한국도로학회 논문집
第10券4號 2008年 12月
pp. 213 ~ 224

Appropriate Boundary Conditions for Three Dimensional Finite Element Implicit Dynamic Analysis of Flexible Pavement

연성포장의 3차원 유한요소해석을 위한 최적 경계조건 분석

유 평 준* Imad L. Al-Qadi** 김 연 복***
Yoo, Pyeong Jun Kim, Yeon Bok

요 지

트럭 축하중에 의한 도로포장체의 응력과 변형은 대부분 다층 탄성 이론에 의해 예측된다. 대부분의 다층 탄성 이론에 의한 이론적 계산값이 연성 포장 재료의 점탄성적 거동특성, 동적 트럭 축하중, 비균등 타이어 접지압 및 형상등을 해석에 고려하지 못하므로, 계측값에 비해 매우 작은 값을 예측하므로서 도로 포장 두께설계가 과소 설계될 우려가 크다. 이와 같은 도로 포장체 구조해석시 이용되는 중요한 변동요소를 포장 재료의 물성 모델 측면, 비균등 접지압 및 형상 측면, 동적 유한요소해석 측면에서 분석하여 이용 가능한 모델을 본 논문에서 제안하였다. 경계조건 및 민감도 분석을 수행을 통한 효과적인 3차원 연성포장의 유한요소해석모델을 결정하는 방법론을 제안하였으며, 최적 유한요소모델 분석결과와 현장에서 취득한 결과와의 상호비교를 통하여 모델의 유의성을 검증하였으며, 동적 접지하중조건, 점탄성 물성 모델 등을 3차원 유한요소 모델에 접목하고, 최적 경계조건을 결정하였다.

핵심용어 : 유한요소법, 연성포장, 경계조건

Abstract

Flexible pavement responses to vehicular loading, such as critical stresses and strains, in each pavement layer, could be predicted by the multilayered elastic analysis. However, multilayered elastic theory suffers from major drawbacks including spatial dimension of a numerical model, material properties considered in the analysis, boundary conditions, and ill-presentation of tire-pavement contact shape and stresses. To overcome these shortcomings, three-dimensional finite element (3D FE) models are developed and numerical analyses are conducted to calculate pavement responses to moving load in this study.This paper introduces a methodology for an effective 3D FE to simulate flexible pavement structure. It also discusses the mesh development and boundary condition analysis. Sensitivity analyses of flexible pavement response to loading are conducted. The infinite boundary conditions and time-dependent history of calculated pavement responses are considered in the analysis. This study found that the outcome of 3D FE implicit dynamic analysis of flexible pavement that utilizes appropriate boundary conditions, continuous moving load, viscoelastic hot-mix asphalt model is comparable to field measurements.

Keywords: finite element, flexible pavement, implicit dynamic analysis, boundary condition

* 정회원 · 한국건설기술원 선임연구원
** 비회원 · University of Illinois at Urbana-Champaign 교수
*** 정회원 · 한국건설기술원 연구위원

1. INTRODUCTION

Pavement researchers have widely used traditional layered linear elastic theory to predict flexible pavement responses to various loading conditions. Using this method, pavement responses to assumed static circular loading area and uniform load distribution are calculated. However, such assumptions result in erroneous pavement responses (Al-Qadi et al., 2005).

In order to overcome the aforementioned drawbacks in the layered linear elastic theory, finite element method has recently been widely utilized in pavement engineering. The finite element method provides versatility and flexibility in analysis to accurately simulate many engineering problems. It can successfully consider various factors in the analysis process such as nonuniform contact stress distribution, linear or nonlinear viscoelastic material behavior, moving wheel loading, interlayer conditions, and infinite foundation. If these factors are successfully incorporated into a numerical model, a developed FE model may then be expanded to other scenarios that were not tested in the field. Hence, numerical results can be used to define pavement damage potential(ABAQUS, 2005; Bathe, 1996). In this study, a three dimensional finite element(3D FE) model was developed and validated with field measurements. The model investigated stresses and strains at each interface layer of a pavement subjected to moving wheel loads. A convergence study was conducted to define the proper FE boundary conditions and to attain stable solutions. The following analytical factors were considered in the 3D FE model:

1) A spatial dimension of the 3D FE model was selected in order to reduce boundary condition errors and keep the size of the FE mesh within acceptable limits of pavement dimension. The 3D FE mesh was designed to give an optimal accuracy with small elements under the loading area and relatively larger elements far from the loading area.
2) Four lateral boundaries and one at the bottom of the flexible pavement were selected after conducting sensitivity analyses between a reference model and an infinite boundary model.
3) All pavement interface layers are assumed as contact layers subjected to a proper interface friction model. The interface condition was based upon the parametric study conducted by Yoo, 2007. The study utilized layer interface models with various Coulomb friction coefficients.
4) The commercial software ABAQUS was selected for FE modeling of pavement structure. This allows for 3D analyses under moving wheel loading, various types of material properties including elastic characteristics for unbound layers, linear viscoelastic material properties for hot-mix asphalt (HMA) layers (Yoo, 2007; ABAQUS, 2005).

