Performance of Doubly Correlated MIMO Channel in OFDM Spatial Multiplexing Systems

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Abstract—In this paper, the capacity of MIMO fading channel in the OFDM-based spatial multiplexing systems is analyzed when there is scattering at both transmitter and receiver. The employed MIMO channel model is spanning from the correlated low-rank case to uncorrelated high-rank case at both transmitter and receiver. The effects of spatial fading correlation on the capacity of MIMO channel is examined when the channel is known and unknown at the transmitter. We also evaluate the impacts of a channel estimation error at the transmitter on the MIMO channel capacity.

Index Terms—Multiple Input Multiple Output (MIMO), Capacity, Orthogonal Frequency Division Multiplexing (OFDM), Spatial Multiplexing

I. INTRODUCTION

Recently, multiple input multiple output (MIMO) wireless communication systems that use spatial diversity and/or array gain at both the transmitter and the receiver have received enormous attention. The use of both multiple transmit and receive antennas offers an additional degree of freedom gain in the system, which can be exploited by spatial multiplexing of information under sufficiently rich scattering environments, and lead to a linear increase in the capacity of a MIMO channel.

Previous works of MIMO channel capacity have mostly assumed that MIMO fading channels have independent and identically distributed (i.i.d.) Gaussian channels [1]. In real propagation channels, the spatial fading is not independent due to the small separation between two antennas and/or scare surrounding scattering environments. In [2] – [6], the effects of spatial fading correlation on the performance of MIMO channels have been examined. However, the channel model used in [5] does not allow for spatial correlation at both transmitter and receiver. On the other hand, the MIMO capacity analysis in [6] has been made when there exists scattering at both transmitter and receiver.

Orthogonal frequency division multiplexing (OFDM) provides robustness to frequency selective fading environments as well as spectral efficiency. This modulation technique can transform a frequency selective channel into a set of frequency flat channels. In [5], the OFDM-based spatial multiplexing systems have been considered to analyze the capacity when the channel is unknown at the transmitter and the spatial correlation is present only at the BS side.

In this paper, the realistic capacity of doubly correlated MIMO channels in the OFDM-based spatial multiplexing systems is analyzed by employing the stochastic channel model, which embraces the uncorrelated high-rank and correlated low-rank channels. This MIMO channel can be regarded to be general in terms of exhibiting spatial fading correlations from the correlated low-rank case to uncorrelated high-rank case at both transmitter and receiver. It is assumed that the receiver has perfect channel knowledge. We study the influence of spatial correlation on the capacity performance of MIMO fading channels in the cases with and without the knowledge of channel side information at the transmitter. In addition, when there is a channel estimation error at the transmitter, we look at how much the MIMO channel capacity is different.

II. STOCHASTIC DOUBLY CORRELATED MIMO RADIO CHANNEL MODEL

We consider the uplink case in a wideband (WB) MIMO system with N_t transmit and N_r receive antennas. Denoting $\mathbf{x}$ as the $N_t \times 1$ transmitted signal vector and $\mathbf{y}$ as the $N_r \times 1$ received signal vector, the MIMO system describing an input-output relationship with i.i.d. additive white Gaussian noise (AWGN) is given by

$$\mathbf{y} = \sum_{l=0}^{L-1} \mathbf{H}_{c,l}\mathbf{x} + \mathbf{n} \tag{1}$$

where the correlated NB MIMO channel can be depicted as a $N_r \times N_t$ channel transfer matrix $\mathbf{H}_{c,l}$ and L is the

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number of multiple paths. $\mathbf{n}$ is the $N_r \times 1$ AWGN vector with zero mean and variance matrix $\sigma_n^2 \mathbf{I}_{N_r}$.

