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# 주파수 선택성 채널에서 불완전한 채널상태정보를 갖는 MIMO 검파 알고리즘의 성능비교

(A Performance Comparison of MIMO Detection Algorithms in Frequency Selective Fading Channel with Imperfect Channel State Information)

임 진\*, 윤 석 현\*

(Jin Ren and Seokhyun Yoon)

요 익

신호의 검파는 무선통신시스템에서 매우 중요한 문제이며 최근에는 다중입력다중출력(MIMO) 통신을 위한 검파 알고리즘에 대한 연구가 많이 진행되었다. 그러나 이러한 연구의 많은 부분이 주파수 비선택적 채널을 가정하였거나 주파수 선택적 채널을 가정하더라도 수신단에서의 채널상태정보는 완전하다는 가정 하에 연구가 수행되었다. 따라서, 본 연구에서는 주파수 선택적 채널에서 불완전한 채널정보를 사용할 때 몇 가지 MIMO 검파 알고리즘에 대해 얻을 수 있는 오류율 성능을 비교해 보고 자 한다.

#### **Abstract**

Signal detection is a key technique in wireless communication system. Recently, several detection algorithms have been developed for multiple-input multiple-output (MIMO) wireless communication systems. However, most research in this area had assumed a flat-fading channel environment and all these techniques are based on the assumption that the channel state information (CSI) at the receiver side is perfect. But in practical situation, the available CSI may be imperfect because of channel estimation errors and/or outdated training. In this paper, we will compare the performance of several detection algorithms in MIMO frequency selective fading channel environment with imperfect CSI.

Keywords: MIMO, detection, Frequency selective fading, CSI

### I. Introduction

Recent advances in information theory suggest that multiple-input multiple-out (MIMO) wireless communication systems can provide potentially very-high-rate data transmission capabilities<sup>[1~2]</sup>. To exploit such potential, a number of space-time

communication schemes have been proposed. including the Bell labs layed space-time (BLAST) system<sup>[1~3]</sup>, the various space-time coding (STC) techniques<sup>[4]</sup>, and the transmitter precoding schemes [5]. The vertical BLAST (V-BLAST) architecture is an example of narrowband MIMO systems now under implementation. In V-BLAST, every transmit antenna radiates an equal-rate, independently encoded stream of data $^{[1\sim2]}$ ; and the receiver employs maximum likelihood/maximum posteriori (ML/MAP) decoders<sup>[6~7]</sup>, linear decoders<sup>[8~9]</sup> and successive interference cancellation (SIC) decoder<sup>[10]</sup>. From the practical point of view, the complexity,

<sup>\*</sup> 학생회원, \*\* 정회원, 단국대학교 전자전기공학부 (Dept. of Electronics and Electrical Engineering, Dankook University)

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decoding latency, and error performance have to consider, so the signal detection technique is improved continuously up to now.

rate wireless transmissions experience frequency-selective propagation effects. Albeit challenging to mitigate, once acquired, these frequency-selective offer fading channels multipath-diversity gains. MIMO system promise to enhance the overall system performance and increase spectral efficiency far beyond single-input single-output Shannon limit. In order to enjoy this huge increase in capacity, we need to develop efficient and reliable receivers for MIMO systems. This design can be challenging, especially in frequency selective channels where the transmitted signal has to be detected in the presence of noise, intersymbol interference (ISI) and multiuser interference (MUI).

MIMO system is a promising technology with multiple antennas at both the transmitter and the receiver, and its capacity increases linearly with the minimum between the numbers of transmit and receive antennas. However, the huge capacity potential is based on the assumption that the channel state information (CSI) is known perfectly to the receiver, which is impossible in practice. Actually, the CSI must be first estimated before the demodulation and decoding, therefore an accurate estimation of CSI is essential to MIMO systems.

The objective of this paper is to compare the performance of MIMO detection algorithms in the presence of Frequency selective fading channel. Channel knowledge is not assumed to be available at the receiver. The estimation of frequency selective MIMO channels is based on the transmission of the pilots.