2. STRUCTURAL PAVEMENT ANALYSIS

Critical pavement responses to loading are often computed using numerical analysis. In the analysis, several assumptions, such as static loading condition, circular loading area, uniform load distribution, and elastic material properties are inevitably introduced. On the other hand, finite element numerical analyses allow considering more advanced conditions such as three dimensional nonuniform contact stresses, frictional interfaces, and HMA linear viscoelastity to have more accurate pavement responses.

The critical pavement responses to loading, which can be used to predict pavement damage, are often used to calculate the allowable number of wheel load repetitions. Pavement failure mechanisms such as fatigue cracking, primary rutting within HMA, subgrade rutting, and surface initiated top-down cracking are strongly related to critical responses at surface, inside or interlayer of a pavement system. Hence, accurate pavement responses determined using advanced numerical pavement analysis is important for quantifying pavement damage.

2.1 Conventional Analyses

Closed-form solutions for a homogeneous mass have been developed for calculating stresses, strains, and deflections. Numerical models developed by Burmister (1945) assume the load uniformly distributed over a circular contact area known as uniform pressure model. This method provides a closed-form solution for calculating stresses for a homogeneous, isotropic, and linear elastic semi-infinite space under a point load.

Based on this method, the vertical stress at the centerline of the load was proposed as follows:

$$\sigma_z = \frac{3P}{2\pi z^2} \tag{1}$$

where,

P : point load, and

z : vertical depth of the point of interest.

A similar solution was defined when the point load is changed to a distributed load resulting in the following closed form solution at axis of symmetry (Huang, 1993):

$$\sigma_z = q\left\{1 - \frac{z^3}{[a^2+z^2]^{3/2}}\right\} \tag{2}$$

where,

q : normal stress (uniform pressure applied over a circular area of radius), and

a : flexible plate radius.

In addition, the multi-layered elastic analysis considers n-layer pavement system in cylindrical coordinate. The modulus of elasticity and Poisson ratio are the elastic material properties in that model. This theory satisfy equilibrium conditions by taking stress components as derivatives of the Airy stress function, $\phi(x,y)$. The compatibility condition in terms of the stresses, σ_{xx} and σ_{yy} , for the special case of no-body force is held as the Eq. (3) (Huang , 1993).

$$\nabla^4\phi = \nabla^2(\sigma_{xx}+\sigma_{yy}) = 0 \tag{3}$$

The biharmonic Eq. (3) is a forth-order differential equation, thus, the stresses and displacements formulate with four constants of integration which must be determined from the boundary and continuity conditions. The external load must be a constant load distributed over a circular area in this case. The closed-form solution and the multi-layered elastic analysis have some limitations in considering the time and temperature dependencies of hot-mix asphalt, various external loading condition, and the spatial geometry of a pavement (Huang, 1993).

Prior to executing pavement responses analyses, contact stress distributions at the surface and within the pavement system needs to be investigated, which are tire type dependent. It should be noted that all closed-form solutions and multi-layered elastic analyses up to date are independent of tire-pavement contact stress and area. Major assumptions in conventional analyses are as follows (Huang, 1993; Ullidtz, 1987):

1) Homogeneous, isotropic, and linear elastic layer.
2) Static loading over a uniform circular area.
3) A semi-infinite subgrade layer with a constant modulus.
4) Compatibility of strains and stresses is satisfied at

all layer interfaces.

5) Mass Inertia and damping force effects are ignored.

In addition, these models ignore effects of nonuniform contact stresses and surface tangential forces at tire-to-pavement surface. However, it was concluded that tire-pavement contact configuration significantly affects the contact stress distribution at pavement surface. Several studies have shown that tire contact stresses are nonuniform; but rather have unique shape and distributions depending on tire configurations (Al-Qadi et al., 2005; Bonaquist, 1992; De Beer, 1997).

Regarding the nonuniform contact stresses, which are varied along the width and length of a tire-rib, should be addressed in computing pavement responses. This may not be accomplished by a conventional approach; but through the finite element method.

2.2 Finite Element Analysis

The structural design of flexible pavements assumes that the vertical component of the contact pressure is uniformly distributed over a circular tire imprint area and is normally equal to the tire inflation pressure. However, considerable experimental evidence suggests that this assumption is invalid (Sebaaly, 1992).

Hence, to incorporate various tire-pavement contact shapes in pavement analysis, not only the spatial dimension of a FE model needs to be defined according to vehicular loading characteristics; but also there is a need to select an effective FE analysis method according to pavement responses of interest and computational efficiencies. On the other hand, element type, size, and boundary conditions must be selected carefully through sensitivity analyses, since the accuracy of an FE calculation highly depends on them.

Three types of FE modeling approaches have been often used for the structural analysis of pavement systems: axisymmetric, two-dimensional (2D) plane strain, and three-dimensional (3D) domain. The general advantages and disadvantages of each approach are as follows:

2.2.1 Axisymmetric FE Approach

This formulation reduces a 3D pavement structure to a 2D by assuming constant properties in all horizontal planes using cylindrical coordinates. Hence, the traffic load is applied over a single circular area. This may results in a serious limitation when a dual-tire assembly is considered (Cho et al., 1996).