The doubly correlated channel coefficients between multiple transmit/receive antennas are generated with a simple stochastic WB MIMO model [7]-[9]. Here, the transmission connection between the MS and BS described by the NB MIMO radio channel is expressed as $\mathbf{H}_{c,l} = \mathbf{A}_l \times \mathbf{B}_l \in \mathbf{C}^{N_r \times N_t}$ where $\times$ denotes an element-by-element multiplication, and elements corresponding to the mth row and nth column of matrices $\mathbf{H}_{c,l} \in \mathbf{C}^{N_r \times N_t}$, $\mathbf{A}_l \in \mathbf{R}^{N_r \times N_t}$, and $\mathbf{B}_l \in \mathbf{C}^{N_r \times N_t}$ are $h_l^{(m)(n)}$, $\sqrt{P_l^{(m)(n)}}$, and $\beta_l^{(m)(n)}$, respectively. Here, $P_l^{(m)(n)} = E\left\{ \left| h_l^{(m)(n)} \right|^2 \right\}$ is an average power of the transmission coefficient at the lth path between transmit antenna n and receive antenna m and the correlated channel coefficients $\beta_l^{(m)(n)}$, $m = 1, 2, \cdots, N_r$, and $n = 1, 2, \cdots, N_t$, with $E\left[\left| \beta_l^{(m)(n)} \right|^2 \right] = 1$ are obtained according to $\mathbf{C}_l = \mathbf{L}_l \, \boldsymbol{\alpha}_l$ where

$$\mathbf{C}_l = \left[\beta_l^{(1)(1)} \; \beta_l^{(2)(1)} \cdots \beta_l^{(N_r)(1)} \; \beta_l^{(1)(2)} \cdots \beta_l^{(N_r)(N_t)} \right]^T \quad (2)$$

$$\boldsymbol{\alpha}_l = \left[\alpha_l^{(1)} \quad \alpha_l^{(2)} \quad \cdots \quad \alpha_l^{(N_r N_t)} \right]^T \quad (3)$$

Here, $\alpha_l^{(p)}$ is a zero-mean complex i.i.d. random variable shaped by the desired Doppler spectrum. So, its amplitude and phase are Rayleigh and uniformly distributed, respectively. $(\cdot)^T$ denotes the transpose. The lower triangular matrix $\mathbf{L}_l$ can be obtained by the Cholesky decomposition of $\mathbf{R}_{MIMO,l} = \mathbf{L}_l \mathbf{L}_l^T$ with

$$\mathbf{R}_{MIMO,l} = \mathbf{R}_{BS,l} \otimes \mathbf{R}_{MS,l} \quad (4)$$

where $\otimes$ denotes a Kronecker product and the symmetric correlation matrices $\mathbf{R}_{BS,l}$ at the MS and $\mathbf{R}_{MS,l}$ at the BS consist of elements $\rho_{BS,l}^{(m_1)(m_2)}$ on the m_1th row and m_2th column and $\rho_{MS,l}^{(n_1)(n_2)}$ on the n_1th row and n_2th column, respectively. The envelope correlation coefficients, $\rho_{BS,l}^{(m_1)(m_2)}\left(|m_1 - m_2| D \right)$ and $\rho_{MS,l}^{(n_1)(n_2)}\left(|n_1 - n_2| D \right)$, where $D = d/\lambda$ with the element spacing d between two adjacent antennas at the BS and wavelength λ are the correlation coefficients between two receive antennas m_1 and m_2 at the BS and between two transmit antennas n_1 and n_2 at the MS. They are dependent on the system and propagation parameters such as the distance between antenna elements, the power azimuth spectrum (PAS) type, azimuth spread (AS) and, mean direction of departure (DOD) and mean direction of arrival (DOA), respectively. Here, the uniform PAS whose AS is determined as $S_l = \Delta\varphi_l / \sqrt{3}$ [9] is defined over $[\varphi_l - \Delta\varphi_l, \varphi_l + \Delta\varphi_l]$ with the mean DOA or mean DOD φ_l and parameter $\Delta\varphi_l$. It is pointed that the parameters used for the lth resolvable path at the BS and MS can be denoted by mean DOA φ_l^{BS}, mean DOD φ_l^{MS}, angular spreads S_l^{BS}, S_l^{MS}, antenna spacings D_{BS} and D_{MS}. Note that other PASs such as a truncated Gaussian and a truncated Laplacian distribution can be considered.

