Network Congestion Control using Robust Optimization Design

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ABSTRACT

Congestion control is one of major mechanisms to avoid dropped packets. Many researchers use optimization theories to find an efficient way to reduce congestion in networks, but they do not consider robustness that may lead to unstable network utilities. This paper proposes a new methodology in order to solve a congestion control problem for wired networks by using a robust design principle. In our particular numerical example, the proposed method provides robust solutions that guarantee high and stable network utilities.

Key Words : Congestion Control, Robust Design, Network optimization, Mean-Squared Error, Network Utility

I. Introduction

Congestion control concerns controlling traffic entry into a network, so as to avoid congestive collapse by attempting to avoid oversubscription of any of the processing or link capabilities of the intermediate nodes and networks and taking resource reducing steps, such as reducing the rate of sending packets. In wired network, congestion control is modeled as a network utility maximization issue in Kelly's work [1]:

$$\begin{aligned} &maximize \sum_{s} U_s(x_s) \\ &subject\ to\ \ Rx \le c \end{aligned} \tag{1}$$

where x, R, and c denote a source rate vector which is the only optimization variables, routing matrix, and link capacity vector, respectively. Utility functions $U_s(x_s)$ are often assumed to be smooth, increasing, concave, and to depend on local rates only, although recent investigations have removed some of these assumptions for applications that they are invalid. Low et al. [3] use Langrangian relaxation to decompose this problem into simpler problems. This decomposition method allows solving the problem in a distributed way. The result of Kelly [1] and Low [3] has been widely accepted and used in wired networks. However, the assumption that link capacity vector is constant is not true in wireless networks. Different from wired networks, there are interferences between links in wireless networks. Therefore, when a link is used to transfer data, some others, which are in its reference region, should not be used. Motivated by Kelly's works, a number of researchers apply this optimization theory in network issues [11][12][13][14][15][16][17][18] which are surveyed in [2],[9],[10]. These works solve many issues such as power control [11], congestion control [11],[18], routing [12],[16], scheduling [12],[17],[18]. Many decomposition methods are proposed to solve problems in distributed ways. The key idea of network optimization is to use limited network resources efficiently to provide the best benefit; therefore, these above works improve network utilities significantly. However, to the customers' point of view, they expect not only high network utilities but also stable services with small variances, which can be solved by robust design.

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The primary goal of robust design is to determine the best design factor settings by minimizing performance variability and product bias, i.e., the deviation from the target value of a product.(Shin and Cho, 2005). Because of their practicability in reducing the inherent uncertainty associated with design factors and system performance, the widespread application of robust design techniques has resulted in significant improvements in product quality, manufacturability and reliability at low cost. To illustrate this method graphically, the probability density functions of two cases A and B as shown in Figure 1 are considered. Denoting as the desired target value T for two cases, Figure 1 clearly shows that the advantage of the mean-squared error (MSE) model; as a result, the variability reduction is achieved by allowing a small magnitude of process bias. From robust design view point, solution B is better than solution A because of its smaller variance.

In this paper, we use robust design to improve network optimization problems. Although we believe that we can apply robust design in any network optimization problem, for the first step, we focus on Kelly's problem. The main contributions of the paper are as follows:

- We extend Kelly's problem with consideration of robustness. In this model, we assume that data rate follows Poisson distribution which is more practical than deterministic assumption in Kelly's model. By presenting MSE, this model objective is both high network utility and small variance. This model is located in section 3.
- The above problem is formulated as a multiple objective optimization problem. Using the multiple-objective optimization problem makes our solution more flexible. We can see in section 4 that priority of each user can be adjusted by the weight vector.
- The numerical analysis in section 4 shows that our solution is high network utility, robust, and flexible.

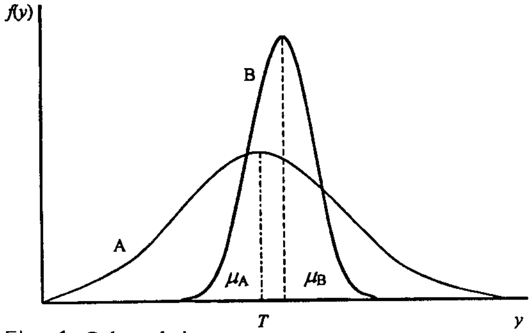

Fig. 1. Robust design

The rest of the paper is organized as follows. Section II presents related work about network optimization and robust design. Our mathematical model is presented in section III. We conduct a numerical example in section IV. Section V concludes the paper and discuss about future works.

