LDPC Codes' Upper Bounds over the Waterfall Signal-to-Noise Ratio (SNR) Region

Kyuhyuk Chung* *Regular Member*

ABSTRACT

This paper presents LDPC codes' upper bounds over the waterfall SNR region. The previous researches have focused on the average bound or ensemble bound over the whole SNR region and showed the performance differences for the fixed block size. In this paper, the particular LDPC codes' upper bounds for various block sizes are calculated over the waterfall SNR region and are compared with BP decoding performance. For different block sizes the performance degradation of BP decoding is shown.

Key Words: Upper bounds, LDPC codes, BP decoding, Weight distributions, Simple bound

I. Introduction

Iterative decoding [1]-[5] for low-density parity-check (LDPC) codes represents a great advancement in communications theory because of their excellent performance. LDPC codes [1], [2] are preferred because of advantage for efficient parallel hardware implementation as well as their excellent performance. Numerous simulations and bounds have demonstrated their remarkable performance.

Recently a proposed simple bound technique [6] showed a tight upper bound on the performance of repeat accumulate codes and LDPC codes by using an ensemble input-output weight distribution based on the uniform interleaver assumption above the cutoff rate.

One previous research [7] stated the upper bound for an LDPC code with a particular parity check matrix using the simple bound and estimated input-output weight distributions. In that paper, however, the upper bounds were calculated for the fixed size and the performance degradation for diffenrent block sizes was relatively neglected.

Therefore, this paper calculates the upper bounds for particular LDPC codes with various block sizes over the waterfall SNR region using the bound technique based on the simple bound [6] with an estimated weight distribution [7]. These bounds show how much iterative decoding, i.e., the Belief-Propagation (BP) decoding[1], [3], is suboptimal from the maximum likelihood (ML) decoding. The remainder of this paper is organized as follows. In Section II, we describe bounding techniques used in this paper. In Section III, we present upper bounds for LDPC codes with particular parity check matrices over the waterfall SNR region. In Section IV, we conclude the paper.

II. Simple Bounding Technique and Estimated Input-Output Distributions

The performance of LDPC codes is close to Shannon's channel capacity limit for moderate to large block sizes, so there is a need for bounds on performance that are useful for rates above the cutoff rate. In [6] such a simple bound on the probability of decoding error for block codes above the cutoff rate is derived in a closed form.. Divsalar's bound is simple because it does not require any integration or optimization in its final

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* 단국대학교 공과대학 컴퓨터학부 컴퓨터공학전공 (khchung@dankook.ac.kr)
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version and the tightness is compared with the existing tight bounds using thresholds c_0 as follows[6].

$$c_0(Divsalar) = c_0(Poltyrev) = c_0(Dolinar)$$

$$= c_0(Duman) \leq c_0(Viterbi) = c_0(Berlekamp)$$

Consider a linear binary (N, K) block code C, where N is the codeword length and K the information frame length.

For a given code, d is the Hamming weight of a codeword and $Q(\cdot)$ is the complementary unit variance Gaussian distribution function. The upper bound [6] on the bit error rate (BER) with ML codeword decoding is given by

$$P_b \leq \sum_{d=d_{\min}}^{N-K+1} \min\{e^{-NE(c,d)}, e^{Ng(\delta)}Q(\sqrt{2cd})\}.$$

In the above inequality, the exponent $E(c, d)$ is defined as follows.

If $c_0(\delta) < c < \frac{e^{2g(\delta)}-1}{2\delta(2-\delta)}$,

$$E(c,d) \equiv \frac{1}{2}\ln[1-2c_0(\delta)f(c,\delta)] + \frac{cf(c,\delta)}{1+f(c,\delta)},$$

otherwise $E(c, d) \equiv -g(\delta) + \delta c$.

Parameters are defined as $\delta \equiv d/N$, $c \equiv r(E_b/N_0)$ with E_b being the energy per information bit and N_0 being the one-sided noise spectral density,

$$c_0(\delta) \equiv (1-e^{-2g(\delta)})\frac{1-\delta}{2\delta},$$

$$f(c,\delta) \equiv \sqrt{\frac{c}{c_0(\delta)} + 2c + c^2} - c - 1,$$

and

$$g(\delta) \equiv \frac{1}{N}\ln\left\{\sum_w \frac{w}{K} A_{w,d}\right\}$$

In the above definitions, $d_{\min}$ is the minimum distance of the code and $A_{w,d}$ denotes the number of codewords for an input sequence weight w and output codeword weight d. In order to apply this simple bound to a particular code, the input-output weight distribution $A_{w,d}$ should be obtained for that particular code, which is usually very complicated. Therefore, an upper bound is obtained using an ML estimated input-output weight distribution [7], which is given by

$$\hat{A} = \binom{K}{w}\frac{k}{N_s}$$

where k is the number of codewords with the Hamming weight d for input sequences of the Hamming weight w among N_s generated sample codewords. In order to calculate an ML estimated input-output weight distribution, a specific encoder is requred. Using Gaussian elimination, it is possible to obtain LDPC encoders. Sample codewords are randomly generated.

