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A Weak Convergence of the Linear Random Field Generated by Associated Randomvariables $\mathbb{Z}^{2\dagger}$

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Abstract

In this paper we show the weak convergence of the linear random(multistochastic process) field generated by identically distributed 2-parameter array of associated random variables. Our result extends the result in Newman and Wright (1982) to the linear 2-parameter processes as well as the result in Kim and Ko (2003) to the 2-parameter case.

Keywords: Weak convergence; linear random field; associated; maximal inequality; two-parameter process.

1. Introduction

A finite collection of random variables, $Y_1, \ldots, Y_n$ is said to be associated if for any two coordinatewise nondecreasing functions f, g on $\mathbb{R}^n$,

$$\mathrm{Cov}(f(Y_1, \ldots, Y_n),\ g(Y_1, \ldots, Y_n)) \geq 0, \tag{1.1}$$

whenever the covariance is defined. An infinite collection is associated if every finite subcollection is associated. This definition was introduced by Esary *et al.* (1967) and has found several applications in reliability theory (see, Barlow and Proschan, 1975). The basic concept actually appears in Harris (1960) in the context of percolation models and it was subsequently applied to the Ising models of statistical mechanics in Fortuin *et al.* (1971). In the statistical mechanics literature (see, *e.g.*, Lebowitz, 1972), which developed independently of reliability theory, associated random variables are said to satisfy the FKG inequalities.

One of the results originating in statistical mechanics which is of particular probabilistic interest concerns a central limit theorem for certain stationary p-parameter arrays, $\{\xi(t_1, \ldots, t_p),\ (t_1, \ldots, t_p) \in \mathbb{Z}^p\}$ of associated random variables (see, Newman, 1980). Let $\{\xi(t_1, \ldots, t_p),\ (t_1, \ldots, t_p) \in \mathbb{Z}^p\}$ be a strictly stationary p-parameter array of finite variance, mean zero, associated random variables such that

$$\sigma^2 = \sum_{(t_1, \ldots, t_p) \in \mathbb{Z}^p} \mathrm{Cov}\big(\xi(0, \ldots, 0),\ \xi(t_1, \ldots, t_p)\big) < \infty. \tag{1.2}$$

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For $0 \le r_1, \ldots, r_p \le 1$, define

$$W_n(r_1, \ldots, r_p) = \frac{1}{\sigma n^{\frac{p}{2}}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p),$$

where $[\cdot]$ denotes the usual greatest integer function and let $W(r_1, \ldots, r_p)$ be the p-parameter Wiener process, a mean zero Gaussian process with

$$\mathrm{Cov}\big(W(r_1, \ldots, r_p),\ W(s_1, \ldots, s_p)\big) = \Pi_{j=1}^{p} \min(r_j,\ s_j). \tag{1.3}$$

Newman and Wright (1982) showed that the finite dimensional distributions of W_n converges in distribution to those of W.

The linear processes are of special importance in time series analysis and they arise in a wide variety of contexts (see, *e.g.*, Hannan, 1970, Chapter 6). Applications to economics, engineering and physical sciences are extremely broad and a vast amount of literature is devoted to the study of theorems for linear processes under various assumptions on random variables.

Kim and Ko (2003) showed a weak convergence of the stationary linear process generated by associated sequence for the case $p = 1$ and extended the result in Newman and Wright (1981) to the linear process.

We are interested in a weak convergence for a linear random fields(multipameter stochastic processes) on the lattice $\mathbb{Z}_+^2$.

In this paper we extend the weak convergence of 2-parameter arrays of associated random variables in Newman and Wright (1982) to the linear random fields by using the generalized Beveridge - Nelson decomposition (see, Marinucci and Poghosyan, 2001; Phillips and Solo, 1992) and the maximal inequality (see, the proof of Theorem 10 in Newman and Wright, 1982).