II. Related works

Currently, motivated by Kelly's works many researchers are trying to improve Kelly's model to apply in different kinds of networks. The hottest trend of this problem is to cooperate between layers in networks to achieve global optimal network utility. Network protocols may instead be holistically analyzed and systematically designed as distributed solutions to some global optimization problems in the form of generalized network utility maximization (NUM) [11]-[18]. Some authors propose methods that allow MAC and transport layer to cooperate [11], [13], [14], [15], and [17] while other authors solve joint MAC, routing, and transport protocol. These works solve many issues such as power control [11], congestion control [11],[18], routing [12],[16], scheduling [12],[17],[18]. Network environment is also diverse such ad hoc networks [12], [15], multi-hop wireless networks [11], [13], [18]... Although there exists many researches in this area but the main objective is how to achieve maximal network utility function. In practice, high network utility, however, is not enough, customers expect its stability. This nice characteristic can be obtained by using robust design.

Pioneered by Dr. Genichi Taguchi, robust design has quickly become popular in industrial because it improves product quality significantly. There are number of strategies for robust design. However, our objective is to maximize network utility and at the same time minimize network utility variance. Therefore, the strategy that minimizes the bias and the variance jointly in [5][6] is suitable for this trade-off. This method is

$$\begin{array}{ll} Minimize & \left((\hat{\mu}-\tau)^2+\hat{\sigma}^2(x)\right) \\ Subject\ to & x \in [x_L, x_U] \end{array} \quad (2)$$

where $(\hat{\mu}-\tau)^2$ denotes bias and $\hat{\sigma}^2(x)$ denotes variance.

Ⅲ. Problem Formulation

It is well-known that Kelly's network optimization is formulated as (1) in [1][3]. In the first step of robust design approach, we extend this basic model. Assume that a network composes a set of sources S and set of links L. Each source $s \in S$ has data rate x_s (packets/s). Different from Kelly's model [1][3], we assume that this data rate follows Poisson distribution with average parameter λ_s and $U_s(x_s)$ denotes network utility function corresponding to x_s. The network utility function $U_s(x_s)$ is assumed to be increasing over x_s. $L(s) \subset L$ is set of links that source s uses to transmit data. For each link $l \in L$, let $S(l) \subset S$ be set of sources that use link l. Each link l has capacity of c_l. Aggregate data rate through a link l cannot exceed its capacity. In Kelly's model, it is

$$\sum_{s \in S(l)} x_s \le c_l, \forall l \in L \quad (3)$$

But in our model, data rate $x_s \sim Poisson(\lambda_s)$ therefore, $E(x_s) = \lambda_s$. And thus,

$$\sum_{s \in S(l)} \lambda_s \le c_l, \forall l \in L \quad (4)$$

For Kelly's problem, they tried to find data rate x that maximizes total network utility functions $\sum_s U_s(x_s)$. In this paper, we extend Kelly's model in two aspects:

- We do not use a simple sum for objective function. We use multiple objective functions instead of a single objective function. This method makes the model more flexible.
- Our objective is to maximize network utility functions to improve network performance and at the same time to minimize variance to guarantee robustness.

At first, we estimate expected value of $U_s(x_s)$. We have $x_s \sim Poisson(\lambda_s)$, i.e., probability function is

$$P(x_s = k) = \frac{e^{-\lambda_s}\lambda_s^k}{k!} \qquad \forall s \in S, k \ge 0$$

That leads to

$$E(U_s(x_s)) = \sum_{k=0}^{\infty} \frac{e^{-\lambda_s}\lambda_s^k}{k!} U_s(k) \quad (5)$$

And variance can be computed by

$$VAR(U_s(x_s)) = E(U_s^2(x_s)) - (E(U_s(x_s)))^2 = \\ = \sum_{k=0}^{\infty} \frac{e^{-\lambda_s}\lambda_s^k}{k!} U_s^2(k) - \left(\sum_{k=0}^{\infty} \frac{e^{-\lambda_s}\lambda_s^k}{k!} U_s(k)\right)^2 \quad (6)$$