2.2.2 2-D Plane Strain FE Approach

Although this approach requires less computational time and memory than 3D domain, it could not accurately simulate actual traffic loadings. It assumes that the longitudinal dimension of a pavement system, which is parallel to traffic direction, has no effect on pavement responses to loading. Some field measurements show that longitudinal strain is significant and may not be neglected in pavement damage analysis (Al-Qadi et al., 2004; Priest, 2005).

2.2.3 3D FE Approach

The 3D FE approach can simulate a pavement system more accurately than the aforementioned approaches. It accommodates various conditions in numerical analysis including nonuniform loading and contact shape like Fig 1 in this study, pavement discontinuities, and infinite and stiff foundations. However, this approach requires relatively longer computational time, memory and pre- and post-processing efforts than others. Usually a trade-off between accuracy and computational time is involved. Although a finer mesh provides a more accurate solution, it needs more computation time (ABAQUS, 2005; Desai 1979).

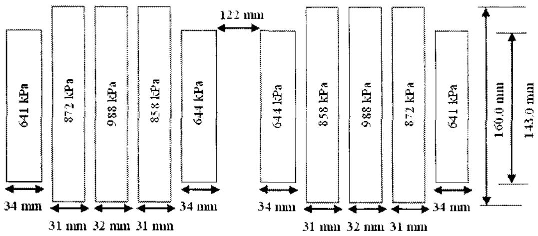


Fig. 1 Nonuniform Maximum Tire Contact Stresses and Contact Shapes: Dual-Tire Assembly (275/80R22.5) at 35.5 kN Wheel Load and 720 kPa Inflation Pressure

2.3 Three Dimensional Finite Element Analyses

2.3.1 Sensitivity Analysis for Vertical Mesh Thickness

The mesh size of a 3D finite element needs to be carefully selected as it directly affects the level of accuracy. When evaluating results of the FE model, two general criteria are recommended to be checked (Bathe, 1996; Desai, 1979):

1) The FE solution has to converge to the continuum model solution. To ensure this criterion, a regular mesh refinement process can be used. The FE solution is then verified against a simplified solution through a layered elastic theory based on general assumptions (e.g., static loading, fully-bonded interface conditions, uniform contact stress, and linear elastic material behavior).
2) A mesh refinement criterion can be held when jumps in stresses across inter-element boundaries become negligible.

In addition, the selection of element thickness is proved to be a more complicated one. At the interface between top layers in a pavement system, the continuity of stresses is highly affected by the selected element thickness. Therefore, a detailed sensitivity analysis needs to be performed. To ensure an acceptable level of accuracy of results and the convergence of the developed 3D FE model, the 3D FE model was refined until specific criteria were satisfied. A sensitivity analysis of the element thickness was conducted to check the jump-stress across inter-element boundaries, while keeping the computation time within reasonable limits.

Within the context of a continuum model, no jumps in vertical stresses should occur at the interface between the layers. Hence, the criterion used in the evaluation of the accuracy of the model is the determination of the vertical stress jump that may occur at the interface. In this analysis, the different element thicknesses ranging from 2.38mm to 38.1mm were considered, Table 1. Table 1 presents the vertical stress jump at the HMA wearing surface (38.1mm)-base layer interface; being the closest to the load, it is the most critical interface. The continuity of stresses at the interface between two layers is highly affected by the selected element thickness. If a minimum 5% accuracy of the applied average contact stress (800kPa) is needed (i.e. 40kPa), it appears that an element thickness of 9.50mm or smaller would provide an acceptable level of accuracy. Another critical factor in the selection of the appropriate element thickness is controlling the computational time and data storage space requirements for the selected 3D element dimension. All FE models were processed on a dual-core personal computer, which is equipped with 8GB memory. A significant increase in computational time and data storage space occurs after refining the mesh size to a thickness less than 9.50mm.

2.3.2 Sensitivity Analysis: Boundary Condition

It is also important to select proper boundary conditions because they highly impact pavement

Table 1. Mesh Size Sensitivity Analysis

Case ID	Element Thickness (mm)	Model Size ([a]DOF)	Number of Elements	Data Storage (Gbytes)	[b]Computation Time (sec)	Jump at [c]WS-[d]BM Interface (kPa)
A	38.10	172,476	42,916	2.35	3145	93.2
B	19.05	212,226	55,656	3.09	4871	52.2
C	9.50	291,726	81,136	4.19	6599	25.6
D	4.76	450,726	132,096	7.26	18565	9.90
E	2.38	760,776	231,468	12.91	30551	6.97

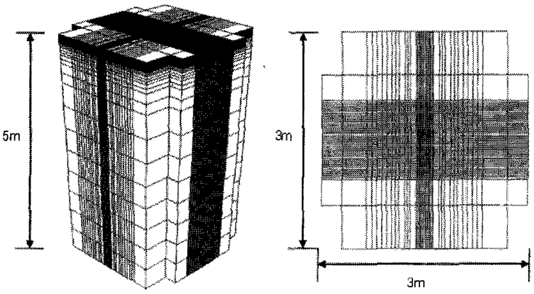
[a]DOF:Degree of Freedom/Number of Equations, [b]Computation time per step(sec)
[c]WS:Wearing Surface of HMA, and [d]BM:Base Mix of HMA

responses. Since a 3D FE model simulates an entire multi-layered pavement system in this study, it was unrealistic to impose any fixed boundary conditions. Instead, infinite boundary conditions were investigated to simulate the far-field region in four vertical sides and a horizontal bottom of the pavement.