2. Decomposition of Bivariate Polynomials

Define a linear random field(multiparameter stochastic process) on $\mathbb{Z}^2$, by

$$X(t_1,\ t_2) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a(i_1,\ i_2)\xi(t_1 - i_1,\ t_2 - i_2), \quad (t_1,\ t_2) \in \mathbb{Z}^2, \tag{2.1}$$

where $\{\xi(t_1,\ t_2)\}$ is a 2-parameter array of identically distributed and associated random variables with $E\xi(t_1,\ t_2) = 0$ and $E(\xi(t_1,\ t_2))^2 < \infty$ and the real number

$$a(i_1, i_2) \ge 0, \quad \text{for all } (i_1,\ i_2), \quad i_1, i_2 \in \mathbb{N} \cup \{0\}. \tag{2.2}$$

To consider the decomposition of bivariate polynomials (see, Marinucci and Poghosyan, 2001) put

$$A(x_1,\ x_2) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a(i_1,\ i_2) x_1^{i_1} x_2^{i_2}, \quad (x_1,\ x_2) \in \mathbb{R}^2, \tag{2.3}$$

where $|x_i| \leq 1,\ i = 1, 2$ and

$$\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{k_1=i_1+1}^{\infty}\sum_{k_2=i_2+1}^{\infty} a(k_1,\, k_2) < \infty. \tag{2.4}$$

Note that (2.4) implies

$$A(1,\, 1) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a(i_1,\, i_2) < \infty.$$

The following lemma extends a result known for $p = 1$ as the Beveridge-Nelson decomposition (*cf.* Phillips and Solo, 1992) to the case $p = 2$.

Lemma 2.1 (Marinucci and Poghosyan, 2001) Let Γ be the class of all subsets γ of $\{1, 2\}$. Let $y_j = x_j$ if $j \in \gamma$ and $y_j = 1$ if $j \notin \gamma$. Then we have

$$A(x_1,\, x_2) = \sum_{\gamma\in\Gamma} \{\Pi_{j\in\gamma}(x_j - 1)\}\, A_\gamma(y_1,\, y_2),$$

where $\Pi_{j\in\phi} = 1$ and

$$A_\gamma(y_1,\, y_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a_\gamma(i_1,\, i_2) y_1^{i_1} y_2^{i_2}, \tag{2.5}$$

$$a_\gamma(i_1,\, i_2) = \sum_{s_1=i_1+1}^{\infty}\sum_{s_2=i_2+1}^{\infty} a(s_1,\, s_2), \tag{2.6}$$

where the sums go over indices s_j, $j \in \gamma$, where as $s_j = i_j$ if $j \notin \gamma$.

It follows from (2.3), (2.5) and (2.6) that $A(1,\, 1) = A_\varnothing(1,\, 1)$.

Let $A_{\{1\}} = A_1,\ A_{\{2\}} = A_2$ and $A_{\{1.2\}} = A_{12}$.

In other words, we have

$$A(x_1,\, x_2) = A(1,\, x_2) + (x_1 - 1)A_1(x_1,\, x_2),$$
$$A(1,\, x_2) = A(1,\, 1) + (x_2 - 1)A_2(1,\, x_2),$$
$$A_1(x_1,\, x_2) = A_1(x_1,\, 1) + (x_2 - 1)A_{12}(x_1,\, x_2),$$

where

$$A_1(x_1,\, x_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{k_1=i_1+1}^{\infty} a(k_1,\, i_2) x_1^{i_1} x_2^{i_2},$$

$$A_{12}(x_1,\, x_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{k_1=i_1+1}^{\infty}\sum_{k_2=i_2+1}^{\infty} a(k_1,\, k_2) x_1^{i_1} x_2^{i_2},$$

hence

$$A(x_1,\, x_2) = A(1,\, 1) + (x_1 - 1)A_1(x_1,\, 1) + (x_2 - 1)A_2(1,\, x_2) + (x_1 - 1)(x_2 - 1)A_{12}(x_1,\, x_2).$$

As in Marinucci and Poghosyan (2001) we also consider the partial backshift operator satisfying

$$B_1\xi(t_1, t_2) = \xi(t_1 - 1, t_2) \text{ and } B_2\xi(t_1, t_2) = \xi(t_1, t_2 - 1), \tag{2.7}$$

which enables us to write (2.1) more compactly as

$$\begin{aligned} X(t_1, t_2) &= \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a(i_1, i_2) B_1^{i_1} B_2^{i_2} \xi(t_1, t_2) \\ &= A(B_1, B_2)\xi(t_1, t_2), \end{aligned} \tag{2.8}$$

where

$$A(B_1, B_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a(i_1, i_2) B_1^{i_1} B_2^{i_2}.$$