In order to separately capture the effects of spatial fading correlations at the BS and MS antennas, the correlated MIMO channel can be modeled as

$$\mathbf{H}_{c,l} = \mathbf{R}_{BS,l}^{1/2} \mathbf{H}_w \mathbf{R}_{MS,l}^{1/2} \quad (5)$$

where $\mathbf{R}_{MS,l}$ and $\mathbf{R}_{BS,l}$, respectively, are the transmit and receive covariance matrices that specify the transmit and receive correlations at the lth path. $\mathbf{H}_w$ is an $N_r \times N_t$ i.i.d. random matrix with $CN(0,1)$ entries.

III. CAPACITY ANALYSIS OF OFDM SPATIAL MULTIPLEXING SYSTEMS

A. Capacity of OFDM Spatial Multiplexing Systems

In the OFDM-based spatial multiplexing system, the transmitted data symbols from multiple transmitter antennas on the nth tone can be expressed as $\mathbf{s}_n = \left[s_n^{(1)} \; s_n^{(2)} \; \cdots \; s_n^{(N_t)} \right]^T$ and the received signal vector is given by

$$\mathbf{z}_n = \mathbf{H}_s^{(n)} \mathbf{s}_n + \mathbf{w}_n, \quad n = 1, 2, \cdots, N \quad (6)$$

Where

$$\mathbf{H}_s^{(n)} = \sum_{l=0}^{L-1} \mathbf{H}_{c,l} e^{-j2\pi (n/N) l} \quad (7)$$

and $\mathbf{w}_n$ is AWGN with zero mean and $E\left[\mathbf{w}_n \mathbf{w}_k^H \right] = \sigma_w^2 \mathbf{I}_{N_r} \delta(n-k)$.

Consider the following vectors:

$$\mathbf{z} = \left[\mathbf{z}_1^T \quad \mathbf{z}_2^T \quad \cdots \quad \mathbf{z}_N^T \right]^T \quad (8)$$

$$\mathbf{s} = \left[\mathbf{s}_1^T \quad \mathbf{s}_2^T \quad \cdots \quad \mathbf{s}_N^T \right]^T \quad (9)$$

$$\mathbf{w} = \left[\mathbf{w}_1^T \quad \mathbf{w}_2^T \quad \cdots \quad \mathbf{w}_N^T \right]^T \quad (10)$$

Moreover, the block-diagonal matrix representing the correlated channel and OFDM modulation is defined as

$$\mathbf{H} = \mathrm{diag}\left\{ \mathbf{H}_s^{(n)} \right\}_{n=1}^{N} \quad (11)$$

Then, the received data vector is given by

$$\mathbf{z} = \mathbf{H}\mathbf{s} + \mathbf{w} \qquad (12)$$

where $\mathbf{w}$ is AWGN with zero mean and $E\left[\mathbf{w}\mathbf{w}^H\right] = \sigma_w^2 \mathbf{I}_{N_r N}$.

The capacity of OFDM spatial multiplexing on the basis of a stochastic MIMO channel given the channel realization $\mathbf{H}$ under an average transmitter power constraint is defined as the following:

$$C = \max_{\mathrm{Tr}(\mathbf{S}) \le P} \frac{1}{N} \log_2 \left[\det \left(\mathbf{I}_{N_r N} + \frac{1}{\sigma_w^2} \mathbf{H}\mathbf{S}\mathbf{H}^H \right) \right] \text{(bits/s/Hz)} \qquad (13)$$

where $\mathbf{S} = \mathrm{diag}\{\mathbf{S}_n\}_{n=1}^N$. Here, $\mathbf{S}_n$ is the $N_t \times N_t$ covariance matrix of input signal vector $\mathbf{s}_n$ and P is the maximum transmit power.

