribution and $u_t = u$ for all t, inequality (2) in Theorem 1 reduces to

$$\overline{B} \leq U_B := \mathrm{E}\left[(\varepsilon_1 - u)^+\right] + \overline{Q^s} \tag{3}$$

From the proof in <Appendix>, one can easily find that the equality of (2) holds only when

$$(\varepsilon_t - u_t)^+ \geq 0 \text{ if } Q_t = 0 \text{ and}$$
$$(\varepsilon_t - u_t)^+ = 0 \text{ if } Q_t > 0.$$

Thus, the average amount of customer backorders is equal to the sum of the shortages of both type components if at most one type of component is in shortage every period.

We conduct numerical experiments to see the effectiveness of the upper bound U_B as u changes (l_t is set to u for all t). We assume the demand follows a moving average process of lag 1; $D_t = \lambda + \varepsilon_t - \theta\varepsilon_{t-1}$. The average demand λ is set to 20 and the i.i.d. forecast error is assumed to have a normal distribution with zero mean and variance σ_ε^2 . As for the standard component supply, we assume that the supplier replenishes the inventory using a revised base-stock policy. The base-stock level is set to 15 and the maximum number of components delivered in one period has a normal distribution with mean 25 and variance 35. We provide detailed description of a revised base-stock policy in Section 4.1.

With the moving average demand process, large σ_ε^2 and θ imply a high demand variance ($\mathrm{Var}[D_1] = (1+\theta^2)\sigma_\varepsilon^2$). In <Table 1>, we obtain the average customer backorders $\overline{B}$ with two demand variance levels through simulation. In both cases the experiment results show that U_B is close to $\overline{B}$ particularly when the backorder level is low. As expected, increasing the quantity adjustment parameter u reduces the average customer backorders. The assembler can offer a better service (less backorders) to the customer when the supplier provides higher flexibility (larger u). However, the supplier flexibility has little marginal value when it is above a certain threshold. In <Table 1>, for example, $\overline{B}$ remains almost the same for $u \geq 15$ with higher and $u \geq 10$ with lower demand variance.

<Table 1> Backorder Levels with Quantity Adjustment Flexibility

	High Variance*			Low Variance**		
u	U_B	$\overline{B}$	β_a (%)	U_B	$\overline{B}$	β_a (%)
0	3.77	3.41	16.09	1.36	1.33	6.63
1	3.31	2.99	14.02	0.92	0.90	4.48
2	2.91	2.64	12.27	0.60	0.59	2.92
3	2.57	2.35	10.81	0.39	0.38	1.87
5	2.07	1.93	8.68	0.17	0.17	0.08
7	1.75	1.67	7.39	0.11	0.11	0.53
10	1.52	1.48	6.48	0.09	0.09	0.46
15	1.42	1.41	6.12	0.09	0.09	0.46
20	1.41	1.41	6.09	0.09	0.09	0.46

Note) * $\sigma_\varepsilon^2 = 35, \theta = 0.5$, ** $\sigma_\varepsilon^2 = 10, \theta = 0.2$.

The result of Theorem 1 is also useful when the customer service level is measured by other metricssuch as the unfill rate which is defined as the fraction of unsatisfied demand;

$$\beta_a := \left(\overline{\lim_{T\to\infty}} \sum_{t=1}^{T} \min\{B_{t+1}, D_t\} / T \right) / \mathrm{E}[D_1].$$

Since B_{t+1} denotes customer demand not satisfied until the beginning of period t, the unsatisfied demand out of D_t can be expressed as $\min\{B_{t+1}, D_t\}$. Thus, β_a represents the fraction of customer demand which is not met immediately on average. From (3), it can be easily shown that $U_B / \mathrm{E}[D_1]$ is an upper bound on β_a. <Table 1> shows that the actual unfill rate β_a obtained by simulation is well approximated by $U_B / \mathrm{E}[D_1]$ when the variance of demand process is small.

4. Estimating the Value of Flexibility

We now describe how to estimate the value of custom component supplier flexibility using two examples. In each example, the assembler adopts a different method to improve the standard component supply : keeping safety stocks or procuring from secondary sources.

4.1 A Case with Safety Stocks of Standard Components

In this case, the assembler keeps inventory to cope with the interruption by standard component shortage. Safety stocks for unusually high demand or low component supply allow the assembler to maintain a desirable level of component shortage.