To reduce the problem size, infinite boundary elements should be located as close as possible to the area of interest. However, this will induce some errors in calculations. This implies that the infinite boundary, which is coupled with finite elements, should be located sufficiently far from the area of interest. Sensitivity analyses were performed to determine the closest distance where infinite boundary elements can be located (ARA, 2004).

The load is modeled as a static, uniform 800 kPa contact stress having a longest radius "r" of 150 mm. Infinite elements were also used to simulate the four vertical sides and the subgrade's support of the pavement system. The subgrade was divided into four layers: the upper three layers were defined by elastic material properties from the backcalculation analysis of FWD deflection measurements of the Virginia Smart Road, while the fourth layer, which was at the bottom of pavement model, was assumed as a bedrock infinite foundation. All calculations were performed using the implicit dynamic analysis method(Elseifi et al., 2006; Yoo, 2007).

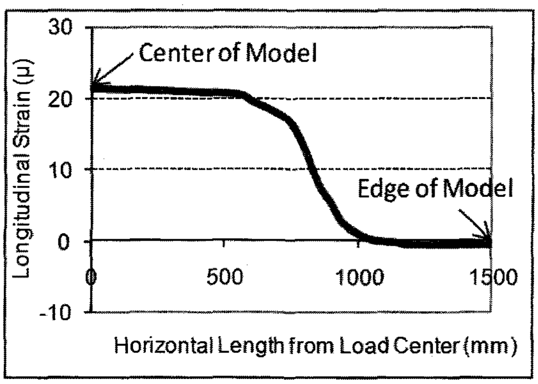
To use a reference model, a large size of pavement, 3m × 3m × 5m, was modeled as shown in Figs 2(a) and (b). Roller boundary conditions were utilized for four sides of this model.



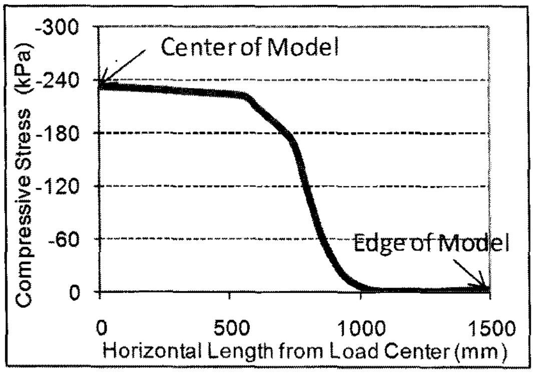
(a) 3D Shape (b) In-Plane Dimension

Fig. 2 Reference Pavement FE Model

The effect of boundary condition on pavement responses for the reference model was first analyzed. Analysis results showed that all pavement responses at the edge of the reference model were negligible, Figs 3 (a) and (b). Hence, pavement responses from this reference model could be used to compare with the critical pavement responses from the reduced model, which has infinite boundary elements.



(a) Longitudinal Strain at the bottom of HMA



(b) Compressive Stress at the top of the Aggregate Base

Fig. 3 Reference Model Responses

To describe the mesh schematic of the infinite boundary element, a symmetric pavement diagram is presented with a half of the 3D FE model in the 2D plane in Fig 4. The infinite boundary elements were located at a distance "Lh" in the horizontal direction and "Lv" in the vertical direction. The relative response comparison between the reference model and the infinite model were made to define the boundaries.

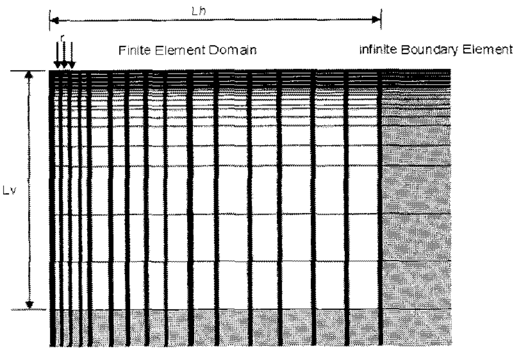
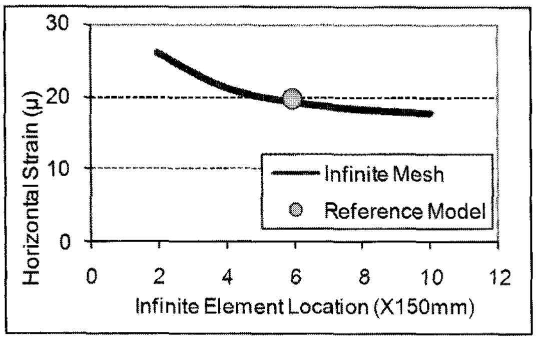
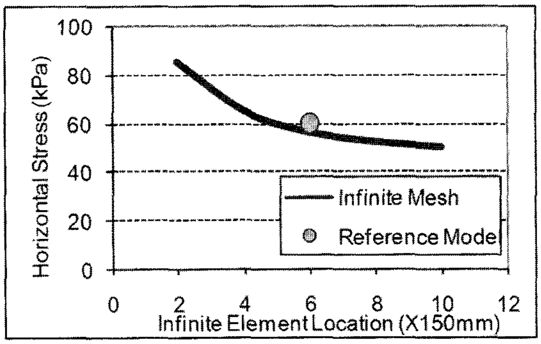


