The Ramp-Rate Constraint Effects on the Generators' Equilibrium Strategy in Electricity Markets

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Abstract – In this paper, we investigate how generators' ramp-rate constraints may influence their equilibrium strategy formulation. In the market model proposed in this study, the generators' ramp-rate constraints are explicitly represented. In order to fully characterize the inter-temporal nature of the ramp-rate constraints, a dynamic game model is presented. The subgame perfect Nash equilibrium is adopted as the solution of the game and the backward induction procedure for the solution of the game is designed in this paper. The inter-temporal nature of the ramp-rate constraints results in the Markov property of the game, and we have found that the Markov property of the game significantly simplifies the subgame perfect Nash equilibrium characterization. Finally, a simple electricity market numerical illustration is presented for the successful application of the approach proposed.

Keywords: Dynamic game theory, Electricity markets, Markov property, Ramp rate constraints, Subgame perfect Nash equilibrium

1. Introduction

Recently, the electricity industry has undergone significant restructuring, and government-owned utilities around the world have become privatized. As restructuring and privatization continue, many studies have been performed considering the unique characteristics of electricity as well as electricity market economics.

Among the many physical characteristics of electricity, much research has been focused on the understanding of the role of transmission networks in a deregulated industry. Borenstein et al. have studied the competitive effects of a transmission line that connects two electricity markets [1]. They have shown that there may be no direct relationship between the competitive effect of a transmission line and the actual line flow. Moreover, with a sufficiently large capacity line, the full benefits of competition can be achieved even in cases where the equilibrium line flow is zero. For a sufficiently large line capacity, the market outcome is equivalent to the case where the markets are merged; that is, where there is unlimited capacity between the markets. Their works have also included an empirical analysis of the California electricity market modeled as a duopoly.

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Willems has studied a very similar market model to that of Borenstein et al. and investigated the role of the network operator for promoting competition among the generators [2]. Quick and Carey have applied the "dominant firm price-leadership model" to assess market power in Colorado's electricity industry and they have demonstrated that strategies exist to reduce the market power [3]. Leautier has studied regulatory contracts for the operators of transmission networks and has proposed a regulatory contract that induces network operators to "optimally" expand the grid [4]. Stoft has investigated market power issues when the generators serve a demand with capacity constrained transmission lines [5]. He has considered the effect on the market power of financial transmission rights (FTRs) and the resulting distribution of the congestion rent. Cho has investigated the competitive equilibrium in electricity markets over a network with finite capacity [6]. He has suggested a tool to check whether an equilibrium is efficient. He has also examined markets for firm transmission rights in a market with a specific structure.

Considering the numerous studies carried out on the transmission network constraints, the physical constraints of generators have been less studied in the context of markets. Baldick and Hogan have applied a supply function equilibrium model to analyze electricity markets with capacity constrained generators [7]. Arroyo and Conejo have described a market clearing tool which considers the minimum up and down time constraints of the generators [8].

One of the most important constraints that considerably affect a generator's economic production is the ramp- rate constraint. Wang and Shahidehpour have proposed an algorithm to solve unit commitment problems considering the ramp-rate constraint in the vertically integrated industry environment [9]. Lee et al. have presented a price-based ramp-rate model [10]. However, they have not considered the strategic interaction of the generators within the market. Shrestha et al. have studied the ramp-rate constraints in deregulated markets. Even though they have addressed the strategic dispatch decision, they have not considered the strategic interaction.

In this paper, therefore, a dynamic game model is proposed in order to consider the strategic interaction of a generator with the ramp-rate constraints. The solution of the game is obtained based on the subgame perfect Nash equilibrium concept [11]. Backward induction approach is adopted for determining the subgame perfect Nash equilibrium of the game. We have found that the inter-temporal nature of the ramp-rate constraints yields the Markov property of the game. By this, we can conclude in this paper that the Markov property of the game significantly reduces the complexity of the subgame perfect Nash equilibrium characterization. As an illustration for the approach proposed, a simple numerical example is presented.