In this subsection, we use a base-stock policy to control standard component inventory. With this type of policy the assembler first checks the current inventory level and customer demand at the beginning of each period and then places an order enough to bring the inventory level up to a predetermined base-stock level. This type of inventory policy is simple to implement and used widely in practice. Also the base-stock policy has been studied in a number of researches, and it is known to minimize the long-run average inventory-backorder cost in certain situations.

Average inventory level is one of the important performance characteristics of an inventory policy. Base-stock policy adjusts the inventory level using base-stock level. Higher base-stock levels increase average inventory level and in turn lower average component shortage. It may incur high inventory holding cost to maintain a low shortage level. If the acceptable level of standard component shortage is relatively high, the assembler can use a lower base-stock level and it saves holding cost for standard component. As shown in Section 3, high flexibility of custom component supplier makes it acceptable to keep a relatively high level of standard component shortage. Thus, when the assembler uses a base-stock policy for standard component supply, the value of custom component supplier flexibility can be estimated through savings in holding cost.

Since holding cost is directly related to inventory level, we first need the distribution of inventory level to compute average holding cost. We consider a standard component inventory system where the maximum number of component the supplier can deliver in a period is limited. The initial inventory level is equal to the base-stock level s and in each period a new order is placed which is of the same size as the customer demand. Unsatisfied order is backlogged and delivered later. In this type of inventory systems, the gap between the base-stock level and inventory level $(s-I_t^s)$ represents the outstanding order, which is ordered but not delivered yet. When the maximum number of component the supplier can deliver in a period is limited, the gap can be viewed as work-in-process inventory in a discrete-time production-inventory system. In heavy traffic the work-in-process inventory has an exponential distribution with parameter $\nu := 2(\mu-\lambda)/(\sigma_D^2+\sigma_V^2)$ when the supplier's capacity is i.i.d. with mean μ and variance σ_V^2 and customer demand is also i.i.d. with mean λ and variance σ_D^2. Thus, the steady-state work-in-process inventory $W_\infty := s-\lim_{t\to\infty} I_t^s$ can be estimated by an exponential random variable.

Toktay and Wein [14] combine the heavy traffic result with a corrected diffusion approximation by Glasserman and Liu [5] to take advantage of both approaches. Specifically, they suggest the following approximation of work-in-process inventory distribution;

$$\Pr\{W_\infty = 0\} = 1-e^{-\nu\beta} \text{ and}$$
$$\Pr\{W_\infty > x\} = e^{-\nu(x+\beta)} \text{ for } x \geq 0 \tag{4}$$

where $\beta = 0.583\sqrt{\sigma_D^2+\sigma_V^2}$ is the correction term. Toktay [13] shows that the modified distribution above is more accurate than the heavy traffic approximation. Even though the heavy traffic results regarding work-in-process have limitation to apply to general manufacturing systems with low traffic, we use the distribution function in (4) to illustrate how the framework suggested in this paper can be used when the standard component inventory is managed by base-stock policy.

For the numerical experiments in this section, we assume that customer demand is i.i.d.; $D_t = \lambda+\varepsilon_t$ where $\lambda = 20$, and $\varepsilon_t \sim N(0,25)$. Supplier's capacity is assumed to follow a normal distribution with $\mu = 22$ (10% more than λ) and $\sigma_V^2 = 25$. <Table 2> shows several combinations of custom and standard component supply performance metrics, $\mathrm{E}[(\varepsilon_1-u)^+]$ and $\overline{Q^s}$, which make U_B equal to 2; that is, to keep the customer backorders within 10% of the average demand. In <Table 2>, we first compute $\mathrm{E}[(\varepsilon_1-u)^+]$ for different level of flexibility. Then the allowable average shortage of standard component $\overline{Q^s}$ is calculated using the trade-off results in (3);

$$\overline{Q^s} = U_B - \mathrm{E}[(\varepsilon_1-u)^+]$$

This is a performance requirement of standard component supply for each flexibility level. The requirement on the standard component supply becomes more strict (less $\overline{Q^s}$) when the custom component supplier provides less flexibility

<Table 2> Component Supply Performance ($U_B = 2$)