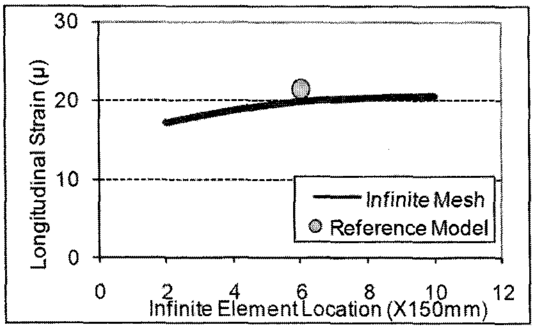
Fig. 4 2D Plane Mesh for Boundary Investigation



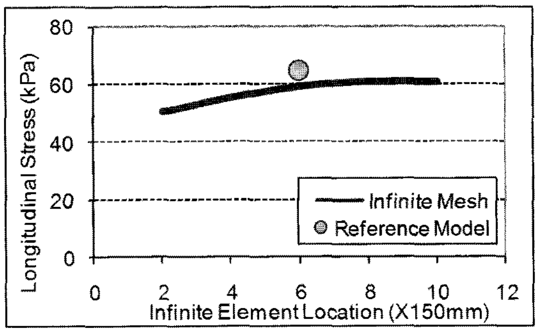
(a) Horizontal Strain at the bottom of HMA



(b) Horizontal Stress at the bottom of HMA



(c) Longitudinal Strain at the bottom of HMA



(d) Longitudinal Stress at the bottom of HMA

Fig. 5 Vertical Infinite Element Boundary

2.3.3 Location of Vertical Side Infinite Boundary

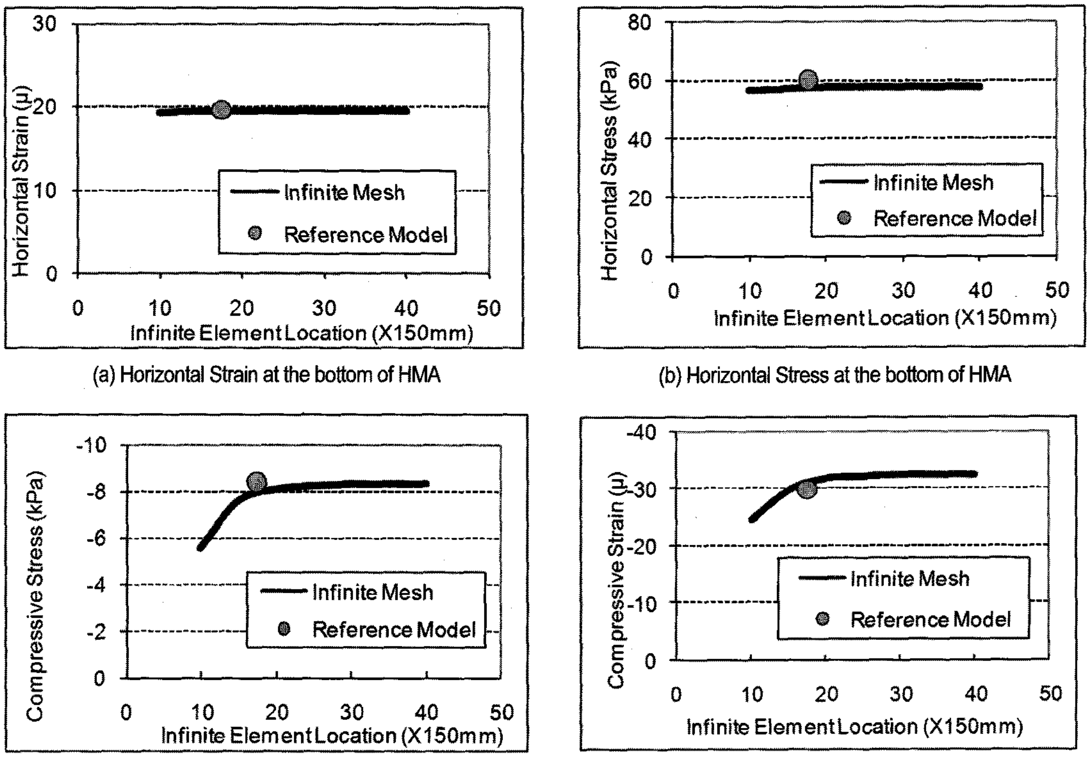
The vertical infinite boundaries were located at a distance "Lh" from the center of the tire load. "Lh" was varied between two and ten times of the maximum length of the loading area (2r ~ 10r), as shown in Figs 5 (a) through (d). The results show consistent convergences to the reference value as "Lh" increases. In all cases, the differences between the reference (a single dot in Figs 5 (a) through (d)) and the infinite model become sufficiently small for "Lh" about six times "r".

2.3.4 Location of Horizontal bottom Infinite Boundary

In addition to the location of four vertical side's infinite elements, the infinite elements for a horizontal bottom of the model were located at a distance "Lv" from the surface of the pavement. "Lv" was varied between ten and forty times of the loading radius (10r ~ 40r), Figs 6(a) through (d).