This paper is organized as follows. Section II describes the electricity market model with explicit representation of the ramp rate constraints. In Section III, a dynamic game model is presented and the market equilibrium is analyzed. Section IV presents a simple numerical illustration of the approach. Finally, the conclusion is provided in Section V.

2. Electricity Market Model

In this paper, we have considered a series of electricity spot markets of which time index is denoted by $t \in \{1,2,\cdots,T\}$. There are N generators in the market and the production cost function $C_i : \Re_+ \to \Re_+$ of generator-i, where $i \in \{1,2,\cdots,N\}$, is of a quadratic form:

$$C_i(q_i) = \frac{1}{2} a_i q_i^2 + b_i q_i + c_i \tag{1}$$

where q_i is generator-i's production, and a_i, b_i, and c_i are parameters. Demand in the market at time-t is assumed to be characterized by an inverse-demand function denoted by $P_t : \Re_+ \to \Re_+$ and is represented by an affine curve with a negative slope:

$$P_t(q) = -\alpha_t q + \beta_t \text{, where } \alpha_t, \beta_t \in \Re_+ \tag{2}$$

where q is transaction quantity in the market. Let $q_{i,0}$ denote the initial production quantity of generator-i. Let also Δq_{iu} and Δq_{id} denote generator-i's ramp-up-rate and ramp-down-rate for the interval between two consecutive time indices, respectively. Then, generator -i's ramp-rate constraints can be written as:

$$q_{i,t-1} - \Delta q_{id} \le q_{i,t} \le q_{i,t-1} + \Delta q_{iu},\ \forall t \in \{1,2,\cdots,T\} \tag{3}$$

where $q_{i,t}$ is generator-i's production quantity at time-t.

3. Market Equilibrium Analysis

3.1 Game Model

In the electricity market modeled in Section I, generators compete against each other by choosing their production quantities (Cournot assumption). The generator ramp-rate constraints described in (3) are inter-temporal constraints in nature and, therefore, we can apply a dynamic game theory in order to fully characterize a generator's strategic interaction in the market. Figure 1 shows the extensive form representation of the dynamic game for the electricity market considered in this paper.

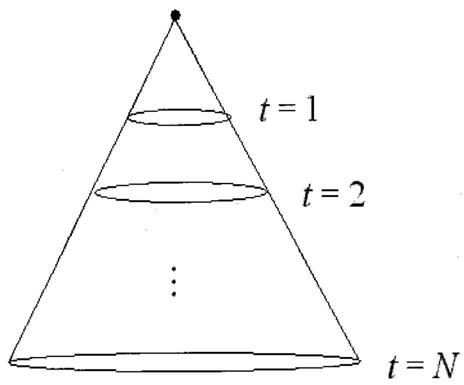

Fig. 1. Extensive form for the electricity market

In this game model, there is a static game embedded in the whole dynamic game at each time-t. That is, at each t, generators compete against each other to serve demand at that time by 'simultaneously' choosing their production quantities.

One of the popular solution concepts in dynamic game theory is the subgame perfect Nash equilibrium. In this paper, the market equilibrium is defined by the subgame perfect Nash equilibrium. In order for every subgame characterization, we have denoted $\Pi_{i,t}$ by the generator-i's payoff function during all the subgames from time-t to T:

$$\Pi_{i,t} = \sum_{\tau=t}^{T} \pi_{i,\tau} = \sum_{\tau=t}^{T} \left(P_\tau(q) q_{i,\tau} - C_i(q_{i,\tau}) \right) \tag{4}$$

where $\pi_{i,t}$ denotes generator-i's profit from the spot market at time-t.

3.2 Equilibrium Analysis

One way to easily characterize a subgame perfect Nash equilibrium is the backward induction and the backward induction has been applied for equilibrium analysis in this paper. The first step of the approach is analyzing the last node subgames at time-T.