The paper is organized as follow. Next section briefly describes the channel model and channel estimation scheme. Several MIMO detection algorithms are introduced in Section III. Section IV presents the numerical results. Conclusions are given in section V.

## II. System Description

### A. Channel model

We consider a frequency selective MIMO-OFDM wireless system with  $N_T$  transmit and  $N_R$  receive antennas, and  $N_{\rm sc}$  subcarriers which employs the Four-quadrature amplitude modulation (4QAM). The Information bits is transmitted into Channel Encoder module, and then input to Bit Interleaver module which adopt Random Interleaver, we can see Fig.1. The transmitted MIMO-OFDM symbols are arranged in the vector

$$\mathbf{s}(k) = \left[\mathbf{s}_1^t(k), \dots, \mathbf{s}_{N_T}^t(k)\right]^T \tag{1}$$

where  $s_j^t(k)$  is the transmitted signal by the j-th transmit antenna on subcarrier k in t-th time interval and  $(\bullet)^T$  denotes the transpose operation. Superscript t for time index is omitted in the sequel when it is clear from context.

Between every transmit antenna m and every receive antenna n there is a complex single-input single-output (SISO) time channel impulse response  $h_{n,m}(l)$  of length l, described by the vector

$$h_{n,m} = [h_{n,m}(0), \cdots, h_{n,m}(L-1)]^T$$
 (2)

The channel is normalized by the condition

$$\sum_{k=0}^{L} E\{|h_{n,m}(l)|^2\} := 1 \ \forall n,m$$
 (3)

The frequency selective SISO channel is defined by

$$H_{n,m} = [H_{n,m}(0), \cdots, H_{n,m}(N_{sc} - 1)]^T$$
 (4)

where  $H_{n,m}$  is the  $h_{n,m}$  channel vector after DFT with elements. $H_{n,m}(k)$  is the DFT of  $h_{n,m}(l)$ .

Assuming the same channel order for all channels, the frequency selective MIMO channel can be described by  $N_{sc}$  complex channel matrices

$$H(k) = \begin{bmatrix} H_{1,1}(k) & \dots & H_{1,N_T}(k) \\ \vdots & \ddots & \vdots \\ H_{N_R,1}(k) & \dots & H_{N_R,N_T}(k) \end{bmatrix}, k = 0, \dots N_{sc} - 1$$
 (5)

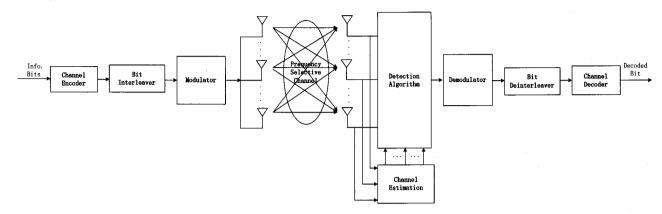


그림 1. 시스템 모델 Fig. 1. System model

of the dimension  $N_R \times N_T$ .

The symbols received by the  $N_R$  antennas are arranged in a vector

$$y(k) = [y_1(k), \dots, y_{N_n}(k)]^T$$
(6)

where  $y_r(k)$  is the received signal by the r-th receive antenna on subcarrier k. y(k) can be expressed with (1), (5) and n(k) as noise vector of length  $N_R$  as

$$y(k) = H(k)s(k) + n(k), k = 0, \dots, N_{sc-1}$$
(7)

We assume additive white Gaussion noise (AWGN) with zero mean and variance  $\sigma_n^2$  per receive antenna, i.e. the covariance matrix of the noise vector is given by

$$R_{nn} = E\{n(k)n^{H}(k)\} = \sigma_n^2 I_{N_R}$$
(8)

where I is the identity matrix and  $(\bullet)^H$  denotes the complex-conjugate (Hermitian) transpose.