In this paper, we are dealing with multiple objective functions, and we have to define system targets. Let X denote feasible region of data rate x which satisfies constraint (3). In Kelly's model, the objective is to maximize aggregate network utility functions. Therefore, we can choose ideal maximum utility functions as our targets for network utility functions. Let $U^T = (U_s^T)_{s \in S}$ be the network utility function target vector where (Fig. 2.)

$$U_s^T = \max_{x \in X} U_s(x_s)$$

Because $U_s(x_s)$ is increasing over x_s therefore

$$U_s^T = \max_{x \in X} U_s(x_s) = U_s(\max_{x \in X} x_s) \quad (7)$$

To guarantee network performance and robust-

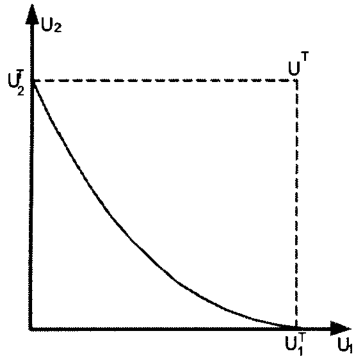

Fig. 2. Ideal network utility target

ness, we minimize the bias and the variance in a joint manner. It can be obtained by minimizing mean squared error (MSE) [4][5][6], i.e.,

$$\begin{aligned} &minimize \quad (MSE_s)_{s \in S} \\ &subject\ to \quad \sum_{s \in S(l)} \lambda_s \le c_l, \forall l \in L \end{aligned} \tag{8}$$

where

$$MSE_s = (E(U_s(x_s)) - U_s^T)^2 + VAR(U_s(x_s)) \tag{9}$$

This problem is a multiple objective optimization problem. There are a number of methods which are presented in [8] but in this paper we use weighted-sums approach because of its simplification. In the numerical example, this method is acceptable because Pareto surface is convex. For decision making, we have to minimize weighted sums as follows:

$$\begin{aligned} &minimize \quad MSE = \sum_{s \in S} w_s MSE_s \\ &subject\ to \quad \sum_{s \in S(l)} \lambda_s \le c_l, \forall l \in L \end{aligned} \tag{10}$$

where $w_s, s \in S$ is positive such that $\sum_{s \in S} w_s = 1$.

MSE_s, which is defined in (9), is a very complicate function. We do not have its closed form in general case. Furthermore, this function is convex neither. Therefore, to solve the problem (10), we divide feasible region of vector λ into a mesh. We will search the mesh to find the optimal solution.

Ⅳ. Numerical Analysis and Discussion

Assume that we have a network as shown in Fig. 3. The network composes of six nodes and five links. Two flows x_1 and x_2 share common links. Capacities of links $(c_1, c_2, c_3, c_4, c_5)$ are $(5, 9, 5, 9, 9)$ which are shown in Fig. 3. We assume that traffic is elastic, i.e., $U_s(x_s) = log(x_s), \forall s \in S$. We conduct this example in MatLab.

Feasible region in decision space is defined by constraint (3), and shown in Fig. 4(a) in this example. By applying equations (5), (6), (7), and (9), we can computer MSE_1 and MSE_2, and thus feasible regions in criterion space is shown in Fig. 4(b). We can see that feasible region in criterion space is convex; therefore, weighted-sums approach is suitable in this case.

First, we assign weights equally, i.e., $w_1 = w_2 = 0.5$. Fig. 5 shows the relationship between MSE and control variable λ. By searching in feasible region to minimize MSE we can find that the optimal operation point is $(\lambda_1, \lambda_2) = (3.6, 5.4)$ as shown in the third row of Table 1.

