Unit Commitment by Separable Augmented Lagrangian Relaxation

Guk-Hyun Moon*, Sung-Kwan Joo*, Kisung Lee† and Jae-Seok Choi**

Abstract – The non-separable quadratic penalty terms create an inherent difficulty when applying the standard augmented Lagrangian relaxation (ALR) method for decomposing the unit commitment problem into independent subproblems. This paper presents a separable augmented Lagrangian relaxation method for solving the unit commitment problem. The proposed method is designed to have a separable structure by introducing the quadratic terms with additional auxiliary terms in the augmented Lagrangian function. Numerical results are presented to validate the effectiveness of the proposed method.

Keywords: dynamic programming, Lagrangian relaxation, unit commitment

1. Introduction

The objective of the unit commitment problem is to find the least-cost option to schedule generator startups and shutdowns while meeting forecasted loads and various generator operating constraints over a given period of time. The unit commitment problem is a complex mixed-integer nonlinear optimization problem which involves a large number of continuous or discrete decision variables and constraints, and complex inter-temporal constraints such as generator minimum up time and minimum down time restrictions.

A survey of unit commitment techniques is provided in [1]. There have been various attempts to solve the unit commitment problem using priority list [2], dynamic programming [2-4], integer and linear programming, mixed-integer programming [5] and the Lagrangian relaxation optimization technique [6,7]. The Lagrangian relaxation optimization technique was initially proposed in [6] in the late 1970's for the unit commitment problem. The use of the Lagrangian relaxation technique allows one to separate the original unit commitment problem into independent subproblems by relaxing the coupling constraints. It has been recognized that the solution obtained by classical Lagrangian relaxation (CLR) method may oscillate around the optimal solution since the duality gap is not reduced to zero in non-convex problems like the unit commitment problem.

The augmented Lagrangian relaxation (ALR) method [10], which augments the Lagrangian function with additional quadratic penalty terms, is proposed to overcome the solution oscillation problem caused by the non-zero duality gap in non-convex problems. However, the quadratic terms added in the Lagrangian function is not separable over an individual generator. The non-separable quadratic penalty terms create an inherent difficulty when applying the ALR method for decomposing the unit commitment problem into independent subproblems.

This paper presents a new augmented Lagrangian relaxation method to solve the unit commitment problem. The proposed method is designed to have a separable structure by introducing the quadratic terms with additional auxiliary terms in the augmented Lagrangian function.

This paper is organized as follows. In Section 2, the unit commitment problem is formulated with the CLR method. Section 3 presents the proposed ALR method to solve the unit commitment problem. Finally, numerical simulation results are presented for validation of the proposed method in Section 4.

2. Problem Formulation

In this section, the unit commitment problem is formulated as a mixed-integer nonlinear optimization problem. The unit commitment is a problem of determining the most economic on/off status of generating units subject to various technical constraints in order to meet forecasted loads over a time horizon. The unit commitment problem can be formulated as the following optimization problem:

$$\min_{P_i^t, U_i^t} \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_i^t) + ST_i^t(1 - U_i^{t-1})] \cdot U_i^t \tag{1}$$

subject to:

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$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t = 0 \tag{2}$$

$$P_{i,\min}^t U_i^t \le P_i^t \le P_{i,\max}^t U_i^t \tag{3}$$

$$U_{i,t} = \begin{cases} 1, & if\ T_{i,on} < T_{i,up,} \\ 0, & if\ T_{i,off} < T_{i,down,} \\ 0\ or\ 1, & otherwise \end{cases} \tag{4}$$

where P_i^t represents the power output of unit i during time period t, and $F_i(P_i^t)$ denotes the fuel cost function of unit i to produce P_i^t. ST_i^t is the start up cost of unit i during period t, and U_i^t represents the on-off status (1 or 0) of the unit i during period t. Equations (2) and (3) represent load constraints and generators limits constraints, respectively. Equation (4) represents minimum up and down time constraints of generators. Note that load constraints are coupling constraints which need to be met across the units.