The above ideas shall be exploited here to establish the weak convergence of the linear field on $\mathbb{Z}^2$. To this aim, we write

$$\xi_\gamma(t_1, t_2) = A_\gamma(L_1, L_2)\xi(t_1, t_2), \tag{2.9}$$

where the operator L_i is defined as $L_i = B_i$ for $i \in \gamma$, $L_i = 1$ otherwise; that is

$$\begin{aligned} \xi_1(t_1, t_2) &= A_1(B_1, 1)\xi(t_1, t_2), \\ \xi_2(t_1, t_2) &= A_2(1, B_2)\xi(t_1, t_2), \\ \xi_{12}(t_1, t_2) &= A_{12}(B_1, B_2)\xi(t_1, t_2). \end{aligned}$$

Remark 2.1 Note that from (2.2), (2.4) and (2.6) we have

$$0 \le \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a_\gamma(i_1, i_2) < \infty. \tag{2.10}$$

Remark 2.2 Note that $\xi_\gamma(t_1, t_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} a_\gamma(i_1, i_2)\xi(t_1 - i_1, t_2 - i_2)$ by (2.5), (2.7) and (2.9) and that $\xi_\gamma(t_1, t_2)'$s are associated by the properties of association since $a_\gamma(i_1, i_2) \ge 0$ (see, Esary *et al.*, 1967).

3. Results

The following Lemma is a moment maximal inequality for associated random variables on $\mathbb{Z}^2$:

Lemma 3.1 (Newman and Wright, 1982) Let $\{\xi(t_1, t_2), (t_1, t_2) \in \mathbb{Z}^2\}$ be a 2-parameter array of mean zero, finite variance, associated random variables. Then we have

$$E\left(\max_{1\le k_1\le m, 1\le k_2\le n} \sum_{t_1=1}^{k_1}\sum_{t_2=1}^{k_2} \xi(t_1, t_2)\right)^2 \le E(S_{m,n}^2), \tag{3.1}$$

where $S_{m,n} = \sum_{t_1=1}^{m}\sum_{t_2=1}^{n} \xi(t_1, t_2)$.

Theorem 3.1 (Newman and Wright, 1982) Let $\{\xi(t_1, t_2), (t_1, t_2) \in \mathbb{Z}^2\}$ be a strictly stationary 2-parameter array of finite variance, mean zero, associated random variables such that

$$\sigma^2 = \sum_{(t_1, t_2) \in \mathbb{Z}^2} \operatorname{Cov}\big(\xi(0, 0), \xi(t_1, t_2)\big) < \infty. \tag{3.2}$$

Then, for $0 \le r_1, r_2 \le 1$

$$W_n(r_1, r_2) \Rightarrow W(r_1, r_2),$$

where $W(\cdot, \cdot)$ denotes 2-parameter Wiener process, *i.e.*, a mean zero Gaussian process with covariance function satisfying

$$E\big(W(r_1, r_2)W(s_1, s_2)\big) = \min(r_1, s_1) \times \min(r_2, s_2)$$

and $\Rightarrow$ denotes weak convergence.

To prove main result we need the following lemmas.

Lemma 3.2 Let $\{\xi(t_1, t_2)\}$ be a 2-parameter array of identically distributed, mean zero associated random variables satisfying condition (3.2) in Theorem 3.1. Assume that (2.2) and (2.4) hold. Then

$$E(\xi_\gamma(t_1, t_2))^2 < \infty, \quad \text{for } \gamma \in \Gamma. \tag{3.3}$$

Proof: From Remarks in Section 2 we have

$$\begin{aligned}\xi_\gamma(0, 0) &= \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a_\gamma(i_1, i_2)\xi(-i_1, -i_2) \\ &= \sum_{i=0}^{\infty} a_\gamma(\phi(i))\xi(-\phi(i)),\end{aligned}$$

where $\phi : \mathbb{Z} \to \mathbb{Z}^2$ and $\{\xi(-\phi(i))\}$ is a sequence of identically distributed and associated random variables. Hence,

$$\begin{aligned}E\big(\xi_\gamma(t_1, t_2)\big)^2 &= E\big(\xi_\gamma(0, 0)\big)^2 \\ &= E\left(\sum_{i=0}^{\infty} a_\gamma(\phi(i))\xi(-\phi(i))\right)^2 \\ &= \left[\left\{E\left(\sum_{i=0}^{\infty} a_\gamma(\phi(i))\xi(-\phi(i))\right)^2\right\}^{\frac{1}{2}}\right]^2 \\ &\le \left[\sum_{i=0}^{\infty} a_\gamma(\phi(i))\left\{E\big(\xi(-\phi(i))\big)^2\right\}^{\frac{1}{2}}\right]^2 \\ &\le C\left[\sum_{i=0}^{\infty} a_\gamma(\phi(i))\right]^2 \\ &< \infty \text{ by } (2.10),\end{aligned}$$

where the first bound follows from Minkowski's inequality and the second bound from condition (3.2). □

