

# Results on Fuzzy Weakly $(r, s)$ -Continuous Mappings on the Intuitionistic Fuzzy Topological Spaces in Šostak's Sense

Won Keun Min

Department of Mathematics, Kangwon National University, Chuncheon, 200-701, Korea

## Abstract

In this paper, we investigate some characterizations for fuzzy weakly  $(r, s)$ -continuous mapping on an intuitionistic fuzzy topological space in Šostak's sense.

**Key words** : fuzzy weakly  $(r, s)$ -continuous, fuzzy  $(r, s)$ -continuous, fuzzy  $(r, s)$ -semiopen, fuzzy  $(r, s)$ -preopen, fuzzy  $(r, s)$ - $\beta$ -open, fuzzy  $(r, s)$ -regular open

## 1. Introduction

The concept of fuzzy set was introduced by Zadeh [13]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3], Chattopadhyay, Hazra and Samanta introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces. Çoker and his colleagues [4,5,6,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. In [5], Çoker and Demirci introduced intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

The concepts of fuzzy  $(r, s)$ -open sets, fuzzy  $(r, s)$ -semiopen sets and fuzzy  $(r, s)$ -preopen sets are introduced in [8,9]. Lee and Kim [10] introduced and studied the concept of fuzzy weakly  $(r, s)$ -continuous mappings. In this paper, we investigate some characterizations for fuzzy weakly  $(r, s)$ -continuous mappings on the intuitionistic fuzzy topological space in Šostak's sense.

## 2. Preliminaries

Let  $I$  be the unit interval  $[0, 1]$  of the real line. A member  $\mu$  of  $I^X$  is called a fuzzy set of  $X$ . For any  $\mu \in I^X$ ,  $\mu^c$  denotes the complement  $1 - \mu$ . By  $\tilde{0}$  and  $\tilde{1}$  we denote constant maps on  $X$  with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let  $X$  be a nonempty set. An intuitionistic fuzzy set  $A$  is an ordered pair

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \} \text{ (simply, } A = (\mu_A, \gamma_A))$$

where the functions  $\mu_A : X \rightarrow I$  and  $\gamma_A : X \rightarrow I$  denote the degree of membership and the degree of nonmembership, respectively, and  $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$  for  $x \in X$ .

An intuitionistic fuzzy point  $x_{(\alpha, \beta)}$  in  $X$  is an intuitionistic fuzzy set

$$x_{(\alpha, \beta)} = (\mu_A, \gamma_A)$$

where the functions  $\mu_A : X \rightarrow I$  and  $\gamma_A : X \rightarrow I$  are defined as follows.

$$(\mu_A(y), \gamma_A(y)) = \begin{cases} (\alpha, \beta), & \text{if } y = x, \\ (0, 1), & \text{if } y \neq x, \end{cases}$$

and  $0 \leq \alpha + \beta \leq 1$ .

An intuitionistic fuzzy point  $x_{(\alpha, \beta)}$  is said to belong to an intuitionistic fuzzy set  $A = (\mu_A, \gamma_A)$  in  $X$ , denoted by  $x_{(\alpha, \beta)} \in A$ , if  $\mu_A(x) \geq \alpha$  and  $\gamma_A(x) \leq \beta$  for  $x \in X$ .

An intuitionistic fuzzy set  $A$  in  $X$  is the union of all intuitionistic fuzzy points which belong to  $A$ .

**Definition 2.1.** ([1]) Let  $A = (\mu_A, \gamma_A)$  and  $B = (\mu_B, \gamma_B)$  be intuitionistic fuzzy sets on  $X$ . Then

- (1)  $A \subseteq B$  iff  $\mu_A \leq \mu_B$  and  $\gamma_A \geq \gamma_B$ .
- (2)  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$ .
- (3)  $A^c = (\gamma_A, \mu_A)$ .
- (4)  $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$ .
- (5)  $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$ .
- (6)  $0_{\sim} = (\tilde{0}, \tilde{1})$  and  $1_{\sim} = (\tilde{1}, \tilde{0})$ .

Let  $f$  be a map from a set  $X$  to a set  $Y$ . Let  $A = (\mu_A, \gamma_A)$  be an intuitionistic fuzzy set of  $X$  and  $B = (\mu_B, \gamma_B)$  an intuitionistic fuzzy set of  $Y$ .