3. Unit Commitment by Lagrangian Relaxation Methods

3.1 Classical Lagrangian Relaxation (CLR) Technique

The classical Lagrangian relaxation (CLR) method attempts to solve the unit commitment problem by relaxing the coupling constraints (2). The coupling constraints with Lagrangian multipliers are added to the objective function (1) to form the following Lagrangian function:

$$L(P,U,\lambda) = \sum_{i=1}^{N}\sum_{t=1}^{T}[F_i(P_i^t) + ST_i^t(1-U_i^{t-1})]\cdot U_i^t + \sum_{t=1}^{T}\lambda^t\cdot(P_{load}^t - \sum_{i=1}^{N}P_i^t U_i^t) \tag{5}$$

The Lagrangian function can be re-written as follows:

$$L(P,U,\lambda) = \sum_{i=1}^{N}\sum_{t=1}^{T}\{F_i(P_i^t) + ST_i^t(1-U_i^{t-1}) - \lambda^t\cdot P_i^t\}\cdot U_i^t \tag{6}$$

where the constant term P_{load}^t can be dropped from the optimization problem.

The following decomposed terms can be minimized separately for each generating unit:

$$\min_{P_i^t, U_i^t} \sum_{t=1}^{T}\{F_i(P_i^t) + ST_i^t(1-U_i^{t-1}) - \lambda^t\cdot P_i^t\}\cdot U_i^t \tag{7}$$

In this research, the each unit's commitment scheduling solutions during the time steps are obtained by forward dynamic programming technique.

3.2 Separable Augmented Lagrangian Relaxation Technique

In this section, the unit commitment problem is relaxed by new augmented Lagrangian relaxation. Similar to the standard ALR method, an additional quadratic term is added to the objective function, but added quadratic term has a separable structure [9]. The following quadratic terms with additional auxiliary terms will be used to form the augmented Lagrangian function.

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{t=1}^{T}\beta\cdot(P_i^t U_i^t\cdot(P_i^t U_i^t - P_i^{t(n)}U_i^{t(n)}) + \sum_{i=1}^{N}\sum_{t=1}^{T}\beta\cdot(\overline{P}_i^t\overline{U}_i^t\cdot(\overline{P}_i^t\overline{U}_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)})\\&+c\cdot\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|P_i^t U_i^t - P_i^{t(n)}U_i^{t(n)}\right\|^2 + c\cdot\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|\overline{P}_i^t\overline{U}_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)}\right\|^2\\&+c\cdot\sum_{i=1}^{N}\sum_{t=1}^{T}\left\|P_i^t U_i^t - \overline{P}_i^t\overline{U}_i^t\right\|^2\end{aligned} \tag{8}$$

Then, the proposed augmented Lagrangian problem at (n+1)-th iteration is formulated as follows:

$$\begin{aligned}L_c(P,U,\overline{P},\overline{U},\lambda) :=& \sum_{i=1}^{N}\sum_{t=1}^{T}[F_i(P_i^t) + ST_i^t(1-U_i^{t-1})]\cdot U_i^t\\&+\sum_{t=1}^{T}\lambda^t\cdot(P_{load}^t - \sum_{i=1}^{N}(P_i^t U_i^t - \overline{P}_i^t\overline{U}_i^t)) + \sum_{t=1}^{T}\sum_{i=1}^{N}\beta\cdot(P_i^t U_i^t\cdot(P_i^t U_i^t - P_i^{t(n)}U_i^{t(n)})\\&+\sum_{t=1}^{T}\sum_{i=1}^{N}\beta\cdot(\overline{P}_i^t\overline{U}_i^t\cdot(\overline{P}_i^t\overline{U}_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)}) + c\cdot\sum_{t=1}^{T}\sum_{i=1}^{N}\left\|P_i^t U_i^t - P_i^{t(n)}U_i^{t(n)}\right\|^2\\&+c\cdot\sum_{t=1}^{T}\sum_{i=1}^{N}\left\|\overline{P}_i^t\overline{U}_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)}\right\|^2 + c\cdot\sum_{t=1}^{T}\sum_{i=1}^{N}\left\|P_i^t U_i^t - \overline{P}_i^t\overline{U}_i^t\right\|^2\end{aligned} \tag{9}$$

where $P_i^{t(n)}, U_i^{t(n)}, \overline{P}_i^{t(n)}, \overline{U}_i^{t(n)}$ are obtained at the previous n-th iteration.

The proposed augmented Lagrangian function is separated into sub-problem at the n-th iteration as follows:

$$\begin{aligned}L_c(P,U,\lambda) :=& \sum_{i=1}^{N}\sum_{t=1}^{T}[F_i(P_i^t) + ST_i^t(1-U_i^{t-1})]\cdot U_i^t\\&+\sum_{t=1}^{T}\sum_{i=1}^{N}\lambda_i^t\cdot P_i^t U_i^t + \sum_{t=1}^{T}\sum_{i=1}^{N}c\cdot\left\|P_i^t U_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)}\right\|^2\end{aligned} \tag{10}$$

where the constant term P_{load}^t can be dropped from the optimization problem.