From Lemmas 3.1 and 3.2 we also have:

Lemma 3.3 Let $\{\xi(t_1, t_2), (t_1, t_2) \in \mathbb{Z}^2\}$ be a 2-parameter array of identically distributed, mean zero associated random variables satisfying (3.2) in Theorem 3.1. Assume that (2.2) and (2.4) hold. Then

$$E\left(\max_{1\le k_1\le m, 1\le k_2\le n} \sum_{t_1=1}^{k_1}\sum_{t_2=1}^{k_2} \xi_\gamma(t_1, t_2)\right)^2 \le E(S_{m,n})^2,$$

where $\xi_\gamma(t_1, t_2)$ is defined in (2.9) and $S_{m,n} = \sum_{t_1=1}^{m}\sum_{t_2=1}^{n} \xi(t_1, t_2)$.

The following theorem is the main result.

Theorem 3.2 Let $X(t_1,t_2)$ be defined as in (2.1) and $\{\xi(t_1,t_2), (t_1,t_2) \in \mathbb{Z}^2\}$ a 2-parameter array of identically distributed, finite variance, mean zero, associated random variables satisfying the condition (3.2) of Theorem 3.1. Assume that (2.2) and (2.4) hold.

Then, for $0 \le r_1, r_2 \le 1$

$$\frac{1}{\sigma n}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} X(t_1, t_2) \Rightarrow A(1, 1)W(r_1, r_2), \tag{3.4}$$

where $\sigma^2 = \sum_{(t_1, t_2)\in\mathbb{Z}^2} \mathrm{Cov}\big(\xi(0, 0), \xi(t_1, t_2)\big) < \infty$.

Corollary 3.1 Let $X(t_1, t_2)$ satisfy model (2.1) and $\{\xi(t_1, t_2)\}$ a 2-parameter array of identically distributed and associated random variables with $E\xi(t_1, t_2) = 0$, $E(\xi(t_1, t_2))^2 < \infty$. If $a(i_1, i_2) = 1$ for $i_1 = i_2 = 0$, $a(i_1, i_2) = 0$ otherwise, then (3.4) holds.

Example 3.1 Let $A(x_1, x_2) = 1 + x_1 + x_1x_2 + x_2^2$ and let

$$\begin{aligned} X(t_1, t_2) &= \xi(t_1, t_2) + \xi(t_1, -1, t_2) + \xi(t_1 - 1, t_2 - 1) + \xi(t_1, t_2 - 2) \\ &= A(B_1, B_2)\xi(t_1, t_2) \end{aligned}$$

for $A(B_1, B_2) = 1 + B_1 + B_1B_2 + B_2^2$. Then $A(1, 1) = 4$ and Theorem 3.2 implies, as $n \to \infty$,

$$(\sigma n)^{-1}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} X(t_1, t_2) \Rightarrow 4W(r_1, r_2).$$

Remark 3.1 Note that if $a(i_1, i_2) = 1$ for $i_1 = i_2 = 0$, $a(i_1, i_2) = 0$ otherwise, then $X(t_1, t_2) = \xi(t_1, t_2)$.

Remark 3.2 Note that Corollary 3.1 is a special case of Theorem 3.2. Hence Theorem 3.2 is an extension of Theorem 3.1.