				Approximation		Simulation		
u	$\mathrm{E}[(\varepsilon_1-u)^+]$	$\overline{Q^s}$	s	$\overline{I^s}$	Δ_h(%)	$\widetilde{I^s}$	$\widetilde{\Delta_h}$(%)	$\overline{B}$
0	1.992	0.008	87.80	54.16	-	78.97	-	2.00
1	1.532	0.468	36.94	18.04	66.68	28.56	63.83	1.91
2	1.149	0.851	29.47	13.05	9.22	21.47	8.98	1.87
3	0.840	1.160	25.59	10.58	4.57	17.90	4.51	1.87
5	0.414	1.586	21.68	8.19	4.40	14.42	4.41	1.90
7	0.181	1.819	19.97	7.19	1.84	12.94	1.87	1.93
10	0.041	1.959	19.04	6.66	0.97	12.15	1.00	1.97
15	0.002	1.998	18.80	6.53	0.26	11.95	0.26	1.98
20	0.000	2.000	18.78	6.52	0.01	11.94	0.01	1.98

(smaller u) and vice versa. The increment of custom component supplier flexibility can be considered to have as much value as the cost savings from the relaxed performance requirements on standard component procurement.

In the next step, we associate the different levels of $\overline{Q^s}$ with standard component supply cost. Using the approximate distribution of work-in-process inventory in (4), we compute the average component shortage and inventory level as follows;

$$\overline{Q^s} = \mathrm{E}\left[(W_\infty - s)^+\right] = \int_s^\infty (x-s)\nu e^{-\nu(x+\beta)}dx$$
$$= \frac{1}{\nu}e^{-\nu(s+\beta)}$$

From the above equation, we get an expression for s in terms of $\overline{Q^s}$;

$$s = -\log(\nu\overline{Q^s})/\nu - \beta \qquad (5)$$

The base-stock level s for an average shortage $\overline{Q^s}$ can be obtained from the above equation. As expected, base-stock levels can be decreased as the desired level of shortage gets lower. The computed base-stock level, for instance, to make $\overline{Q^s}$ = 0.008 is about 87.80. This is the estimated base-stock level of standard component to maintain the average backorders as low as 10% of demand ($U_B = 2$) when the custom component suppler does not provide any flexibility. If the suppler provides flexibility level of $u=1$ (accepting 5% more order quantities than average demand), then the constraint on standard component shortage can be relaxed to $\overline{Q^s}$ = 0.468 and the assembler can lower the base-stock level down to s = 36.94 .

Lowered base-stock levels bring savings in holding cost of standard component. Assume holding cost is linear to inventory levels, we compute average inventory level $\overline{I^s}$ using the approximate distribution in (4);

$$\overline{I^s} = \mathrm{E}\left[(s - W_\infty)^+\right] = \int_0^s (s-x)\nu e^{-\nu(x+\beta)}dx$$
$$= \left(s - \frac{1}{\nu}\left(1-e^{-\nu s}\right)\right)e^{-\nu\beta}$$

As expected, the assembler carries more inventory (large $\overline{I^s}$) when the base-stock level is high (large s). Using (5), we can also see the impact of component shortage level on the average inventory level;

$$\overline{I^s} = \overline{Q^s} - \frac{1}{\nu}e^{-\nu\beta}\left(\log(\nu\overline{Q^s}) + \nu\beta + 1\right)$$

Since

$$\frac{d\overline{I^s}}{d\overline{Q^s}} = 1 - \frac{1}{\nu}e^{-\nu\beta}\frac{1}{\overline{Q^s}} \qquad (6)$$
$$= 1 - e^{\nu s} \le 0 \qquad (7)$$

the average inventory level decreases as the allowable backorder level increases.

For a given U_B, we call u^0 a *base* flexibility if $u^0 = \max\{u : \mathrm{E}[(\varepsilon_1 - u)^+] = U_B, u \ge 0\}$. With base flexibility, the upper bound on the average customer backorders can be achieved only when there are no backorders of standard component ($\overline{Q^s} = 0$). In the case there is no solution to $\mathrm{E}[(\varepsilon_1 - u)^+] = U_B$, we use 0 as a base flexibility.