The image of  $A$  under  $f$ , denoted by  $f(A)$ , is an intuitionistic fuzzy set in  $Y$  defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

The inverse image of  $B$  under  $f$ , denoted by  $f^{-1}(B)$ , is an intuitionistic fuzzy set in  $X$  defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A smooth fuzzy topology [11] on  $X$  is a map  $T : I^X \rightarrow I$  which satisfies the following properties:

- (1)  $T(\tilde{0}) = T(\tilde{1}) = 1$ .
- (2)  $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$  for  $\mu_1, \mu_2 \in I^X$ .
- (3)  $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$  for  $\mu_i \in I^X$ .

The pair  $(X, T)$  is called a smooth fuzzy topological space.

An intuitionistic fuzzy topology on  $X$  is a family  $T$  of intuitionistic fuzzy sets in  $X$  which satisfies the following properties:

- (1)  $0_\sim, 1_\sim \in T$ .
- (2) If  $A_1, A_2 \in T$ , then  $A_1 \cap A_2 \in T$ .
- (3) If  $A_i \in T$  for all  $i$ , then  $\bigcup A_i \in T$ .

The pair  $(X, T)$  is called an intuitionistic fuzzy topological space.

Let  $I(X)$  be a family of all intuitionistic fuzzy sets of  $X$  and let  $I \otimes I$  be the set of the pair  $(r, s)$  such that  $r, s \in I$  and  $0 \leq r + s \leq 1$ .

**Definition 2.2.** ([6]) Let  $X$  be a nonempty set. An intuitionistic fuzzy topology in Šostak's sense (SoIFT for short)  $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$  on  $X$  is a map  $T : I(X) \rightarrow I \otimes I$  which satisfies the following properties:

- (1)  $\mathcal{T}_1(0_\sim) = \mathcal{T}_1(1_\sim) = 1$  and  $\mathcal{T}_2(0_\sim) = \mathcal{T}_2(1_\sim) = 0$ .
- (2)  $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$  and  $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$ .
- (3)  $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$  and  $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$ .

The  $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$  is said to be an intuitionistic fuzzy topological space in Šostak's sense (SoIFTS for short). Also, we call  $\mathcal{T}_1(A)$  a gradation of openness of  $A$  and  $\mathcal{T}_2(A)$  a gradation of nonopenness of  $A$ .

**Definition 2.3.** ([10]) Let  $x_{(\alpha, \beta)}$  be an intuitionistic fuzzy point in an SoIFTS  $(X, \mathcal{T}_1, \mathcal{T}_2)$  and  $(r, s) \in I \otimes I$ . Then an intuitionistic fuzzy set  $A$  is said to be a fuzzy  $(r, s)$ -neighborhood of  $x_{(\alpha, \beta)}$  if there is a fuzzy  $(r, s)$ -open set  $B$  in  $X$  such that  $x_{(\alpha, \beta)} \in B \subseteq A$ .

### 3. Main Results

**Definition 3.1.** ([10]) Let  $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$  be a mapping from an SoIFTS  $X$  to another SoIFTS  $Y$  and  $(r, s) \in I \otimes I$ . Then  $f$  is said to be fuzzy weakly  $(r, s)$ -continuous if for each fuzzy  $(r, s)$ -open set  $B$  of  $Y$ ,  $f^{-1}(B) \subseteq \text{int}(f^{-1}(cl(B, r, s)), r, s)$ .

**Theorem 3.2.** Let  $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$  be a mapping from an SoIFTS  $X$  to another SoIFTS  $Y$  and  $(\alpha, \beta), (r, s) \in I \otimes I$ . Then  $f$  is a fuzzy weakly  $(r, s)$ -continuous mapping if and only if for every intuitionistic fuzzy point  $x_{(\alpha, \beta)}$  and each fuzzy  $(r, s)$ -neighborhood  $V$  of  $f(x_{(\alpha, \beta)})$ , there exists a fuzzy  $(r, s)$ -neighborhood  $U$  of  $x_{(\alpha, \beta)}$  such that  $f(U) \subseteq cl(V, r, s)$ .