## B. Channel estimation

In this paper, the channel estimation scheme exploits the pilots in order to jointly estimate the components of CSI between each transmit and receive antenna. In order to do this, pilot signal with length  $N_p \cdot N_{sc}$  OFDM symbols for one antenna is needed. The pilots design is shown in Fig.2. The received signal at subcarrier k for all the receive antennas during the training period can be written as:

$$\widetilde{Y}_{p}(k) = H(k)P(k) + \widetilde{N}(k), k = 0,...,N_{sc} - 1$$
 (9)

where

$$\widetilde{Y}_{P}(K) = \begin{bmatrix} \widetilde{y_{1,1}}(k) & \cdots & \widetilde{y_{1,N_p}}(k) \\ \vdots & \ddots & \vdots \\ \widetilde{y_{N_R}}_1(k) & \cdots & \widetilde{y_{N_R}}_{N_p}(k) \end{bmatrix}$$

is a  $N_R \times N_p$  matrix representing the received signal at the  $N_R$  antennas, subcarrier k during the training period,

$$P(k) = \begin{bmatrix} P_{1,1}(k) & \dots & P_{1,N_P}(k) \\ \vdots & \ddots & \vdots \\ P_{N_T1}(k) & \dots & P_{N_TN_p}(k) \end{bmatrix}$$

is a  $N_T \times N_P$  matrix representing the pilot signals at subcarrier k, and

$$\widetilde{N}(k) = \begin{bmatrix} \widetilde{N_{1,1}}(k) & \cdots & \widetilde{N_{1,N_P}}(k) \\ \vdots & \ddots & \vdots \\ \widetilde{N_{N_R}}_1(k) & \cdots & \widetilde{N_{N_R}}_{N_P}(k) \end{bmatrix}$$

of size  $N_R \times N_P$  represents the AWGN noise.  $N_P$  is the number of pilot symbol and a integer multiple of  $N_T$ , i.e.  $N_P = q \cdot N_T$ . By increasing  $q \cdot (N_p)$ , we can obtain better channel estimate.

Least Square channel Estimation: The linear Least Square (LS) channel estimate is given by [11]

$$\widehat{\boldsymbol{H}}_{LS}(k) = \widetilde{\boldsymbol{Y}}_{p}(k)\boldsymbol{P}(k)^{\dagger} \tag{10}$$

where  $P(k)^{\dagger} = P(k)^{H}(P(k)P(k)^{H})^{-1}$  is the pseudo

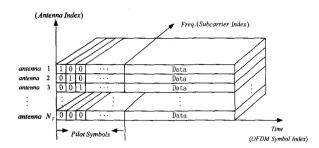


그림 2. 채널추정을 위한 직교 파일럿 신호 Fig. 2. Orthogonal pilots design for channel estimation.

inverse of P(k). P(k) is the transmitted orthogonal pilot matrix of all transmit antennas.  $\widetilde{Y}_p(k)$  is the received pilot signal.  $(\bullet)^H$  and  $(\bullet)^{-1}$  denote the complex-conjugate (Hermitian) transpose and the inverse, respectively.

Minimum Mean Square Error Channel Estimation: The minimum Mean Squre Error (MMSE) channel estimate is given by [11]

$$\widehat{H_{MMSE}}(k) = \widetilde{Y}_{p}(k) \left[ P(k)^{H} R_{hh} P(k) + \sigma_{n}^{2} N_{R} I \right]^{-1} P(k)^{H} R_{hh}$$
(11)

where  $R_{hh} = E\{H(k)H(k)^H\}$  for all k is the the correlation matrix of channel vector H. At the receiver, the channel may change from measurement to measurement and the past estimates of H cannot be used to calculate  $R_{hh}$ , so we can know  $E\{H(k)H(k)^H\} = I_{N_R}$  from (3). Now  $\widehat{H_{MMSE}}$  only depends on the known pilot symbols and the noise power  $\sigma_n^2$ .