The following separated terms can be minimized separately for each generating unit:

$$\begin{aligned}&\sum_{t=1}^{T}[\{F_i(P_i^t) + ST_i^t(1-U_i^{t-1})\}\cdot U_i^t\\&+\lambda_i^t\cdot P_i^t U_i^t + c\cdot\left\|P_i^t U_i^t - \overline{P}_i^{t(n)}\overline{U}_i^{t(n)}\right\|^2]\end{aligned} \tag{11}$$

In this research, the each unit's commitment scheduling solutions during the time steps are obtained by forward dynamic programming technique.

3.3 Algorithm for the proposed separable ALR method

In this section, the algorithm procedure of the proposed method for the unit commitment problem is presented. The proposed ALR algorithm can be summarized as follows:

-Step 1) Initialize the primal variables $P^{(0)}$ and $U^{(0)}$ with arbitrary values. Set the dual variables $\lambda^{(0)}$ to 0. Choose the algorithmic constant parameters β and c between 0 and 3.

-Step 2) Solve the following primal optimization sub-problem:

$$\min_{P_i,U_i^t} L_c(P,U,\lambda) \quad \text{subject to constraints (3) ,(4).}$$

Then, each unit's commitment scheduling solution is obtained by forward dynamic programming.

-Step 3) Update the dual variables by solving the dual optimization problem.

-Step 4) Stop if the computed dual gap is less than a specified tolerance value:

$$\frac{L_c(P^{t(n)},U^{t(n)},\lambda^{t(n)})-\phi_c(\lambda^{(n)})}{\phi_c(\lambda^{(n)})} \le \varepsilon \tag{12}$$

Otherwise, increase the count number (i.e., n = n+1) and move to Step 5.

-Step 5) Compute the step length for Lagrangian multiplier at n+1-th iteration using the GRS method described in [14]. The load mismatch information is used to calculate the step length for Lagrangian multiplier.

-Step 6) Update dual variables by using the following formula with the step length $\alpha^{t(n+1)}$ found in step 5:

$$\lambda^{t(n+1)} = \lambda^{t(n)} + \alpha^{t(n+1)} \cdot l^{t(n)} \tag{13}$$

Go back to Step 2 and repeat the algorithm.

4. Numerical Examples

The proposed separable ALR method has been tested on the modified IEEE-30 bus system with six units and the modified one-area Reliability Test System-96 with 32 units [15] in order to demonstrate its performance.

4.1 Case 1: Modified IEEE-30 Bus System

A modified IEEE-30 bus system with six units is developed for Case 1. Table 1 shows the characteristics and operating constraints of generators for Case 1. The hourly forecasted load information is also given in the second column of Table 2. The proposed ALR method is used to solve the unit commitment problem. For the purpose of comparison, the CLR method is applied to solve the same unit commitment problem. Using the forward dynamic programming technique, the optimal unit commitment schedules are determined for each generator. The duality gap tolerance, which is used for the stopping criterion, is set to $\varepsilon = 10^{-1}$ for Case 1.

Table 2 shows the results of the unit commitment schedule obtained by CLR and proposed separable ALR methods. Comparison of results obtained by the CLR and proposed separable ALR methods in Table 2 shows that several generators in time stages 3, 4, 21, and 22 have different commitment schedules. Fig. 1 shows the evolution of total costs computed by the CLR and proposed separable ALR methods during the iterative process. It can be seen from Table 3 that the total costs of solution obtained by the proposed separable ALR method