*Proof.* Let  $x_{(\alpha, \beta)}$  be an intuitionistic fuzzy point in  $X$  and  $V$  a fuzzy  $(r, s)$ -neighborhood of  $f(x_{(\alpha, \beta)})$ ; then there exists a fuzzy  $(r, s)$ -open set  $B$  such that  $f(x_{(\alpha, \beta)}) \subseteq B \subseteq V$ . Since  $f$  is a fuzzy weakly  $(r, s)$ -continuous mapping,

$$\begin{aligned} f^{-1}(B) &\subseteq \text{int}(f^{-1}(cl(B, r, s)), r, s) \\ &\subseteq \text{int}(f^{-1}(cl(V, r, s)), r, s). \end{aligned}$$

Set  $U = f^{-1}(cl(V, r, s))$ ; then

$$x_{(\alpha, \beta)} \in f^{-1}(B) \subseteq \text{int}(U, r, s) \subseteq U.$$

Hence  $U$  is a fuzzy  $(r, s)$ -neighborhood of  $x_{(\alpha, \beta)}$  and  $f(U) \subseteq cl(V, r, s)$ .

For the converse, let  $V$  be a fuzzy  $(r, s)$ -open set in  $Y$ . By hypothesis, for each  $x_{(\alpha, \beta)} \in f^{-1}(V)$ , there exists a fuzzy  $(r, s)$ -neighborhood  $U_{x_{(\alpha, \beta)}}$  of  $x_{(\alpha, \beta)}$  such that  $f(U_{x_{(\alpha, \beta)}}) \subseteq cl(V, r, s)$ . Now we can say there exists a fuzzy  $(r, s)$ -open set  $G_{x_{(\alpha, \beta)}}$  such that

$$x_{(\alpha, \beta)} \in G_{x_{(\alpha, \beta)}} \subseteq U_{x_{(\alpha, \beta)}} \subseteq f^{-1}(cl(V, r, s))$$

for each  $x_{(\alpha, \beta)} \in f^{-1}(V)$ .

Thus we have

$$\begin{aligned} f^{-1}(V) &\subseteq \bigcup \{G_{x_{(\alpha, \beta)}} : x_{(\alpha, \beta)} \in f^{-1}(V)\} \\ &\subseteq f^{-1}(cl(V, r, s)). \end{aligned}$$

Since  $\bigcup \{G_{x_{(\alpha, \beta)}} : x_{(\alpha, \beta)} \in f^{-1}(V)\}$  is a fuzzy  $(r, s)$ -open set, we have  $f^{-1}(V) \subseteq \text{int}(f^{-1}(cl(V, r, s)), r, s)$ .  $\square$

**Theorem 3.3.** Let  $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$  be a mapping from an SoIFTS  $X$  to another SoIFTS  $Y$  and  $(\alpha, \beta), (r, s) \in I \otimes I$ . Then the following statements are equivalent:

- (1)  $f$  is a fuzzy weakly  $(r, s)$ -continuous mapping.
- (2) For each intuitionistic fuzzy point  $x_{(\alpha, \beta)}$  and each fuzzy  $(r, s)$ -open set  $V$  containing  $f(x_{(\alpha, \beta)})$ , there exists a fuzzy  $(r, s)$ -open set  $U$  containing  $x_{(\alpha, \beta)}$  such that  $f(U) \subseteq cl(V, r, s)$ .
- (3)  $cl(f^{-1}(\text{int}(F, r, s)), r, s) \subseteq f^{-1}(F)$  for each fuzzy  $(r, s)$ -closed set  $F$  in  $Y$ .
- (4)  $cl(f^{-1}(\text{int}(cl(B, r, s), r, s)), r, s) \subseteq f^{-1}(cl(B, r, s))$  for each fuzzy intuitionistic fuzzy set  $B$  in  $Y$ .
- (5)  $f^{-1}(\text{int}(B, r, s)) \subseteq \text{int}(f^{-1}(cl(\text{int}(B, r, s), r, s)), r, s)$  for each fuzzy intuitionistic fuzzy set  $B$  in  $Y$ .
- (6)  $cl(f^{-1}(V), r, s) \subseteq f^{-1}(cl(V, r, s))$  for a fuzzy  $(r, s)$ -open set  $V$  in  $Y$ .

*Proof.* (1)  $\Leftrightarrow$  (2) From Theorem 3.2, it is obvious.