# III. Detection Algorithms

## A. Linear detector

For Zero forcing (ZF) detector, we assume that the channel matrix H is invertible and estimate the transmitted data symbol vector as [12]

$$\hat{\mathbf{s}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H} \mathbf{y} = \mathbf{H}^{\dagger} \mathbf{y} \tag{12}$$

where † represents pseudo inverse.

We examine another linear detection algorithm to the problem of estimating a random vector s on the basis of observation y is to choose a matrix B that minimizes the mean square error

$$\epsilon^2 = E[(s - \hat{s})^T(s - \hat{s})] = [(s - By)^T(s - By)]$$

The solution of the linear Minimum Mean Square Error (MMSE) is given by [12]

$$B = (\frac{1}{SNR} I_{N_R} + H^H H)^{-1} H^H$$
 (13)

where the superscript H denotes the complex-conjugate transpose. And the estimate of the transmitted data is

$$\hat{s} = B \times y \tag{14}$$

#### B. MAP detector

The soft-bit value associated with the mth bit of the modulation symbol transmitted from the ith transmit antenna element is determined by the log-likelihood function defined in [13] as

$$L_{im} = \log \frac{\sum_{\check{s} \in \Omega^{(b_i(m)=1)}} P\{y | \check{s}, \mathbf{H}\}}{\sum_{\check{s} \in \Omega^{(b_i(m)=1)}} P\{y | \check{s}, \mathbf{H}\}}$$

$$\tag{15}$$

where  $\check{s}$  is the transmitted vector, consisting of MQAM subsymbol.  $\Omega$  is the set of all possible dimensional candidate symbol vectors of the  $N_T$  -antenna-based transmitted signal.  $\Omega^{(b_i(m)=x)}$ , x=0 or 1 denotes the subset of the set  $\Omega$  of modulation constellation points, which comprises the bit value b equal 1 or 0 at the mth bit position of ith symbol.

However, the direct calculation of the accumulate *a* posteriori conditional probabilities in the numerator and denominator of Equation (15) may have an excessive complexity in practice. Fortunately, as advocated in [14], the expression in Equation (15) can be closely approximated as follows

$$L_{im} \approx \log \frac{P\{y|s_{im}^{1}, \mathbf{H}\}}{P\{y|s_{im}^{0}, \mathbf{H}\}}$$

$$(16)$$

where we define

$$\widetilde{s_{im}^b} = \arg\max_{\check{s} \in \Omega} P\{y | \check{s}, \mathbf{H}\}, b = 0, 1$$
(17)

$$P\{\check{s}|y,H\} = Aexp\left[-\frac{1}{\sigma_n^2}||y - H\check{s}||^2\right]$$
 (18)

where A is a constant, which is independent of any of the values  $\check{s}^{[14]}$ . As suggested by the nature of Equation (16), the detection process employing the objective function determined by Equations (16) and (17) is usually referred to as the Maximum A Posteriori (MAP) probability detector.

A practical approximate of MAP detector, called Max-Log-MAP can be derived as follows. Substituting Equation (18) into (15) yields

$$L_{im} = \log \frac{\sum_{\check{s} \in \Omega} \exp\left[-\frac{1}{\sigma_n^2} ||y - H\check{s}||^2\right]}{\sum_{\check{s} \in \Omega} \exp\left[-\frac{1}{\sigma_n^2} ||y - H\check{s}||^2\right]}$$
(19)

where  $\Omega^x$ ,  $\mathbf{x}=0$ , 1 is the same expression of  $\Omega^{(b_l(m)=x)}$ ,  $\mathbf{x}=0$  or 1. Note Equation (19) involves two summations over  $2^{bN_T-1}$  exponential functions. The may be closely approximated by a substantially simpler expression, namely by

$$L_{im} \approx \frac{1}{\sigma_n^2} \left[ ||y - H\widetilde{s_{im}}||^2 - ||y - H\widetilde{s_{im}}||^2 \right]$$
 (20)

where we have

$$\widetilde{s_{im}^b} = \arg\min_{\check{s} \in O} ||y - H\check{s}||, b = 0, 1$$
(21)

## IV. Numerical Results

A link level simulation was performed to give an insight into the error performance of MIMO Detection Algorithms in Frequency selective fading channel with perfect or imperfect CSI. Table I gives the simulation parameters used.