At the same time, we implement Low's solution [3] for Kelly's problem. Data rate vector (x_1, x_2) converges to $(4.5, 4.5)$. As presented in the previous section, Kelly uses deterministic approach while we use stochastic approach. Therefore, to compare solutions of our problem and Kelly's problem, we have to convert them into same criteria. Note that if we have data rate of $(4.5, 4.5)$ in Kelly's model, then average data rate is $(4.5, 4.5)$ too. Thus the solution of Kelly's problem is equivalent to operation point of

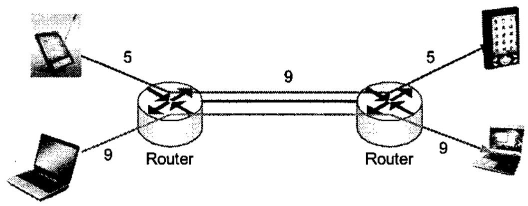

Fig. 3. Network architecture

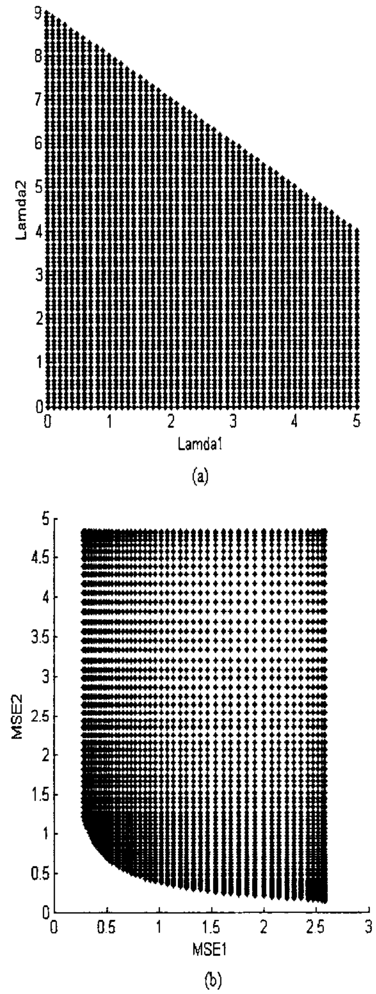

Fig. 4. (a) Feasible region in decision space. (b) Feasible region in criterion space

$(\lambda_1, \lambda_2) = (4.5, 4.5)$ in our model. And we can compute other criteria in the second row of Table 1. Looking at the last column of Table. 1, we can see that the solution of our model have MSE of 0.5943, less than 10% in comparison with the one of Kelly's model.

If we change weights that are assigned to users, we can obtain different solutions as shown in Table 2. A weight for a user shows how important the user is. A higher weight is assigned for more important user. Indeed, when $w_2 = 0.95$, i.e., the second user is very important, all network resources are used by the second user $(\lambda_1 = 0, \lambda_2 = 9)$. When w_1 increases, the first user has more network resource. And when $w_1 = 0.95$, the first user have the first priority to use network resource and the second user use remaining resource. This result shows the flexibility of our solution when we can use weight vector w to control priority of each user.

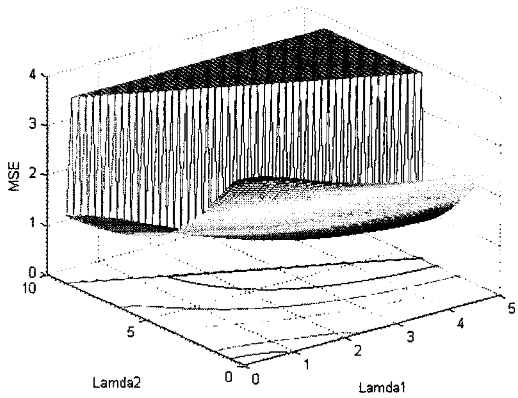

Fig. 5. Mean square error

Changing weight vector w gives us different couple (MSE_1, MSE_2). The curve that connects these couples is Pareto surface which is shown in Fig. 6.