(1)  $\Rightarrow$  (3) Let  $F$  be any fuzzy  $(r, s)$ -closed set of  $Y$ . Then  $1_{\sim} - F$  is a fuzzy  $(r, s)$ -open set in  $Y$  and by (1),

$$\begin{aligned} f^{-1}(1_{\sim} - F) &\subseteq \text{int}(f^{-1}(\text{cl}(1_{\sim} - F, r, s)), r, s) \\ &= \text{int}(f^{-1}(1_{\sim} - \text{int}(F, r, s)), r, s) \\ &= \text{int}(1_{\sim} - f^{-1}(\text{int}(F, r, s)), r, s) \\ &= 1_{\sim} - \text{cl}(f^{-1}(\text{int}(F, r, s)), r, s). \end{aligned}$$

Hence we have  $\text{cl}(f^{-1}(\text{int}(F, r, s)), r, s) \subseteq f^{-1}(F)$ .

(3)  $\Rightarrow$  (4) Let  $B$  be any intuitionistic fuzzy set in  $Y$ . Since  $\text{cl}(B, r, s)$  is a fuzzy  $(r, s)$ -closed set in  $Y$ , by (3),

$$\text{cl}(f^{-1}(\text{int}(\text{cl}(B, r, s)), r, s)) \subseteq f^{-1}(\text{cl}(B, r, s)).$$

(4)  $\Rightarrow$  (5) Let  $B$  be any intuitionistic fuzzy set of  $Y$ . Then,

$$\begin{aligned} f^{-1}(\text{int}(B, r, s)) &= 1_{\sim} - (f^{-1}(\text{cl}(1_{\sim} - B, r, s))) \\ &\subseteq 1_{\sim} - \text{cl}(f^{-1}(\text{int}(\text{cl}(1_{\sim} - B, r, s)), r, s), r, s) \\ &= \text{int}(f^{-1}(\text{cl}(\text{int}(B, r, s)), r, s), r, s). \end{aligned}$$

Hence,

$$f^{-1}(\text{int}(B, r, s)) \subseteq \text{int}(f^{-1}(\text{cl}(\text{int}(B, r, s)), r, s), r, s).$$

(5)  $\Rightarrow$  (6) Let  $V$  be any fuzzy  $(r, s)$ -open set of  $Y$ . Then by (5),

$$\begin{aligned} 1_{\sim} - f^{-1}(\text{cl}(V, r, s)) &= f^{-1}(\text{int}(1_{\sim} - V, r, s)) \\ &\subseteq \text{int}(f^{-1}(\text{cl}(\text{int}(1_{\sim} - V, r, s)), r, s), r, s) \\ &= \text{int}(1_{\sim} - (f^{-1}(\text{int}(\text{cl}(V, r, s)), r, s)), r, s) \\ &= 1_{\sim} - \text{cl}(f^{-1}(\text{int}(\text{cl}(V, r, s)), r, s), r, s) \\ &\subseteq 1_{\sim} - \text{cl}(f^{-1}(V), r, s). \end{aligned}$$

Hence we have

$$\text{cl}(f^{-1}(V), r, s) \subseteq f^{-1}(\text{cl}(V, r, s)).$$

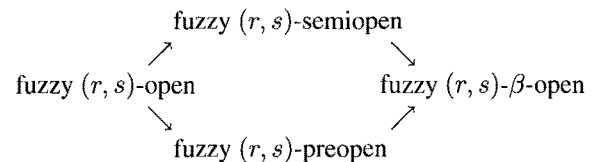
(6)  $\Rightarrow$  (2) Let  $V$  be a fuzzy  $(r, s)$ -open set containing  $f(x_{(\alpha, \beta)})$ . By (6),

$$\begin{aligned} x_{(\alpha, \beta)} &\in f^{-1}(V) \\ &\subseteq f^{-1}(\text{int}(\text{cl}(V, r, s)), r, s) \\ &= 1_{\sim} - f^{-1}(\text{cl}(1_{\sim} - \text{cl}(V, r, s)), r, s) \\ &\subseteq 1_{\sim} - \text{cl}(f^{-1}(1_{\sim} - \text{cl}(V, r, s)), r, s) \\ &= \text{int}(f^{-1}(\text{cl}(V, r, s)), r, s). \end{aligned}$$

Set  $U = \text{int}(f^{-1}(\text{cl}(V, r, s)), r, s)$ . Then  $U$  is a fuzzy  $(r, s)$ -open set satisfying  $f(U) \subseteq \text{cl}(V, r, s)$ . □