Fig. 3 shows the bit error probability curves for MMSE detector (MM), Maximum A Posteriori detector (MAP). The simulation environment is 4×4 antenna configuration, 4QAM constellation, and forward error correction (FEC) code, frequency selective fading with perfect CSI. From the

표 1. 모의실험 파라미터 Table 1. Simulation Parameters.

System Parameter	Parameter Value
antennas	4×4
FEC Type	Convolutional code
Modulation	4QAM
Coding Rate	2/3 1/2 1/3
Number of subcarriers	128
Number of frame	600
Number of pilots	g
Number of data	6000
SNR duration[dB]	-6~12

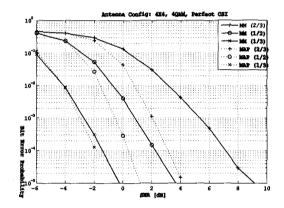


그림 3. MMSE 검출기 및 MAP 검출기의 BER 성능비교; 4x4 안테나 구성, 4QAM 변조, 완벽한 CSI 가정

Fig. 3. A comparison of BER performances: MMSE detector (MM), Maximum A Posteriori detector (MAP), respectively; 4×4 antenna configuration, frequency selective fading with perfect CSI, 4QAM.

simulation result, we can see the detection algorithms performance in 1/3 rate is better than others and MAP detection performance is better than others in different coding rate.

The BER curves in Fig.4 for the different detection algorithms in frequency selective fading with imperfect CSI. The channel estimation method is Least Square algorithm (LS). During -6 dB to -2 dB in SNR duration, the BER curves of the detection algorithm overlap approximately. After SNR is -2 dB, MAP detection performance is best in the different coding rate. The BER of MMSE detection with 1/3 coding rate is worse than the BER of MAP

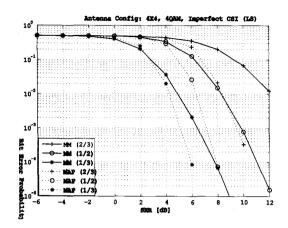


그림 4. MMSE 검출기 및 MAP 검출기의 BER 성능비교; 4x4 안테나 구성, 4QAM 변조, 최소자승(LS) 채널추정

Fig. 4. A comparison of BER performances: MMSE detector (MM), Maximum A Posteriori detector (MAP), respectively; 4×4 antenna configuration, frequency selective fading with imperfect CSI (LS), 4QAM.

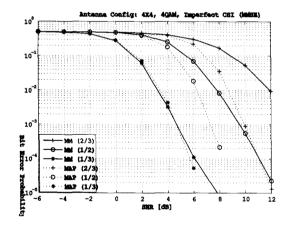


그림 5. MMSE 검출기 및 MAP 검출기의 BER 성능비교; 4x4 안테나 구성, 4QAM 변조, 최소평균자승오차 (MMSE) 채널추정

Fig. 5. A comparison of BER performances: MMSE detector (MM), Maximum A Posteriori detector (MAP), respectively; 4×4 antenna configuration, frequency selective fading with imperfect CSI (MMSE), 4QAM

detection in the same case by 1 dB at a BER of  $10^{-3}$ . Moreover the performance in imperfect CSI is not better than in perfect CSI. At a BER of  $10^{-3}$ , the gap between the MAP detection with imperfect CSI and perfect CSI in Fig.3 is 8 dB approximately.