The base flexibility in this example is 0. Even if the custom component supplier is not flexible at all the assembler has a way to control average backorders below the target level. However, it imposes the assembler a high holding cost of standard component (we refer to this holding cost as *base* holding cost). In <Table 2>, we use Δ_h to denote marginal holding cost savings in a fraction of base holding cost. Thus, in this section, the marginal value of flexibility Δv^c is given by

$$\Delta v^c = \Delta_h \times \text{ base holding cost}$$

In <Table 2>, when the custom component supplier provides flexibility as much as $u=1$ the approximated inventory level drops down to 18.04, a 66.68% reduction in average inventory level (Δ_h = (54.16-18.04)/54.16 ≈ 66.68%). If holding cost is linear in inventory level, such flexibility of the custom component supplier has the same value as 66.68% of the base holding cost. When the flexibility is improved further up to the level of $u=2$, the holding cost is reduced by 9.22%(≈ (18.04-13.05)/54.16) more. As expected from (6), the marginal cost saving diminishes as flexibility level increases. For instance, compared with $u=15$, flexibility level of $u=20$ saves only additional 0.01% of base holding cost.

The estimation of supplier flexibility in this section is based on the approximate inventory level proposed by Toktay and Wein [14]. Admittedly, the computed inventory level may be different from the observed value from actual operations. We conduct a simulation experiment to see how much the difference is. Using SIMAN simulation language, we imitate the operations of players in the supply chain, customer demanding final product, assembler ordering custom and standard components, and suppliers delivering the orders. The last three columns in <Table 2> show the simulation results. As expected, the average backorder level is no more than the upper bound ($\widetilde{B} \leq 2$). The average inventory level ($\widetilde{I^s}$) is higher than the approximated one ($\overline{I^s}$). For example, with base flexibility (i.e. $u=0$), the approximation underestimates average inventory level by 24.81 (=78.97-54.16). Apparently, this gap is significantly large and it might be inappropriate to estimate the holding cost using the approximate inventory level distribution. However, the percentage of inventory level reduction does not show much difference between the approximated (Δ_h) and observed ($\widetilde{\Delta_h}$) ones. This phenomenon may be explained by (7). With an approximate distribution of W_∞ in (4), the derivative of average inventory level with respect to $\overline{Q^s}$ depends on base-stock level s. Regardless of the correction term, the change of $\overline{I^s}$ according to $\overline{Q^s}$ would remain close to the derivative in (7) as long as W_∞ is well approximated by an exponential random variable with parameter ν. The simulation experiments show that the inventory level, i.e. holding cost, would decrease by 63.83% when the custom component supplier provides flexibility level of $u=1$. The saving is estimated to be 66.68% and the estimation error is about 4% ($=|\widetilde{\Delta_h}-\Delta_h|/\widetilde{\Delta_h}$). In most instances in <Table 2>, the estimation error is less than 5%. Thus, our framework seems to estimate effectively the value of flexibility in case the assembler uses a revised base-stock policy.

4.2 A Case with Secondary Sources of Standard Components

In the previous subsection, we illustrate how our framework can be used to estimate the value of flexibility when the assembler keeps inventory of standard component. The same framework can be applied to other cases where the standard component is procured by different methods.

While there is usually a single supplier for custom component, there may exist multiple suppliers for standard component. It is common that the assembler has secondary sources such as spot markets for standard component. Since it is an occasional transaction, procuring from other sources would cost more than the price charged by the contracted supplier. When the contracted supplier fails to fulfil orders, the assembler may obtain the same or substitutable standard component from the secondary sources. Thus, using the trade-offs explained in the previous section, the value of flexibility can be estimated through purchasing cost from the secondary sources, which is saved by improved flexibility.

In this subsection, we assume that the standard component is mainly supplied from a primary source. But the primary supply channel is not 100% reliable (due to transportation mode failure, for instance). We denote the reliability of the primary supply channel by $0 \leq p \leq 1$. Thus, there is a probability $1-p$ of no component delivery that is independent and identical in every period. We assume that the assembler may wait until the next period when the primary supplier fails to deliver an order, or it may cancel the order and procure the component from the secondary source. We use p' to denote the probability that the assembler purchases standard component from the secondary source in such cases. If the primary channel is unavailable in two periods in a row, the assembler is assumed to place an order from the secondary source in order to avoid a large backlog of customer orders. We further assume that whenever it orders from the primary or secondary source, the assembler orders as much as the sum of current demand and backorders. If the supply source is available whether it is primary or secondary, the order of standard components is delivered immediately in full.