**Definition 3.4.** Let  $A$  be an intuitionistic fuzzy set in an SoIFTS  $(X, \mathcal{T}_1, \mathcal{T}_2)$  and  $(r, s) \in I \otimes I$ . Then  $A$  is said to be

- (1) *fuzzy  $(r, s)$ -semiopen* [8] if there is a fuzzy  $(r, s)$ -open set  $B$  in  $X$  such that  $B \subseteq A \subseteq \text{cl}(B, r, s)$ ,
- (2) *fuzzy  $(r, s)$ -preopen* [9] if  $A \subseteq \text{int}(\text{cl}(A, r, s), r, s)$ ,
- (3) *fuzzy  $(r, s)$ -regular open* [10] if  $A = \text{int}(\text{cl}(A, r, s), r, s)$ ,
- (4) *fuzzy  $(r, s)$ - $\beta$ -open* if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A, r, s), r, s), r, s)$ .



The following examples show that the converses of the above diagram may not be true.

**Example 3.5.** Let  $X = \{x, y\}$  and  $A_1, A_2$  and  $A_3$  be intuitionistic fuzzy sets of  $X$  defined as

$$A_1(x) = (0, 0.8), \quad A_1(y) = (0.3, 0.5);$$

$$A_2(x) = (0.8, 0), \quad A_2(y) = (0.3, 0.5);$$

and

$$A_3(x) = (0.8, 0), \quad A_3(y) = (0.6, 0.3);$$

Define  $\mathcal{T} : I(X) \rightarrow I \otimes I$  by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then  $(\mathcal{T}_1, \mathcal{T}_2)$  is an SoIFT on  $X$ . Since  $\text{cl}(A_3, \frac{1}{2}, \frac{1}{3}) = 1_{\sim}$ , clearly  $A_3$  is a fuzzy  $(\frac{1}{2}, \frac{1}{3})$ - $\beta$ -open set but it is not fuzzy  $(\frac{1}{2}, \frac{1}{3})$ -semiopen.

**Example 3.6.** Let  $X = \{x, y, z\}$  and  $A_1, A_2$  and  $A_3$  be intuitionistic fuzzy sets of  $X$  defined as

$$A_1(x) = (0, 0.8), \quad A_1(y) = (0.3, 0.6), \quad A_1(z) = (0.3, 0.6);$$

$$A_2(x) = (0.8, 0), \quad A_2(y) = (0.6, 0.3), \quad A_2(z) = (0.6, 0.3);$$

and

$$A_3(x) = (0.8, 0), \quad A_3(y) = (0.5, 0.3), \quad A_3(z) = (0.5, 0.3).$$

Consider an SoIFT  $\mathcal{T} : I(X) \rightarrow I \otimes I$  on  $X$  defined as follows.

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Since  $A_3 \subseteq \text{cl}(\text{int}(\text{cl}(A_3, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3})$  and  $\text{int}(\text{cl}(A_3, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) \subseteq A_3$ ,  $A_3$  is a fuzzy  $(\frac{1}{2}, \frac{1}{3})$ - $\beta$ -open set but not a fuzzy  $(\frac{1}{2}, \frac{1}{3})$ -preopen set.

**Theorem 3.7.** Let  $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$  be a mapping from an SoIFTS  $X$  to another SoIFTS  $Y$  and  $(r, s) \in I \otimes I$ . Then the following statements are equivalent:

- (1)  $f$  is a fuzzy weakly  $(r, s)$ -continuous mapping.
- (2)  $cl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$  for each fuzzy  $(r, s)$ -open set  $G$  in  $Y$ .
- (3)  $cl(f^{-1}(int(cl(V, r, s), r, s)), r, s) \subseteq f^{-1}(cl(V, r, s))$  for each fuzzy  $(r, s)$ -preopen set  $V$  in  $Y$ .
- (4)  $cl(f^{-1}(int(K, r, s)), r, s) \subseteq f^{-1}(K)$  for each fuzzy  $(r, s)$ -regular closed set  $K$  in  $Y$ .
- (5)  $cl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$  for each fuzzy  $(r, s)$ - $\beta$ -open set  $G$  in  $Y$ .
- (6)  $cl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$  for each fuzzy  $(r, s)$ -semiopen set  $G$  in  $Y$ .