Pavement responses at the bottom of HMA did not significantly vary as the depth of the pavement increases from 10r to 40r, Figs 6(a) and (b). However, compressive stress and strain at top of subgrade were significantly affected by the depth of infinite element boundary, Figs 6 (c) and (d). The compressive responses showed consistent convergences to the reference value as "Lv" increased. The difference between the reference and infinite models become insignificantly small when "Lv" was about fifteen times the loading radius (15r).

The location of lower infinite boundary element recommended the depth where the maximum compressive stress in the subgrade becomes insignificantly



(a) Horizontal Strain at the bottom of HMA

(b) Horizontal Stress at the bottom of HMA

(c) Compressive Stress at the top of Subgrade

(d) Compressive Strain at the top of Subgrade

Fig. 6 Horizontal Infinite Element Boundary at the bottom small, 1% or less of the applied contact stress. A suggested distance by ARA (2004) for the bottom infinite boundary ranges 76mm to 1200mm (ARA, 2004). The maximum difference of the compressive stress in Fig 6(c) resulted in about 2kPa, which is about 0.25% of applied contact stress of 800 kPa.

In addition, infinite boundary elements can be advantageous in reducing the model size, memory, and computation time, Table 2.

Table 2. Characteristics of Analysis Domain

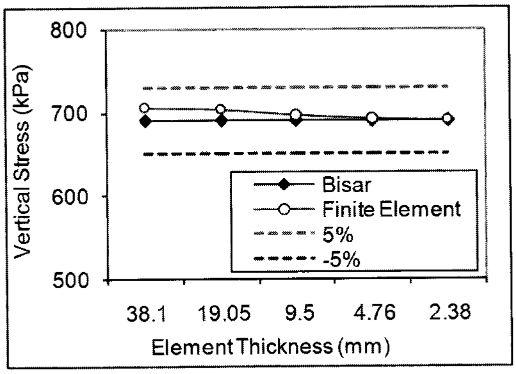
Case ID	Model Size (DOF)	Number of Elements	Memory Used (Gbytes)	Computation Time (sec)
Reference Model	298,152	83,688	2.88	8,728
Infinite Boundary	215,186	59,996	2.09	5,233

2.3.5 Response Analysis: Multi-Layered Elastic vs. 3D FE

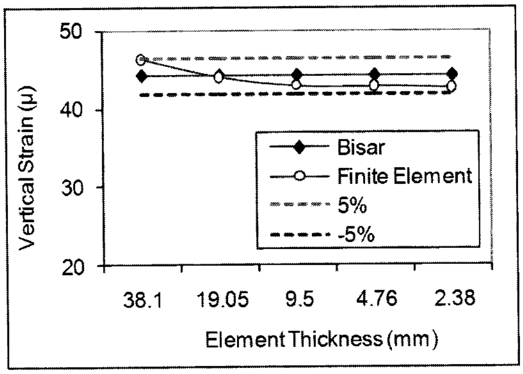
In order to evaluate the accuracy of the FE analysis, a 3D FE model was designed utilizing the same vehicular loading, material conditions, and static analysis for a multi-layered elastic software BISAR-3. BIASR may provide the closest to the closed-form solution. Results presented in Fig 7(a) show the convergence of the vertical stress at the bottom of the HMA wearing surface; while Fig 7(b) presents the convergence of the vertical strain at the bottom of the HMA base layer. The figure presents the mesh refinement between multi-layered elastic analysis and finite element analysis.

Results from static FE models do not appear to exactly converge to the layered elastic solution, which is determined by the BISAR, as the mesh is more refined. Using the same accuracy criterion applied to assess the jump in the vertical stress across layer interfaces, the limit of accuracy shown in Figs 7(a) and (b) indicate that the element thickness of 19.05mm or smaller should provide a level of accuracy of 5% or better, when compared to the BISAR's solution.

The agreement between the 3D FE model and the multilayered theory solution for these cases establishes the adequacy of the geometry and mesh size for the FE model. Hence, it can be determined that an element thickness of 9.5mm for HMA layers would be sufficient for the executable computational time while providing an acceptable stress-jump in the interface.



(a) Vertical Stress at the bottom of the HMA Wearing Surface (38mm)



(b) Vertical Strain at the bottom of the HMA Base Layer (188mm)

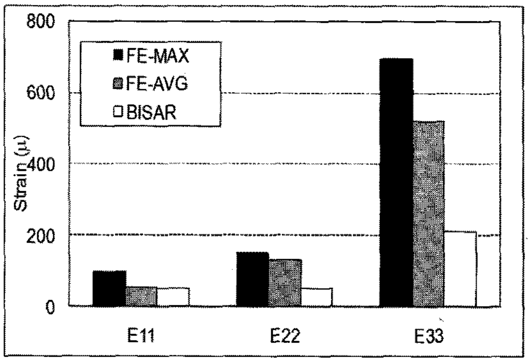
Fig. 7 Sensitivity Analysis of Vertical Mesh Thickness

After establishing the convergence between two analyses methods through elastic analyses, the material properties for each layer are needed to be characterized.