Ⅲ. Upper Bounds and Simulation Results

We consider general regular LDPC codes. The parity check matrices are generated randomly. The number of 1's per column in the parity check matrices is 3 and the number of 1's per row in the parity check matrices is 6. The code block sizes (N, K) are (2000,1000), (1000,500) and (500,250). The rate for these LDPC codes is 1/2. In Figure 1, the upper bounds are compared with the Bit-Error Rate (BER) curves of the iterative decoding performance at the fixed iterations of 50

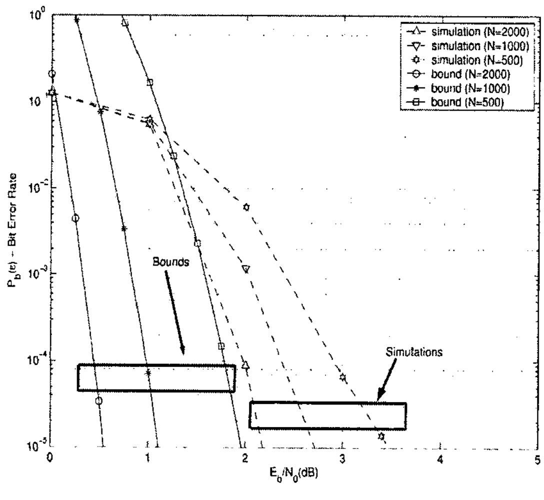

Fig. 1. Comparison of upper bounds with BP decoding performances for (N, K)=(2000,1000), (1000,500), and (500,250). (Solid lines are bounds and dashed lines are simulations.)

Table 1. Comparison of upper bounds to BP decoding performances in E_b/N_0 dB required for BER= 10^{-5}

N	2000	1000	500
Bound	0.55	1.12	1.96
BP	2.12	3.08	4.06
Degradation	1.57	1.96	2.10

for three code block sizes using the BP decoding algorithm [1],[2]. For an ML estimated input-output weight distribution, the number of randomly generated sample codewords N_s is 10000 for each $w=1,2,\cdots,K$. It is interesting that the slopes of the bound curves are steeper than those of BP decoding curves. This implies that it is not easy for BP decoding to approach ML decoding even lower BERs with the given block sizes. In Table 1, at a BER of 10^{-5}, specific E_b/N_0 dB performance degradation of BP decoding from bounds is shown for various block sizes. It is observed that as a block length becomes larger, performance degradation becomes smaller. BP decoding is currently worse than the proposed upper bound and no wonder, even worse than ML decoding performance.

Ⅳ. Conclusion

This paper presents particular LDPC codes' upper bounds for various block sizes over the waterfall SNR region. These bounds are compared with BP decoding performance. It is shown by simulation results that for various block sizes, how much BP decoding performance is suboptimal from ML decoding performance. When ML decoding is possible, specific E_b/N_0 dB gains from BP decoding are predicted for various block sizes. Based on the results, it is conjectured that for shorter lengths, the gain is larger. For future research, it might be meaningful to find out how long the block size is required to approach ML decoding significantly with BP decoding.

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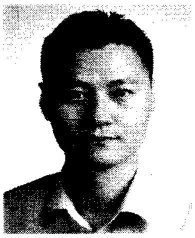
정 규 혁 (Kyuhyuk Chung) 정회원



1997년 2월 성균관대학교 전자공학과 졸업

1998년 12월 University of Southern California 전기공학 석사

2003년 12월 Univ. of Southern California 전기공학 박사

1999년 8월~2000년 5월 미국 Integrated Device Technology, Inc., Member of Technical Staff

2001년 5월~2002년 5월 미국 TrellisWare Technology, Inc., Senior Engineer

2004년 2월~2005년 8월 LG전자 이동통신기술연구소 표준화그룹 선임연구원

2005년 9월~현재 단국대학교 공과대학 컴퓨터학부 컴퓨터공학전공 조교수

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