B. Capacity by Uniform Power Allocation

When the channel is unknown at the transmitter but perfectly known at the receiver, the total available power is optimally selected to be allocated uniformly across all transmit antennas. Assuming that statistically independent data symbols are transmitted from different antennas, we can set $\mathbf{S}_n = (P/N_t N)\mathbf{I}_{N_t}$. Then the ergodic capacity results in

$$C = E_{\mathbf{H}} \left[\frac{1}{N} \sum_{n=1}^{N} \log_2 \left(\det \left(\mathbf{I}_{N_r} + \frac{\rho}{N_t N} \mathbf{\Lambda}^{(n)} \mathbf{H}_w \mathbf{H}_w^H \right) \right) \right] \qquad (14)$$

where $\rho = P/\sigma_w^2$ denotes the signal to noise (SNR) and

$$\mathbf{\Lambda}^{(n)} = \mathrm{diag}\left\{ \lambda_i^{(n)} \left(\mathbf{H}_s^{(n)} \mathbf{H}_s^{(n)^H} \right) \right\}_{i=1}^{n_{\min}} \qquad (15)$$

Here, $\lambda_i^{(n)}$ denotes the ith eigenvalue of $\mathbf{H}_s^{(n)} \mathbf{H}_s^{(n)^H}$ or correlation matrix $\mathbf{R}^{(n)}$ and $n_{\min} = \min(N_t, N_r)$.

The MIMO channel can be transformed into a parallel channel consisting of a set of $n_{\min}$ non-interfering subchannels. Then the capacity of MIMO channel is computed by the sum of the capacities of the $n_{\min}$ subchannels as the following:

$$C = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{n_{\min}} \log_2 \left(1 + \frac{\rho}{N_t N} \lambda_i^{(n)} \right) \quad \text{(bits/s/Hz)} \qquad (16)$$

Note that the ith eigenvalue on the nth tone is the power gain of the ith subchannel on the nth tone.

C. Capacity by Waterfiling Power Allocation

When the channel is known at the transmitter, an optimum power allocation method by waterfilling algorithm can be used to maximize the capacity subject to the total transmitted power constraint $\sum_{n=1}^{N} \sum_{i=1}^{n_{\min}} P_i^{(n)} = P$. The power $P_i^{(n)}$ allocated to the ith subchannel on the nth tone is determined as the following solution:

$$P_i^{(n)} = \left(\mu - \sigma_w^2 / \lambda_i^{(n)} \right)^+ \qquad (17)$$

where $z^+ = \max(z, 0)$ and μ is a parameter chosen such that the total transmitted power constraint is satisfied. Then the optimal capacity can be given by

$$C = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{n_{\min}} \log_2 \left(1 + \rho_i^{(n)} \lambda_i^{(n)} \right) \qquad (18)$$

where $\rho_i^{(n)} = P_i^{(n)} / \sigma_w^2$.

D. Impact of Channel Estimation Error

Now we investigate the effect of channel estimation errors on the capacity of MIMO channel when the transmit power is allocated by the waterfilling algorithm. The channel matrix can be expressed as $\hat{\mathbf{H}}_s^{(n)} = \mathbf{H}_s^{(n)} + \Delta\mathbf{H}_s^{(n)}$ where $\mathbf{H}_s^{(n)}$ consists of perfect channel estimated entries and $\Delta\mathbf{H}_s^{(n)}$ contains error contributions modeled as $CN(0, \sigma_\varepsilon^2)$ components due to imperfect channel estimation. In this case, the weights to be applied to the transmitting antennas are given by $\mathbf{V}_e^{(n)}$, the matrix of eigenvectors of $\hat{\mathbf{H}}_s^{(n)}$. Then the capacity by a waterfilling power allocation scheme can be rewritten as

$$C_e = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{n_{\min}} \log_2 \left(1 + \rho_{i,e}^{(n)} \lambda_i^{(n)} \right) \qquad (19)$$