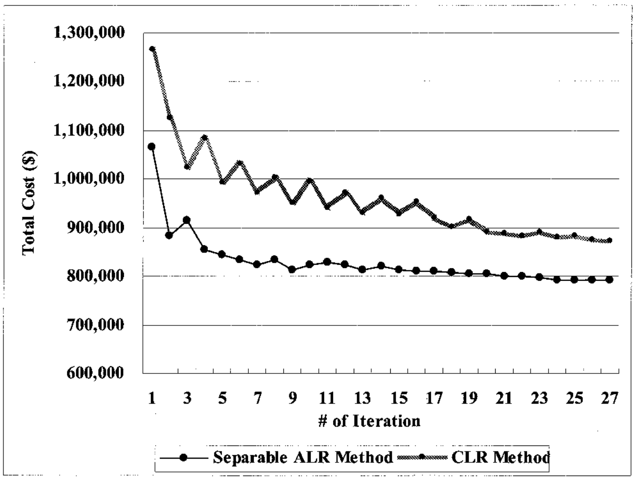

Fig. 1. Evaluation of total cost for Case 1

Table 1. Generator information for Case 1

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
a_i($/h)	400	350	600	700	650	650
b_i($/MWh)	15	14	11	10	8	10
c_i($/MW²h)	0.0025	0.0030	0.0009	0.0006	0.0005	0.0008
ST_i($)	750	800	1300	1500	1500	1100
$P_i^{\min}(MW)$	100	150	200	300	400	400
$P_i^{\max}(MW)$	500	550	800	1000	1200	700
min up time (h)	2	2	5	8	8	4
min down time (h)	3	2	4	5	7	4

Table 2. Solutions of unit schedule of 24 stages for Case 1

Stage (h)	Load (MW)	Unit (1=on, 0=off)					
		1	2	3	4	5	6
1	1700	0	1	0	1	1	0
2	1900	0	1	0	1	1	0
3	2000	0	0	0	1	1	1
4	1950	0	0	0	1	1	1
5	2200	0	0	0	1	1	1
6	2400	0	0	1	1	1	1
7	2800	0	0	1	1	1	1
8	3050	0	0	1	1	1	1
9	3100	0	0	1	1	1	1
10	3250	0	1	1	1	1	1
11	3300	1	1	1	1	1	1
12	3400	1	1	1	1	1	1
13	3500	1	1	1	1	1	1
14	3650	1	1	1	1	1	1
15	3550	1	1	1	1	1	1
16	3300	0	1	1	1	1	1
17	3100	0	0	1	1	1	1
18	2700	0	0	1	1	1	1
19	2600	0	0	0	1	1	1
20	2550	0	0	0	1	1	1
21	2300	0	0	0	1	1	1
22	2150	0	0	0	1	1	1
23	2000	0	1	0	1	1	0
24	1850	0	1	0	1	1	0

Table 3. Comparison of total costs for Case 1

CLR with SubGradient (a)	Proposed separable ALR (b)	Solution Improvement ((a-b)/a)
$872,445	$792,506	9.16%

are about 9.16% less than those obtained by the CLR method.

4.2 Case 2: Modified IEEE One-Area Reliability Test System-96

A modified one-area Reliability Test System-96 with 32 units is developed for Case 2. Table 4 shows the characteristics and constraints of generators for Case 2. The hourly forecasted load information is also given in the second column of Table 5. For Case 2, the duality gap tolerance is set to $\varepsilon = 2\times10^{-1}$. The solutions of the unit commitment schedule obtained by CLR and separable ALR methods are given in Table 5. Fig. 2 shows the evolution of total costs computed by the CLR and separable ALR methods for Case 2 during the iterative process. It can be seen from Table 6 that the total costs of solution obtained by the separable ALR method are about 8.76% less than those obtained by the CLR method.

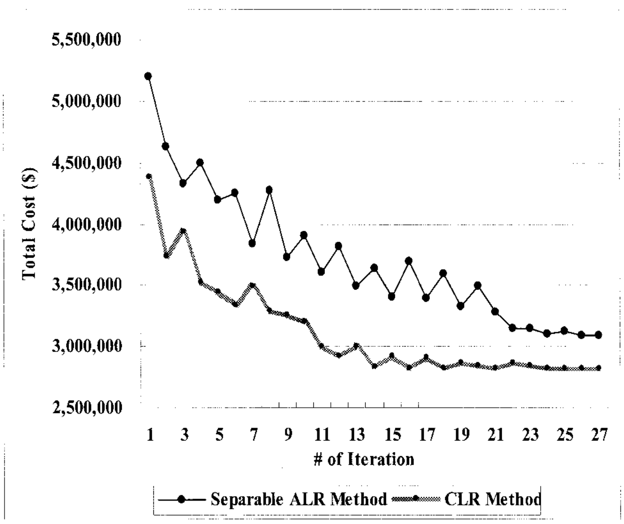

Fig. 2. Evaluation of total costs for Case 2

Table 4. Generator information for Case 2

	Unit 1-2	Unit 3-4	Unit 5-6	Unit 7-8	Unit 9-11	Unit 12-14
$a_i(\$/h)$	800	750	600	650	600	300
$b_i(\$/MWh)$	10	8	12	12	10	14
$c_i(\$/MW^2h)$	0.00045	0.00050	0.00085	0.00070	0.00095	0.0014
$ST_i(\$)$	1200	1050	900	950	700	300
$P_i^{min}(MW)$	170	200	180	120	150	80
$P_i^{max}(MW)$	920	700	760	500	400	330
min up time (h)	6	5	3	4	4	2
min down time (h)	5	4	4	4	3	2