*Proof.* (1)  $\Rightarrow$  (2) Let  $G$  be a fuzzy  $(r, s)$ -open set of  $Y$ ; then by Theorem 3.3 (3), we have  $cl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$ .

(2)  $\Rightarrow$  (3) Let  $V$  be a fuzzy  $(r, s)$ -preopen set in  $Y$ . Then  $V \subseteq int(cl(V, r, s), r, s)$ . Set  $A = int(cl(V, r, s), r, s)$ . Since  $A$  is a fuzzy  $(r, s)$ -open set, from (2), it follows

$$cl(f^{-1}(int(cl(A, r, s), r, s)), r, s) \subseteq f^{-1}(cl(A, r, s)).$$

Since  $cl(A, r, s) = cl(V, r, s)$ , we have

$$cl(f^{-1}(int(cl(V, r, s), r, s)), r, s) \subseteq f^{-1}(cl(V, r, s)).$$

(3)  $\Rightarrow$  (4) Let  $K$  be a fuzzy  $(r, s)$ -regular closed set of  $Y$ . Since  $int(K, r, s)$  is a fuzzy  $(r, s)$ -preopen set, by (3),

$$\begin{aligned} cl(f^{-1}(int(cl(int(K, r, s), r, s), r, s)), r, s) \\ \subseteq f^{-1}(cl(int(K, r, s), r, s)). \end{aligned}$$

Since  $int(K, r, s) = int(cl(int(K, r, s), r, s), r, s)$  and  $K = cl(int(K, r, s), r, s)$ , we have

$$cl(f^{-1}(int(K, r, s)), r, s) \subseteq f^{-1}(K).$$

(4)  $\Rightarrow$  (5) Let  $G$  be a fuzzy  $(r, s)$ - $\beta$ -open set. Then  $G \subseteq (cl(int(cl(G, r, s), r, s), r, s)$  and  $cl(G, r, s)$  is a fuzzy  $(r, s)$ -regular closed set. Hence by (4), we have

$$cl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s)).$$

(5)  $\Rightarrow$  (6) It is obvious.

(6)  $\Rightarrow$  (1) Let  $V$  be a fuzzy  $(r, s)$ -open set; then since  $V$  is a fuzzy  $(r, s)$ -semiopen set, by (6) and  $V \subseteq int(cl(V, r, s), r, s)$ , we have

$$\begin{aligned} cl(f^{-1}(V), r, s) \subseteq cl(f^{-1}(int(cl(V, r, s), r, s)), r, s) \\ \subseteq f^{-1}(cl(V, r, s)). \end{aligned}$$

Hence,  $f$  is a fuzzy weakly  $(r, s)$ -continuous mapping.  $\square$

## References

- [1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87–96.
- [2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.
- [3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237–242.
- [4] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81–89.
- [5] D. Çoker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Šostak's sense*, BUSEFAL **67** (1996), 67–76.
- [6] R. Ertürk and M. Demirci, *On the compactness in fuzzy topological spaces in Šostak's sense*, Mat. Vesnik **50** (1998), no. 3-4, 75–81.
- [7] H. Gürçay, D. Çoker, and A. Haydar Eş, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. **5** (1997), 365–378.
- [8] E. P. Lee, *Semiopen sets on intuitionistic fuzzy topological spaces in Sostak's sense*, J. Fuzzy Logic and Intelligent Systems **14** (2004), 234–238.
- [9] S. O. Lee and E. P. Lee, *Fuzzy  $(r,s)$ -preopen sets*, International J. Fuzzy Logic and Intelligent Systems **5** (2005), 136–139.
- [10] S. J. Lee and J. T. Kim, *Fuzzy  $(r,s)$ -irresolute maps*, International J. Fuzzy Logic and Intelligent Systems **7** (2007), 49–57.
- [11] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371–375.
- [12] A. A. Ramadan, S. E. Abbas, and A. A. Abd El-latif, *Compactness in intuitionistic fuzzy topological spaces*, International Journal of Mathematics and Mathematical Sciences (2005), 19–32.
- [13] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338–353.

---

**Won Keun Min**

Professor of Kangwon National University  
 Research Area: Fuzzy topology, General topology  
 E-mail : wkmin@kangwon.ac.kr