Where

$$\rho_{i,e}^{(n)} = P_{i,e}^{(n)} / \sigma_w^2 \qquad (20)$$

$$P_{i,e}^{(n)} = \left(\mu - \sigma_w^2 / \lambda_{i,e}^{(n)} \right)^+ \qquad (21)$$

Here, $\lambda_{i,e}^{(n)}$ is the ith eigenvalue of $\hat{\mathbf{H}}_s^{(n)} \hat{\mathbf{H}}_s^{(n)^H}$.

IV. NUMERICAL RESULTS

In the simulations, the MIMO channel capacities based on the OFDM spatial multiplexing systems with $N = 512$ subcarriers were obtained from 200 independent Monte-Carlo runs. It is assumed that a 4×4 WB MIMO fading channel with two resolvable paths ($L = 2$) is considered. We consider three cases with different spatial correlations, which are classified according to the values of propagation and system parameters at the MS and BS. First one is a doubly correlated MIMO channel with spatial correlations at both MS and BS sides (called a double correlation). Second one (called a single correlation) is a MIMO channel with a spatial decorrelation at MS and a spatial correlation at the BS in which case the propagation and system parameters at BS side uses the same as those employed in the the BS of the first doubly correlated channel. Finally, when the spatial fading channels at the BS receiver and MS transmitter are uncorrelated (called a double decorrelation) the capacity results are included for the purpose of comparisons with the correlated cases.