At time-T, generator-i's subgame payoff function is its profit $\pi_{i,T}$ at time-T and the profit can be defined as:

$$\pi_{i,T} = \left(-\alpha_T \sum_{j=1}^{N} q_{j,T} + \beta_T \right) q_{i,T} - C_i(q_{i,T}) \tag{5}$$

In (5), the first term is the revenue of the generator-i and the second term is the cost of the generator-i. Due to the ramp-rate constraint, the possible production choice $q_{i,T}$ is restricted according to generator-i's production quantity $q_{i,T-1}$ at the previous time-(T-1). More generally, the ramp-rate constraint of generator-i at time-t can be expressed as:

$$\underline{q}_{i,t} \le q_{i,t} \le \overline{q}_{i,t}, \quad t \in \{1, 2, \cdots, T\} \tag{6}$$

where $\underline{q}_{i,t} = \max(0, q_{i,t-1} - \Delta q_{id})$, $\overline{q}_{i,t} = \min(\overline{q}_i, q_{i,t-1} + \Delta q_{iu})$, and $\overline{q}_i$ is the maximum generation of generator-i.

Due to the ramp-rate constraint, there are three cases for characterizing the best response $q_{i,T}^{BR}$ of generator-i for the subgame at time-T. Since $\pi_{i,T}$ is a concave function and $\frac{\partial \pi_{i,T}}{\partial q_{i,T}}$ is an increasing function with respect to $q_{i,T}$, the best response $q_{i,T}^{BR}$ can be expressed as:

$$q_{i,T}^{BR} = \begin{cases} \underline{q}_{i,T}, & if \ \frac{\partial \pi_{i,T}}{\partial q_{i,T}}\left(q_{1,T}, \cdots, \underline{q}_{i,T}, \cdots, q_{N,T}\right) < 0 \\ \overline{q}_{i,T}, & if \ \frac{\partial \pi_{i,t}}{\partial q_{i,t}}\left(q_{1,T}, \cdots, \overline{q}_{i,T}, \cdots, q_{N,T}\right) > 0 \\ -\frac{\alpha_T}{2\alpha_T + a_i} \sum_{j=1, j \ne i}^{N} q_{j,T} + \frac{\beta_T - b_i}{2\alpha_T + a_i}, & otherwise \end{cases} \tag{7}$$

The third row of (7) can be derived by the first-order necessary condition for optimality:

$$\frac{\partial \pi_{i,T}}{\partial q_{i,T}} = -\alpha_T \sum_{j=1}^{N} q_{j,T} + \beta_T - \alpha_T q_{i,T} - a_i q_{i,T} - b_i = 0 \tag{8}$$

Therefore, the Nash equilibrium strategy profile $q_T^{Nash} = \left[q_{1,T}^{Nash}, \cdots, q_{N,T}^{Nash} \right]$ for the spot market at time-T can be determined by simultaneously solving (7) for all generators. Moreover, the Nash equilibrium payoff profile $\Pi_T^{Nash} = \left[\Pi_{1,T}^{Nash}, \cdots, \Pi_{N,T}^{Nash} \right]$ for the subgame at time-T can also be obtained using the determined equilibrium strategy profile q_T^{Nash}. In this point, an important observation is that the equilibrium strategy profile q_T^{Nash} for the subgame at time-T would be a function of the production quantity profile $q_{T-1} = \left[q_{1,T-1}, \cdots, q_{N,T-1} \right]$ at the previous time-(T-1). This holds, in general, for the subgame from time-t to T. That is, the equilibrium strategy profile q_t^{Nash} for the subgame from time-t to T would be a function of the production quantity profile $q_{t-1} = \left[q_{1,t-1}, \cdots, q_{N,t-1} \right]$ at time-(t-1).