<Table 3> shows numerical experiment results with a secondary source for standard component. For the numerical experiments in this subsection, we assume that demand follows a moving average process; $D_t = \lambda + \varepsilon_t - \theta\varepsilon_{t-1}$ where $\lambda = 20$, $\theta = 0.2$, and $\varepsilon \sim N(0, 25)$. The target level of average backorders is set to 5% of average demand (i.e. $U_B = 1$).

As in the previous subsection, we first compute $\mathrm{E}[(\varepsilon_1 - u)^+]$ for different flexibility parameter u. The base flexibility for the symmetric flexibility example in <Table 3> is $u^0 = 2.5$ (i.e. $\mathrm{E}[(\varepsilon_1 - 2.5)^+] = U_B = 1$). For $u < u^0$, $\mathrm{E}[(\varepsilon_1 - u)^+]$ is greater than the target backorder level even if there is no standard component shortage. For this reason, we only consider $u \geq u^0$ for the remaining of this subsection.

Once $\mathrm{E}[(\varepsilon_1 - u)^+]$ is computed, then the allowable average shortage of standard component $\overline{Q^s}$ is calculated using the trade-off results in Theorem 1. In the next step, we asso-

<Table 3> Component Supply Performance ($U_B = 1$)

u	$E[(\varepsilon_1 - u)^+]$	$\overline{Q^s}$	p'(%)	f'(%)	q'	v^c	$\overline{B}$
2.5	1.000	0.000	100.00	10.00	2.000	-	0.93
3	0.853	0.147	92.60	9.26	1.867	0.21	0.92
4	0.604	0.396	79.80	8.02	1.643	0.57	0.92
6	0.282	0.718	62.76	6.41	1.353	1.04	0.93
10	0.042	0.958	49.69	5.21	1.137	1.38	0.97
14	0.003	0.997	47.53	5.01	1.102	1.44	0.99
20	0.000	1.000	46.37	5.00	1.100	1.45	0.99

ciate the different levels of $\overline{Q^s}$ with standard component supply cost. When the availability of the primary supply channel is independent of the customer demand process, it can be easily shown that average standard component shortage $\overline{Q^s}$ is equal to

$$\overline{Q^s} = \frac{(1-p)(1-p')}{p+(1-p)(2-p')} \cdot \lambda \qquad (8)$$

If the primary supplier is always reliable ($p=1$), there would be no standard component shortage ($\overline{Q^s}=0$). If the assembler always resorts on the secondary source immediately when the primary supplier fails ($p'=1$), the standard component shortage still remains to be 0.

As expected, if the primary supply channel is always reliable (i.e. $p=1$), then there would be no standard component shortage at all (i.e. $\overline{Q^s}=0$). Since $\partial \overline{Q^s}/\partial p < 0$ for $0 \le p < 1$, the component shortage increases as the chance of primary source failure increases. For $p<1$, (8) is equivalent to the following equation

$$p' = 1 - \overline{Q^s}/((1-p)(\lambda - \overline{Q^s}))$$

which shows that p' is a decreasing function of $\overline{Q^s}$. Recall that p' has the meaning of the frequency to rely on the secondary sources when the primary source is not available. When the allowable standard component shortage ($\overline{Q^s}$) is small the assembler should procure from the secondary source more often, and it would cost the assembler more.

In a similar way as for (8), one can derive expressions for the frequency (f') and average quantity (q') of standard component procurements from the secondary source;

$$f' = 1 - p - \overline{Q^s}/\lambda \quad \text{and} \qquad (9)$$

$$q' = (1-p)\lambda - p\overline{Q^s}$$

Note that q' is averaged over all periods including those periods when the assembler does not order from the secondary sources. If the primary supplier never fails to deliver ($p=1$), then no standard component shortage occurs ($\overline{Q^s}=0$) and both f' and q' would be 0. When $p<1$, f' and q' are decreasing functions of $\overline{Q^s}$. This implies that the frequency and quantity of secondary source procurements reduce when more standard component shortage is allowed. Also (9) shows that f' and q' is linear in $\overline{Q^s}$.

<Table 3> lists p', f' and q' for $p=0.9$. With the base flexibility ($u=2.5$), in order to prevent standard component shortage ($\overline{Q^s}=0$) the assembler should order from the secondary supplier whenever the primary supply channel fails ($p'=100\%$). The frequent reliance on the secondary supply channel ($f'=10\%$) increases the average quantity of standard component from the secondary source ($q'=2.0$). The procurement from the secondary source, which usually incurs additional costs such as higher unit price, can be reduced if the custom component supplier is more flexible. <Table 3> shows how much q' can be decreased by contracting a custom component supplier who provides higher flexibility.