Fig. 5 compares the performance of MMSE and MAP detector with different coding rate in frequency selective fading with imperfect CSI. Minimum Mean Square Error (MMSE) method is used for channel

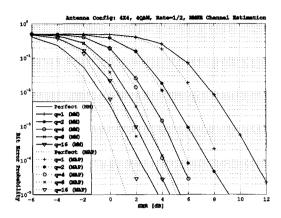


그림 6. 파일럿 심볼의 갯수(q)에 따른 MMSE 검출기 및 MAP 검출기의 BER 성능비교; 4x4 안테나 구성, 4QAM 변조, 1/2 부호화율, 최소평균자승오차 (MMSE) 채널추정

Fig. 6. A comparison of BER performances: Perfect CSI (Perfect) and Different parameter q of pilots of MMSE detector (MM), Perfect CSI (Perfect) and Different parameter q of pilots of Maximum A Posteriori detector (MAP), respectively;  $4\times4$  antenna configuration, 4QAM, Rate=1/2, MMSE Channel Estimation

estimation. The result represents the performance of detection algorithms with MMSE channel estimation is better than LS channel estimation in Fig.4. The loss of the MAP detection with LS channel estimation and 1/3 coding rate in Fig.4 compared to the MAP detection with MMSE channel estimation and 1/3 coding rate in Fig.5 is about 1 dB at a BER of 10<sup>-2</sup>. The SNR range of -6dB to -2dB, the performance of the detection algorithms is kept in a high Error probability above 10<sup>-1</sup>, so it's not easy to differentiate. From -2 dB to 12 dB in SNR range, we can see the MMSE detection performance is worse than others and the MAP detection have the best performance.

Our simulation parameters are  $4\times4$  antenna configuration, 4QAM constellation, rate equals 1/2 and MMSE Channel Estimation. The BER performances of Perfect CSI (Perfect) and Different parameter q of pilots of MMSE detector (MM), Perfect CSI (Perfect) and Different parameter q of pilots of Maximum A Posteriori detector (MAP) are shown in Fig.6. We can see the BER performace of Perfect CSI is better than Different parameter q of

pilots for MMSE or MAP detector and the BER performance of MMSE detector is worse than MAP detector. Moreover the BER performance is better by increasing the parameter q of pilots. When the parameter q value is larger, the number of pilots will increase and it can obtain better channel estimate, so the BER performance is better surely.

#### V. Conclusion

In this paper, we have presented the performance comparison of several MIMO detection algorithms in frequency selective fading channel, especially with perfect or imperfect channel estimation at the receiver. We introduced the several MIMO detection algorithms like ZF, MMSE and MAP, the channel estimation method like LS, MMSE, and the structure of pilots we adopted. Through computer simulation, we compared the bit error rate with a convolutional code of rate 2/3, 1/2 and 1/3 respectively and also compared the performance with different number of pilots under LS or MMSE channel estimation method. From our comparison, we can know the BER performance of MAP detection algorithm is better than others under every code rate by the computer simulation. The bit error rate gap gets less between perfect and imperfect channel state information by increasing the number of pilots. Furthermore, we will more MIMO detection algorithms in frequency selective fading channel with impact channel state information to find a detection algorithm has more advantages and will research in the performance of the detection in Multiuser further.

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## --- 저 자 소 개 -



Ren Jin(학생회원) 2001년 BS in Electronic and Information eng., South-Central Univ. for Nationality.

2005년 전북대 전자정보공학 석사 2008년 현재 단국대학교 전자전기 공학과 박사과정.

<주관심분야 : 채널추정, 시공간부호, MIMO, OFDM, 스마트안테나>



윤 석 현(정회원) 1992년 성균관대학교 전자공학과 학사. 1999년 성균과대학교

1999년 성균관대학교 전자공학과 석사.

2003년 New Jersey Inst. of Tech. Electrical & Computer Eng. 박사.

1999년 한국전자통신연구소 방송기술부 선임연구원 2003년~2005년 삼성전자 정보통신총괄

책임연구원

2008년 현재 단국대학교 전자전기공학부 조교수 <주관심분야 : 무선통신, MIMO, OFDM>