This explicitly shows the nature of the inter-temporal dynamics of the ramp-rate constraints. Since the ramp-rate constraints restrict a generator's production quantities at only two consecutive times, the subgame equilibrium has the Markov property. That is, the subgame equilibrium strategy profile q_t^{Nash} is only dependent on the previous production profile q_{t-1}, but not on the production profiles before time-(t-1), $q_1, \cdots, q_{t-2}$. Therefore, the Markov property of the game dynamics significantly reduces the complexity of characterization of the subgame perfect Nash equilibrium. That is, the subgame perfect Nash equilibrium strategy profile at time-t, q_t^{Nash}, can be represented as:

$$q_t^{Nash}(q_1, \cdots, q_{t-1}) = q_t^{Nash}(q_{t-1}) \tag{9}$$

Suppose that we have the solution for the subgame from time-*t* to *T*. Then, the subgame equilibrium strategy profile q_{t-1}^{Nash} can be obtained by simultaneously solving:

$$\forall i \in \{1,\cdots,N\},$$
$$q_{i,t-1}^{Nash} = \arg\max_{q_{i,t-1}} \Pi_{i,t-1}\left(q_{1,t-1}^{Nash},\cdots,q_{i,t-1},\cdots,q_{N,t-1}^{Nash}\right) \quad (10)$$

where $\Pi_{i,t-1} = \pi_{i,t-1} + \Pi_{i,t}\left(q_{1,t}^{Nash},\cdots,q_{N,t}^{Nash}\right)$. Since we have determined the equilibrium for the subgame at time-*T*, by using the backward induction procedure in (10), the subgame perfect Nash equilibrium strategy profile q^{Nash} can be obtained as:

$$q^{Nash} = \left[\left(q_{1,1}^{Nash},\cdots,q_{1,T}^{Nash}\right),\cdots,\left(q_{N,1}^{Nash},\cdots,q_{N,T}^{Nash}\right)\right] \quad (11)$$

The subgame perfect equilibrium path can also be obtained by determining the actual equilibrium outputs sequentially from the initial production profile $q_0 = \left[q_{1,0},\cdots,q_{N,0}\right]$.

4. Numerical Illustration

In order for an illustration, a simple market model is considered. There are two generators with the same cost function:

$$C_i(q_i) = \frac{1}{2}q_i^2 + q_i, \quad i = 1,2 \quad (12)$$

The ramp rates for two generators are:

$$\Delta q_{1u} = \Delta q_{1d} = 0.1,$$
$$\Delta q_{2u} = \Delta q_{2d} = 0.05. \quad (13)$$

The considered time horizon is set to 2, that is, $t \in \{1,2\}$. The inverse demand functions are:

$$P_1(q) = -2q + 4,$$
$$P_2(q) = -q + 2. \quad (14)$$

The initial production quantity profile is $q_0 = [0.25,\ 0.25]$. Following the backward induction procedures, first consider the subgame at $t = 2$. The equilibrium strategies for the subgame are determined as:

$$q_{1,2}^{Nash} = \begin{cases} q_{1,1} - 0.1, & if \ \ q_{1,1} > \dfrac{1.3 - q_{2,2}^{Nash}}{3} \\ q_{1,1} + 0.1, & if \ \ q_{1,1} < \dfrac{0.7 - q_{2,2}^{Nash}}{3} \\ \dfrac{1 - q_{2,2}^{Nash}}{3}, & otherwise \end{cases} \quad (15)$$

$$q_{2,2}^{Nash} = \begin{cases} q_{2,1} - 0.05, & if \ \ q_{2,1} > \dfrac{1.15 - q_{1,2}^{Nash}}{3} \\ q_{2,1} + 0.05, & if \ \ q_{2,1} < \dfrac{0.85 - q_{1,2}^{Nash}}{3} \\ \dfrac{1 - q_{1,2}^{Nash}}{3}, & otherwise \end{cases} \quad (16)$$