When the flexibility parameter u is increased to 4, the amount of standard component from the relatively high-cost supply source can be reduced by 0.133 (= 2.0-1.867) per period. Besides, the order frequency from the secondary source is diminished by 0.74% (= 10.0%-9.26%). Additional increment of flexibility to $u=6$ brings up a more reduction in q' (0.224 = 1.867-1.643). However, improving flexibility above a certain level does not have a significant effect any more. For instance, improvement of flexibility from $u=14$ to 20 reduces q' only by 0.002 (= 1.102-1.100).

To measure the cost increment due to relying on the secondary source, we denote the ordering cost and additional unit price of the secondary supply channel by K and w, respectively. The average additional cost per period equals

$$Kf' + wq' = (1-p)(K+w\lambda) - (K/\lambda + \wp)\overline{Q^s} \qquad (10)$$

Since f' and q' have a linear relationship with the component shortage $\overline{Q^s}$, so does the secondary source procurement cost in (10). With $u^0 > 0$, $\overline{Q^s}$ becomes zero by the definition of base flexibilities and the procurement cost of (10) gets equal to $(1-p)(K+w\lambda)$. For a flexibility higher than the base level (i.e. for $u > u^0$), standard component shortage can grow and the assembler still can deliver the target customer service level. The standard component procurement cost can be reduced by

$$(1-p)(K+w\lambda)-[(1-p)(K+w\lambda)-(K/\lambda+\wp)\overline{Q^s}] \\ =(K/\lambda+\wp)\overline{Q^s} \qquad (11)$$

The cost savings are attributed to the improved flexibility of the custom component supplier. The assembler may be willing to pay the custom component supplier an extra amount as long as it is no more than the cost reduction in (11). Thus, the value of flexibility defined earlier as additional unit price the assembler is willing to pay for flexibility improvement can be estimated by

$$v^c=\frac{(K/\lambda+\wp)\overline{Q^s}}{\lambda} \qquad (12)$$

<Table 3> lists the estimated flexibility values v^c when it costs $400 to place an order (i.e. $K=400$) and additional $10 per unit (i.e. $w=10$) to buy standard component from the secondary source. To the custom component supplier who is offering $u=3$ flexibility level (0.5 more than the base level), for instance, the assembler may be willing to pay $0.21 more than the unit price set for the base flexibility. The improved flexibility saves the assembler almost the same amount of cost by making more standard component shortage acceptable. As observed in the previous subsection, supplier flexibility has higher marginal values in the low range. In <Table 3>, flexibility improvement from $u=10$ to 14 level induces a less increment of v^c than the initial improvement from the base flexibility to $u=4$ flexibility level).

5. Concluding Remarks

In this paper, we identify the trade-offs between custom and standard component supply channels in assemble-to-order systems. Using the trade-offs, we have suggested a framework to estimate of the value of custom component supplier flexibility. We illustrate that our framework can be applied to estimate effectively the value of flexibility. Our analysis provides the assembler with a robust way to create price guidelines for negotiating flexibility with the custom component supplier.

Note that the framework suggested in this paper has no assumption on how the assembler procures the standard component. As long as the assembler can assess how much it costs to keep the average standard component backorders under a certain level, the assembler can estimate the value of the custom component supplier flexibility by following a procedure illustrated in this paper.

We consider quantity adjustment flexibility which are represented by upward and downward adjustment parameters. In Section 3, we find that the downward flexibility has little influence on backorders since we get an upper bound on the average backorders of customer demand without using the downward adjustment parameter. Thus, the value of flexibility estimated by the framework in this paper is not so much the value of downward flexibility as the value of upward flexibility. While they do not affect customer service level considerably, downward adjustment parameters may have influence on the assembly system in different ways. In our model, the parameters $\{l_t\}$ relates to the committed but not claimed orders and may be associated with a certain operation cost of the assembler. This opens up a possibility for new approaches to evaluate the supplier flexibility. It may involve an optimizing problem of minimizing costs as in Tsay and Lovejoy [16]. In addition, many researchers have studied optimal order policies with demand forecast updates (Lovejoy [8], Sethi and Sorger [11], Sethi et al. [12], and Huang et al. [6]). They studied what are the optimal order quantities and when is the optimal time to commit them. Regarding supplier flexibility, another future research area is to determine projected orders that the assembler takes advantage of available information about future demand.