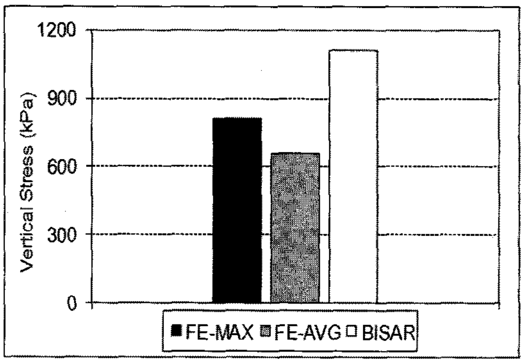
Since HMA only behaves elastically at low temperatures, the elastic theory to describe the HMA's behavior at intermediate and high temperatures often results in an underestimation of pavement responses. Hot-mix asphalt may exhibit close to viscoelastic or viscous behavior at high temperatures or under slow loading speeds. The Prony series, which is equivalent to the generalized Maxwell solid mechanical model, was utilized to predict the linear viscoelastic responses at HMA layers as presented by Al-Qadi et al. 2005 (Al-Qadi et al., 2005; Elseifi et al., 2006). Al-Qadi et al. (2005) have done the creep tests for the Prony series at three temperatures of 5, 25, and 40℃. The linear viscoelastic material properties for HMA at 25℃ were only utilized for 3D FE analysis for the relative comparison with the field measurement, which was shifted to the 25 ℃ in this paper.

Figs 8 and 9 show that there are significant differences between the multilayered elastic analysis and the 3D FE analysis at 25℃. The differences in stresses (Maximum stress of FE model: FE-MAX, Average stress of FE model: FE AVG, and Multilayered elastic analysis: BISAR) at the bottom of wearing surface(Fig 8(b)) and at the bottom of HMA(Fig 9(b)) are relatively smaller than the differences in the strains(Fig 8(a) and Fig 9(a)), because the 3D FE model considers viscoelastic strains rather than the elastic strain only.

For the verification purpose of the developed 3D FE model, Figs 10(a) and (b) presented the measured and

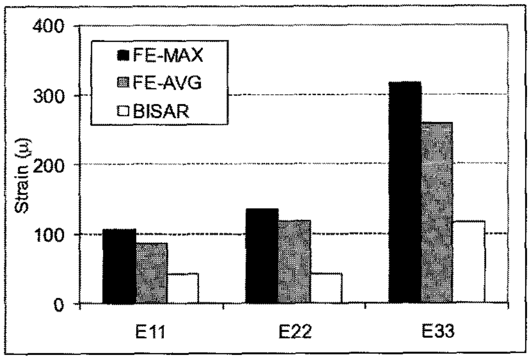


(a) Transverse(E11), Longitudinal(E22), and Compressive(E33) Strains

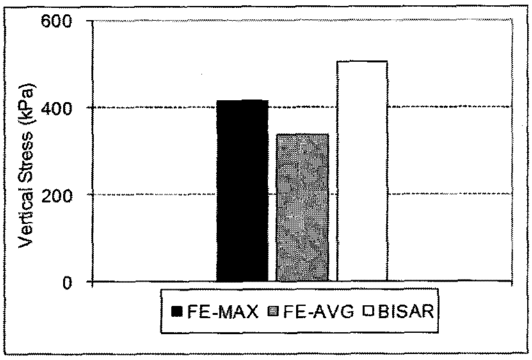


(b) Vertical Stress

Fig. 8 Response Comparison at the bottom of Wearing Surface(38.1mm)



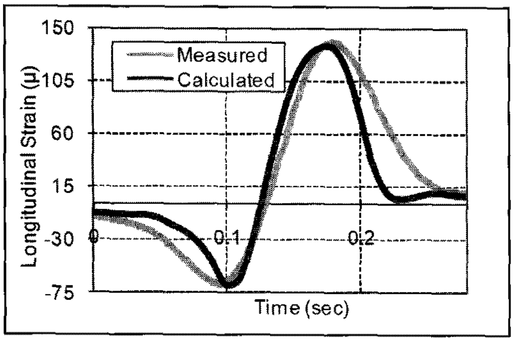
(a) Transverse(E11), Longitudinal(E22), and Compressive(E33) Strains



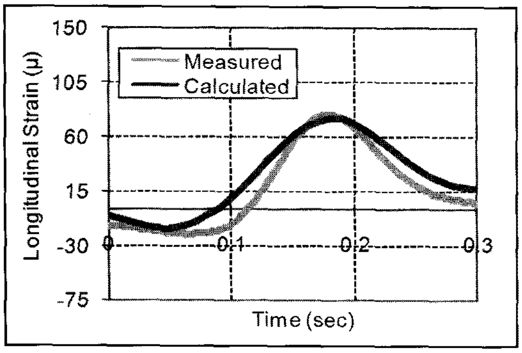
(b) Vertical Stress

Fig. 9 Response Comparison at the bottom of HMA(188mm) calculated longitudinal strains at the bottom of HMA wearing surface and at the bottom of HMA base layer at loading parameters of 35.5 kN of wheel load, 720 kPa of tire inflation pressure like Fig 1, and 8 km/h speed, respectively.