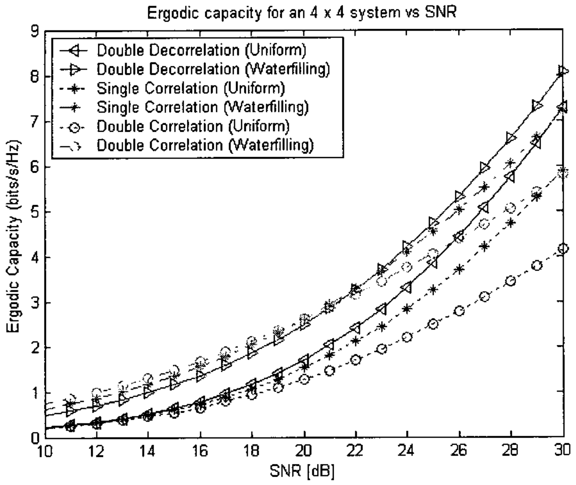

Fig. 1. Ergodic capacity vs SNR

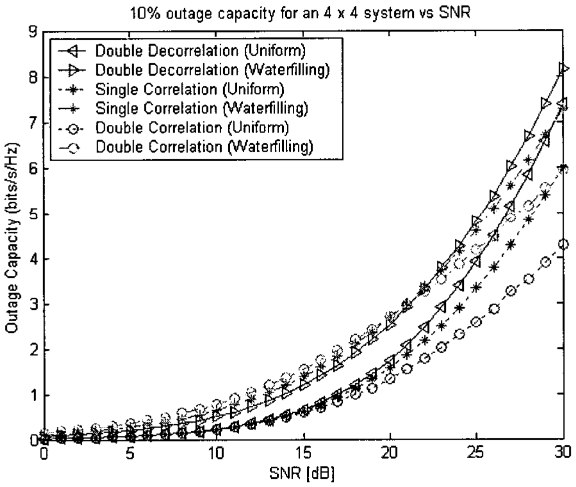

Fig. 2. 10% outage capacity vs SNR

Figs. 1 and 2, respectively, show the ergodic capacities and 10% outage capacities of MIMO channels. The first case with a double correlation uses the following parameters: mean DOAs $\varphi_1^{BS}=30^\circ$, $\varphi_2^{BS}=-20^\circ$, mean DODs $\varphi_1^{MS}=10^\circ$, $\varphi_2^{MS}=-30^\circ$, angular spreads $S_1^{BS}=6^\circ$, $S_2^{BS}=3^\circ$, $S_1^{MS}=2^\circ$, $S_2^{MS}=4^\circ$, antenna spacings $D_{BS}=1$ and $D_{MS}=0.5$. It is observed that the capacity of a doubly correlated MIMO channel under uniform power allocation is substantially lower than that of a doubly uncorrelated one, especially in the high SNR range. Meanwhile, it is seen that the waterfilling gain in the doubly correlated channel is higher than in the doubly uncorrelated case and single correlated one. It is also found that the waterfilling capacity of the doubly correlated channel in the range of less than SNR=21dB is slightly better than the doubly decorrelated and single correlated cases. On the other hand, in the range of more than SNR=21dB, the doubly decorrelated channel under waterfilling power allocation provides the best capacity.

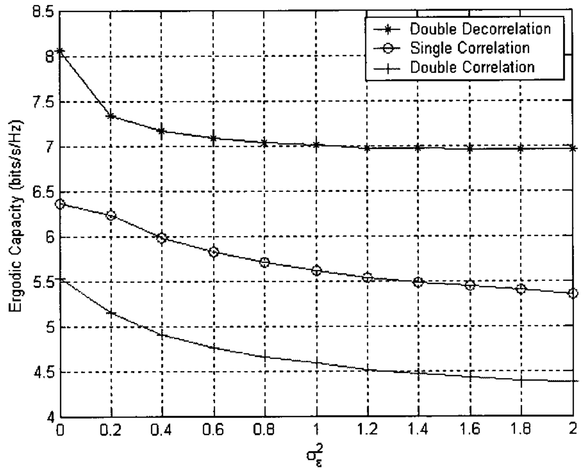

Fig. 3. Effect of channel estimation error on waterfilling capacity

Now we evaluate the impacts of channel estimation errors on the waterfilling ergodic capacities. Fig. 3 shows the variation of ergodic capacities for given mean DOAs $\varphi_1^{BS}=30^\circ$, $\varphi_2^{BS}=-20^\circ$, mean DODs $\varphi_1^{MS}=10^\circ$, $\varphi_2^{MS}=-30^\circ$, angular spreads $S_1^{BS}=5^\circ$, $S_2^{BS}=3^\circ$, $S_1^{MS}=2^\circ$, $S_2^{MS}=4^\circ$, antenna spacings $D_{BS}=0.5$, $D_{MS}=0.5$ and SNR=30dB as a function of the variance σ_ε^2, which is occurred by the channel estimation error. As the variance increases, the capacity in the doubly correlated channel case decreases.

V. CONCLUSIONS

The capacities of doubly correlated MIMO channels in the OFDM spatial multiplexing systems are investigated on the basis of the stochastic channel model. Spatial fading correlation spanning from the correlated case to uncorrelated case at the transmitter and receiver is considered to study its effect on the capacity performance of MIMO fading channels. The capacity under uniform and waterfilling power allocation strategies in the high SNR range for the doubly correlated channel is much worse than the doubly decorrelated and single correlated cases. Since a channel estimation error known at the transmitter has an effect on the waterfilling power allocation, we experience the loss of the MIMO channel capacity. As the variance of the channel estimation error component for SNR=30dB increases, the capacity of the correlated channel decreases.

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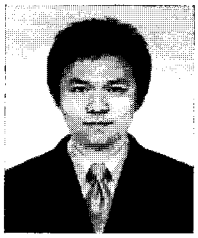

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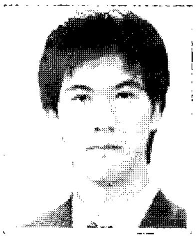

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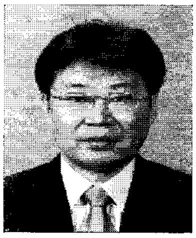

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