We develop the framework of evaluating custom component supplier's flexibility for an assembly system producing a final assembly from one custom and one standard component. When the final assembly is composed of more than one standard component, our framework still can be applied with a proper modification of the definition Q_t^s. However, in the later part of the framework, one needs to assess the effect of increased allowance in standard component supply due to the improved custom component supplier's flexibility, and incorporating multiple standard components make this assessment much complicated. More research work is required to extend the application of the framework suggested in this paper to multi-component assembly systems.

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<Appendix> Proof of Theorem 1

We first prove that the following equation holds

$$R_t^s - D_t = B_t - (\varepsilon_t - u_t)^+ \tag{12}$$

for all t, where R_t^s denotes the number of standard components required in period t.

The number of standard component required is computed at the beginning of each period based on customer demand including any backorders and the number of available custom component

$$R_t^s = \min\{D_t + B_t, I_t^c + O_t\}. \tag{13}$$

With the assumption that the assembler orders as much as it needs in each period, the order quantity of custom component in period t has the following expression.

$$O_t = \max\{\min\{D_t + B_t - I_t^c, \widetilde{O}_t + u_t\}, (\widetilde{O}_t - l_t)^+\}. \tag{14}$$

The equation (14) for the actual order amount of custom component involves the projected order information given to the supplier one period ahead. According to the assumption on the assembler's ordering policy, the projected order quantity is as much as what the assembler expects to need in the next period, i.e. $\widetilde{O}_t = \widetilde{D}_t + B_t - I_t^c$. Note that, when it gives $\widetilde{O}_t$ to the supplier in period $t-1$, the assembler has the necessary information B_t and I_t^c because the projected order is provided after all the procurement activities in period $t-1$. After replacing $\widetilde{D}_t$ with $D_t - \varepsilon_t$, we get an expression for O_t as follows. From the flexibility constraint (1),

$$O_t = \begin{cases} D_t + B_t - I_t^c - (\varepsilon_t - u_t)^+ & \text{if } \varepsilon_t \geq 0 \\ D_t + B_t - I_t^c + (\varepsilon_t + l_t)^- & \text{if } \varepsilon_t < 0. \end{cases}$$

Thus, (13) reduces to

$$R_t^s = D_t + B_t - (\varepsilon_t - u_t)^+$$

and (12) holds always.

The next step of the proof is to show the following inequality;

$$\sum_{t=1}^{T}(R_t^s - D_t) \leq \sum_{t=1}^{T} Q_t^s. \tag{15}$$

If (15) holds, then with (12) it finishes the proof of (2). Since the assembly quantity P_t is equal to $\min\{R_t^s, I_t^s\}$, the expression for backorders reduces to

$$\begin{aligned} B_{T+1} &= \sum_{t=1}^{T}(D_t - P_t) \\ &= \sum_{t=1}^{T}(D_t - R_t^s + (R_t^s - I_t^s)^+) \geq 0, \end{aligned}$$

and we have

$$\sum_{t=1}^{T}(R_t^s - D_t) \leq \sum_{t=1}^{T}(R_t^s - I_t^s)^+. \tag{16}$$

The inventory level is updated according to the following equation;

$$I_t^s = \sum_{k=1}^{t} D_k - Q_t^s - \sum_{k=1}^{t-1} P_k. \tag{17}$$

The amount of replenishment up to period t is equal to $\sum_{k=1}^{t} D_k - Q_t^s$ because each period the assembler places a new order of standard component as much as customer demand. The amount of retrieval from the inventory equals the total production of finished-products, $\sum_{k=1}^{t-1} P_k$. Since $B_t = \sum_{k=1}^{t-1}(D_k - BP_k)$, the equation (17) reduces to

$$I_t^s = D_t + B_t - Q_t^s. \tag{18}$$

From (16) and (18), we get (11);

$$\sum_{t=1}^{T}(R_t^s - D_t) \leq \sum_{t=1}^{T}(R_t^s - D_t - B_t + Q_t^s)^+ \leq \sum_{t=1}^{T} Q_t^s.$$