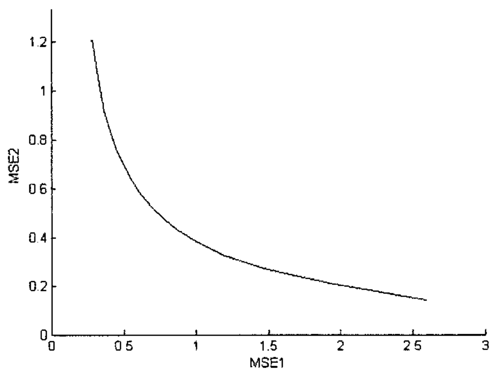

Fig. 6. Pareto surface

Table 1. Comparison between our model and Kelly's model

Model	λ_1	λ_2	$E(U_1)$	$E(U_2)$	$Var(U_1)$	$Var(U_2)$	MSE_1	MSE_2	MSE
Kelly	4.5	4.5	1.3787	1.3787	0.2945	0.2945	0.3477	0.9644	0.6560
Our Model	3.6	5.4	1.1395	1.5789	0.335	0.2504	0.5558	0.6327	0.5943

Table 2. Solution list with different weights

λ_1	λ_2	$E(U_1)$	$E(U_2)$	$Var(U_1)$	$Var(U_2)$	MSE_1	MSE_2	w_1	w_2	MSE
0	9	0	2.1351	0	0.1372	2.5903	0.141	0.05	0.95	0.2635
1.2	7.8	0.2945	1.9812	0.1933	0.1639	1.9225	0.2105	0.1	0.9	0.3817
1.8	7.2	0.5233	1.8944	0.2902	0.1808	1.4699	0.2726	0.15	0.85	0.4522
2.2	6.8	0.6745	1.8321	0.3279	0.1938	1.202	0.3271	0.2	0.8	0.5021
2.5	6.5	0.7836	1.7827	0.3431	0.2044	1.0252	0.3762	0.25	0.75	0.5385
2.8	6.2	0.8878	1.7309	0.3492	0.2159	0.8699	0.4333	0.3	0.7	0.5643
3	6	0.9543	1.6949	0.3491	0.224	0.7782	0.4763	0.35	0.65	0.582
3.2	5.8	1.0185	1.6576	0.3463	0.2324	0.6956	0.5236	0.4	0.6	0.5924
3.4	5.6	1.0802	1.619	0.3415	0.2413	0.6216	0.5757	0.45	0.55	0.5963
3.6	5.4	1.1395	1.5789	0.335	0.2504	0.5558	0.6327	0.5	0.5	0.5943
3.8	5.2	1.1965	1.5374	0.3272	0.2599	0.4977	0.6953	0.55	0.45	0.5866
4	5	1.2512	1.4942	0.3185	0.2696	0.4468	0.7638	0.6	0.4	0.5736
4.2	4.8	1.3038	1.4494	0.3091	0.2795	0.4026	0.8388	0.65	0.35	0.5552
4.4	4.6	1.3543	1.4028	0.2994	0.2895	0.3645	0.9207	0.7	0.3	0.5314
4.6	4.4	1.4028	1.3543	0.2895	0.2994	0.3322	1.01	0.75	0.25	0.5016
4.8	4.2	1.4494	1.3038	0.2795	0.3091	0.3051	1.1074	0.8	0.2	0.4656
5	4	1.4942	1.2512	0.2696	0.3185	0.2829	1.2134	0.85	0.15	0.4225
5	4	1.4942	1.2512	0.2696	0.3185	0.2829	1.2134	0.9	0.1	0.3759

V. Conclusion

This paper deals with congestion control in wired networks. Our mathematical model bases on Kelly's model with some extensions. First, we use stochastic method that makes our model more practical. Second, we minimize *MSE*, i.e., we regulate between high network utility and small network utility variance. Third, the problem is formulated as a multiple objective optimization; therefore, we can adjust user priority flexibly by assigning weight. The numerical example shows the efficiencies of our methods: robustness, high network utility, and flexibility.

Although this paper extends the simplest network optimization in wired networks but we believe that this method is suitable for other network optimization problems of other networks. To apply this method into other network optimization problems, we should consider more the characteristics of traffic (follows Poisson distribution or not), characteristics of networks (interference in wireless networks, multiple channels, multiple radios ...). These issues are very interesting topic for future works. Furthermore, an efficient mathematical (distributed, low complex) should be more investigated.

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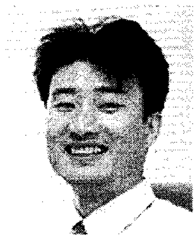

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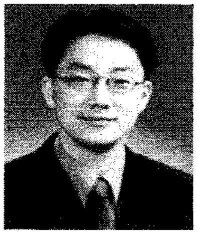

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