4. Proof of Theorem 3.2

Proof: From Theorem 3.1 we have

$$\frac{1}{\sigma n}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]}\xi(t_1,t_2) \Rightarrow W(r_1,r_2). \tag{4.1}$$

From condition (3.2) and Lemma 3.3, there exists a positive constant C such that

$$E\left(\max_{1\le k_1\le m,\ 1\le k_2\le n}\sum_{t_1=1}^{k_1}\sum_{t_2=1}^{k_2}\xi_\gamma(t_1,\, t_2)\right)^2 \le Cmn. \tag{4.2}$$

If we apply Lemma 2.1 to the backshift binomial $A(B_1,\, B_2)$, then the following equality holds almost surely:

$$\begin{aligned}X(t_1,\, t_2) &= A(1,\, 1)\xi(t_1,\, t_2) + (B_1-1)A_1(B_1,\, 1)\xi(t_1,\, t_2)\\ &\quad + (B_2-1)A_2(1,\, B_2)\xi(t_1,\, t_2) + (B_1-1)(B_2-1)A_{12}(B_1,\, B_2)\xi(t_1,\, t_2)\end{aligned}$$

which implies that, for $0 \le r_1,\, r_2 \le 1$

$$\begin{aligned}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]}X(t_1\,,t_2) &= \sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]}A(1,\, 1)\xi(t_1,\, t_2) - \sum_{t_2=1}^{[nr_2]}\xi_1\left([nr_1],\, t_2\right) + \sum_{t_2=1}^{[nr_2]}\xi_1(0,\, t_2)\\ &\quad - \sum_{t_1=1}^{[nr_1]}\xi_2(t_1,\, [nr_2]) + \sum_{t_1=1}^{[nr_1]}\xi_2(t_1,\, 0) - \xi_{12}(0,\, [nr_2]) + \xi_{12}(0,\, 0)\\ &\quad - \xi_{12}([nr_1],\, 0) + \xi_{12}([nr_1],\, [nr_2])\\ &= \sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]}A(1,\, 1)\xi(t_1,\, t_2) + R_n(r_1,\, r_2).\end{aligned} \tag{4.3}$$

Note that $\xi_1(\cdot,\cdot)$, $\xi_2(\cdot,\cdot)$ and $\xi_{12}(\cdot,\cdot)$ are associated (see, Remarks in Section 2).

From Markov's inequality and (4.2), for $0 \le r_1,\, r_2 \le 1$,

$$\begin{aligned}P\left\{\max_{0\le r_1,r_2\le 1} n^{-1}\left|\sum_{t_2=1}^{[nr_2]}\xi_1\left([nr_1],\, t_2\right)\right| > \delta\right\} &\le \frac{E\max_{0\le r_1,r_2\le 1}\left(\sum_{t_2=1}^{[nr_2]}\xi_1([nr_1],\, t_2)\right)^2}{n^2\delta^2}\\ &\le Cn^{-1} = o(1),\end{aligned} \tag{4.4}$$

as $n \to \infty$. We can also apply exactly the same argument to establish

$$P\left\{\max_{0\le r_1,\, r_2\le 1} n^{-1}\left|\sum_{t_1=1}^{[nr_1]}\xi_2(t_1,\, [nr_2])\right| > \delta\right\} = o(1), \quad \text{as } n \to \infty. \tag{4.5}$$

By Lemma 3.2 we have for $0 \le r_1,\, r_2 \le 1$

$$E\big(\xi_{12}([nr_1],\, [nr_2])\big)^2 < \infty$$

and hence by the same argument as above we also have

$$P\left\{\max_{0\le r_1, r_2\le 1} n^{-1}\,|\xi_{12}([nr_1],\, [nr_2])| > \delta\right\} = o(1), \quad \text{as } n \to \infty. \tag{4.6}$$

Thus, we have

$$\sup_{0\le r_1,\, r_2\le 1} |n^{-1}R_n(r_1,\, r_2)| = o_p(1),$$

which yields

$$(\sigma n)^{-1}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} X(t_1,\, t_2) \Rightarrow A(1,\, 1)W(r_1,\, r_2), \quad \text{as } n \to \infty$$

by Theorem 4.1 of Billingsley (1968). □

5. Concluding Remarks

It seems interesting to consider the possibility to apply the same ideas for the case where the innovations $\xi(t_1,\, t_2)$ have a martingale-difference structure of dependence. This issue also will be investigated.

In this paper we consider a weak convergence for the linear random field generated by associated random variables for the case $p = 2$ under finite second moment condition. In the future we will study weak convergence for the linear random fields for the case $p \ge 3$ under the stronger moment condition by using invariance principle for p-parameter arrays of associated random variables in Bulinski and Keane (1996).

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