From (15) and (16), the equilibrium strategy profile for the subgame can be obtained as:

$$q_2^{Nash} = [0.25, 0.25] \quad (17)$$

The equilibrium payoff profile for the subgame can also be determined as:

$$\Pi_2^{Nash} = [0.0938, 0.0938] \quad (18)$$

Now, consider the subgame from $t = 1$ to $t = 2$. The payoff profile for this subgame is:

$$\Pi_1 = \left[\Pi_{1,1},\ \Pi_{2,1}\right]$$
$$= \left[\pi_{1,1} + 0.0938,\ \pi_{2,1} + 0.0938\right] \quad (19)$$

By finding Nash equilibrium for this subgame considering the initial production quantity profile, the subgame perfect Nash equilibrium strategy profile can be obtained as:

$$q^{Nash} = \left[\left(q_{1,1}^{Nash},\ q_{1,2}^{Nash}\right),\ \left(q_{2,1}^{Nash},\ q_{2,2}^{Nash}\right)\right]$$
$$= \left[(0.35,\ 0.25),\ (0.3,\ 0.25)\right] \quad (20)$$

5. Conclusions

This paper proposed a game theoretic approach for studying the strategic interaction of generators in the deregulated electricity markets considering their ramp-rate constraints. In this paper, a dynamic game model has been proposed in order to consider the strategic interaction of the generators with their ramp-rate constraints. The subgame perfect Nash equilibrium has been adopted as the solution of the game. Furthermore, backward induction approach has been applied to determining the subgame perfect Nash equilibrium of the game. Since the inter-temporal nature of the ramp- rate constraints yields the Markov property of the game, we have concluded that the Markov property of the game significantly reduces the complexity of the subgame perfect Nash equilibrium characterization.

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References

[1] S. Borenstein, J. Bushnell, and S. Stoft, "The competitive effects of transmission capacity in a deregulated electricity industry," *RAND Journal of Economics*, Vol. 31, pp. 294-325, 2000.

[2] B. Willems, "Modeling Cournot competition in an electricity market with transmission constraints," *The Energy Journal*, Vol. 23, pp. 95-125, 2002.

[3] D. M. Quick and J. M. Carey, "An analysis of market power mitigation strategies in Colorado's electricity industry," *The Energy Journal*, Vol. 22, pp. 55-77, 2001.

[4] T. O. Leautier, "Regulation of an electric power transmission company," *The Energy Journal,* Vol. 21, pp. 61-92, 2000.

[5] S. E. Stoft, "Financial transmission rights meet Cournot: How TCC's curb market power," *The Energy Journal,* Vol. 20, pp. 1-23, 1999.

[6] I.-K. Cho, I.-K., "Competitive equilibrium in a radial network," *RAND Journal of Economics,* Vol. 34, pp. 438-460, 2003.

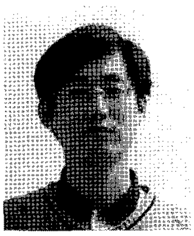
[7] R. Baldick and W. Hogan, "Capacity constrained supply function equilibrium models of electricity markets: stability, non-decreasing constraints, and function space iterations," University of California Energy Institute Working paper PWP-089, Dec. 2001.

[8] J. M. Arroyo and A. J. Conejo, "Multiperiod auction for a pool-based electricity market," *IEEE Trans. on Power Systems*, Vol. 17, No. 4, pp. 1225-1231, Nov. 2002.

[9] C. Wang and S. M. Shahidehpour, "Optimal generation scheduling with ramping costs," *Power Industry Computer Application Conference*, pp. 11-17, May 1993.

[10] F. N. Lee, L. Lemonidis, and K.-C. Liu, "Price-based ramp-rate model for dynamic dispatch and unit commitment," *IEEE Trans. on Power Systems*, Vol. 9, No. 3, pp. 1233-1242, Aug. 1994.

[11] A. Mas-Colell and M. D. Whinston, *Microeconomic Theory*, Oxford University Press, Inc., 1995.



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