	Unit 15-19	Unit 20-21	Unit 22-23	Unit 24-29	Unit 30-31	Unit 32
$a_i(\$/h)$	400	700	650	200	750	800
$b_i(\$/MWh)$	14	9	11	11	12	7
$c_i(\$/MW^2h)$	0.00125	0.00060	0.0008	0.0015	0.00065	0.0004
$ST_i(\$)$	300	950	850	800	1000	1300
$P_i^{min}(MW)$	100	200	220	100	150	300
$P_i^{max}(MW)$	450	600	650	300	700	1000
min up time (h)	1	4	3	1	5	7
min down time (h)	2	4	4	2	5	7

Table 5. Solutions of unit schedule of 24 stages for Case 2

Stage (h)	Load (MW)	Unit (1=on, 0=off) (1 - 32)
1	5500	1 1 1 0 1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 1 0 0 0 0 0 1 0 1
2	5700	1 1 1 0 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0 1 0 1
3	6500	1 1 1 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0 0 1 0 1
4	6200	1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1 1 0 0 1 1 0 0 1 1 0 0 0 0 1 0 1
5	7000	1 1 1 1 0 1 1 1 1 1 1 1 0 0 1 1 0 0 0 1 1 1 0 1 1 1 0 0 0 1 0 1
6	7500	1 1 1 1 0 0 1 0 1 1 1 0 0 0 1 1 0 0 0 1 1 1 0 1 1 1 0 0 0 1 0 1
7	9200	1 1 1 1 0 0 1 0 1 1 1 0 0 0 1 0 0 0 0 1 1 1 0 1 1 1 0 0 0 1 0 1
8	9500	1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 0 0 0 0 1 1 1 1 1 1 1 0 0 0 1 0 1
9	11000	1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 0 0 0 0 1 1 1 1 1 1 1 1 0 0 1 1 1
10	11600	1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1
11	12000	1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1
12	12500	1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1
13	14200	1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1
14	13800	1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1
15	13500	1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 1 1 1 1 1 0 0 0 1 1 1
16	12900	1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 0 0 1 1 1 1 1 1 0 0 0 0 1 1 1
17	12200	1 1 1 1 1 1 1 0 1 1 0 1 0 0 1 1 1 0 0 1 1 1 1 1 1 0 0 0 0 1 1 1
18	11500	1 1 1 1 0 0 1 0 1 1 0 1 0 0 1 1 0 0 0 1 1 1 0 1 1 0 0 0 0 1 1 1
19	9000	1 1 1 1 0 0 1 0 1 0 0 1 1 0 1 0 0 0 0 1 1 1 0 1 1 0 0 0 0 1 1 1
20	8500	1 1 1 1 0 0 1 0 1 0 0 0 1 0 1 0 0 0 0 1 0 0 0 1 1 0 0 0 0 1 1 1
21	8000	1 1 1 1 0 0 1 1 1 0 0 0 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 0 0 1 1 1
22	7600	1 1 1 1 1 1 1 1 0 0 0 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 1
23	6800	1 1 1 0 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 1 0 1
24	5900	1 1 1 0 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 1 0 1

Table 6. Comparison of total costs for Case 2

CLR with SubGradient (a)	Proposed separable ALR (b)	Solution Improvement ((a-b)/a)
$3,088,579	$2,817,947	8.76%

5. Conclusions

In the paper, the separable ALR-based decomposition method is presented to solve the unit commitment problem. The proposed ALR method is designed to have a separable structure by introducing the separable quadratic terms with additional auxiliary terms in the augmented Lagrangian function. It is demonstrated in the numerical examples that the proposed separable ALR method has a great potential for improving the solution of the unit commitment problem.

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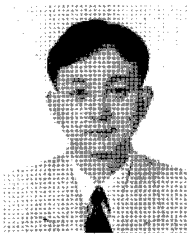

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