Both the measured and calculated longitudinal strains show typical longitudinal strain response. As the load approaches the measuring point, strains are in compression, and then change the direction into tension as the load is leaving. The model prediction of longitudinal strain is in reasonable agreement with field measurements.



(a) Maximum Strain at the bottom of the HMA Wearing Surface (38.1 mm)



(b) Maximum Strain at the bottom of the HMA Base Layer (188 mm)

Fig. 10 Comparison between Measured and Calculated Longitudinal Strains at 25℃

3. CONCLUSION

To develop an effective 3D FE model for a numerical analysis of flexible pavement, the 3D mesh development and boundary condition analyses are presented. Sensitivity analyses of the 3D mesh size, infinite boundary condition, and time-dependent history of calculated pavement responses were conducted. In order to overcome some drawbacks in the conventional multilayer elastic approaches, the developed 3D FE model successfully considered various factors in the analysis such as nonuniform contact stress distribution, viscoelastic HMA behavior, moving wheel loading, and infinite boundaries and foundation.

The locations of the vertical infinite boundaries were recommended at a distance at least six times of longest loading radius from the center of the tire load (6r). The location of infinite boundary at the bottom of the pavement model was recommended the depth where the maximum compressive stress in the subgrade becomes insignificantly small, 1% or less of the pavement-tire contact stress. A suggested distance for the bottom infinite boundary was about 15 times the loading radius (15r).

Both measured and calculated longitudinal strains showed typical longitudinal strain response. As the load approaches the measuring point, strains are in compression, and then change the direction into tension as the load is leaving. The calculated longitudinal strains were in reasonable agreement with field measurements. Since HMA only behaves elastically at low temperatures, the multi-layered elastic analysis to describe the HMA's behavior at intermediate temperatures (25℃) resulted in significant underestimation of pavement responses.

REFERENCES

ABAQUS/*Standard User's Manual Version 6.5* (2005), Hibbit, Karlsson & Sorenson, Inc., Pawtucket, RI.

ARA, Inc (2004), ERES Division: Appendix RR finite element procedures for flexible pavement analysis, National Cooperative Highway Research Program, *Transportation Research Board*, Washington, D.C.

Al-Qadi, I. L., Loulizi, A., Elseifi, M. A., and Lahouar, S. (2004), "The Virginia Smart Road: The impact of pavement instrumentation on understanding pavement performance," *J. of Association of Asphalt Paving Technologists*, Vol 73, 427-466.

Al-Qadi, I.L., Elseifi, M.A. and Yoo, P.J. (2005), "Characterization of pavement damage due to different tire configurations," *Journal of the Association of Asphalt Paving Technologists*, Vol. 84, pp. 921-962.

Bathe, K.J. (1996), *Finite element procedures, Prentice Hall, Englewood Cliffs*, NJ, 1996.

Bonaquist, R. (1992), "An assessment of the increased damage potential of wide-based single tires, *7th International Conference on Asphalt Pavements*," pp1-15.

Burmister, D. M. (1945), "The general theory of stresses and displacements in layered soil systems," *Journal of Applied Physics*, Vol. 16, 84-94, 126-127, 296-302.

Cho, Y.H., McCullough, B. F., and Weissmann, J. (1996), "Considerations on finite-element method application in pavement structural analysis," *Transportation Research Record 1539, Transportation Research Board*, Washington, D.C., 96-101.

Priest, A.L., Timm, D.H., and Barrett W.E. (2005), "Mechanistic comparison of wide-base single vs. standard dual tire configuration," NCAT Report 05-03, *National Center for Asphalt Technology*, Auburn University, Auburn, Alabama.

De Beer, M. and Fisher, C. (1997), "Contact stresses of pneumatic tires measured with the vehicle-road surface pressure transducer Array (VRSPTA) system for the University of California at Berkeley and Nevada Automotive Test Center," Vol 1/2., *University of California, Berkeley, CA.*

Desai, C. S. (1979), *Elementary finite element method, Prentice Hall*, Inc., Englewood Cliffs, New Jersey.

Elseifi, M. A., Al-Qadi, I. L., and Yoo, P. J. (2006), "Viscoelastic modeling and field validation of flexible pavements," *J. of Eng. Mech., ASCE*, Vol. 132, No. 2, 172-178.

Huang, Y. H. (1993), *Pavement analysis and design, 1st ed.*, Prentice Hall, NJ.

Sebaaly P.E. (1992), "Pavement damage as related to tires, pressures, axle Loads, and configurations," *ASTM Special Technical Publication*, No. 1164, Philadelphia, Pennsylvania, pp 54-58.

Ullidtz, P. (1987), *Pavement Analysis, University of Denmark.*

Yoo, P. J., Al-Qadi, I.L., Elseifi, M.A., and Janajreh, I., (2006), "Flexible pavement responses to different loading amplitudes considering layer interface condition and lateral shear forces," *The Internal Journal of Pavement Engineering*, Vol. 7, March, pp 73-86.

Yoo, P. J. (2007), "Flexible pavement dynamic response analysis and validation for various tire configurations," *PhD Dissertation, University of Illinois at Urbana Champaign*, pp 97-224.

접 수 일 : 2008. 8. 1
심 사 일 : 2008. 8. 1
심사완료일 : 2008. 10. 30