# **Hybrid Differential Evolution Technique for Economic Dispatch Problems**

# T. Jayabarathi<sup>†</sup>, V. Ramesh\*, D. P. Kothari\*, Kusuma Pavan\* and Mithun Thumbi\*\*

Abstract – This paper is aimed at presenting techniques of hybrid differential evolution for solving various kinds of Economic Dispatch (ED) problems such as those including prohibited zones, emission dispatch, multiple fuels, and multiple areas. The results obtained for typical problems are compared with those obtained by other techniques such as Particle Swarm Optimization (PSO) and Classical Evolutionary Programming (CEP) techniques. The comparison of the results proves that hybrid differential evolution is quite favorable for solving ED problems with no restrictions on the shapes of the input-output functions of the generator.

Keywords: Combined Economic Emission Dispatch, Economic Dispatch, Hybrid Differential Evolution, Prohibited Zones

#### 1. Introduction

Economic dispatch is an optimizing scheme for a generation system to determine the best generation schedule for a given load demand in terms of cost, environmental pollution effects, and losses. economic dispatch problems and their solutions by non-conventional methods have been very widely discussed in the literature. Evolutionary Programming based solutions for the ED problem with prohibited zones has been discussed in [1-4]. Sudhakaran et al. proposed refined genetic algorithms for solving Combined Economic and Emission Dispatch (CEED) problems [5]. Immanuel et al. [6] solved the same CEED problem by using the particle swarm optimization algorithm. Jeyakumar et al. [7] applied PSO for solving four different types of economic dispatch problems. Basu [8] proposed Hopfield Neural Networks (HNN) for CEED problems. Lee et al. [9] applied adaptive HNN for load dispatch problems. These references are typical and by no means exhaustive. Developments in the field of Evolutionary Computation have resulted in new tools for solving optimization problems in electric power engineering. Storn and Price [10] introduced the Differential Evolution technique for minimizing real functions. The Hybrid Differential Evolution (HDE) technique was employed by Chiou [11] for solving large-scale economic dispatch problems.

Other applications of the same technique can be found in [12-18]. It is a simple method based on stochastic searches.

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In this paper, Section 2 states the various types of ED problems. Section 3 provides the basic algorithm of hybrid differential evolution followed by applications to the problems in the previous section. Section 4 gives the results for typical problems and compares them with other methods. Finally, Section 5 provides the conclusions.

## 2. Problem Formulation

Basically four types of problems are considered here, [19] namely ED with prohibited zones of generator operation (POZ), ED with piecewise quadratic functions for multiple fuels (PQCF), combined economic emission dispatch (CEED), and multi-area economic dispatch (MAED).

# 2.1 Formulation of the problem of ED with POZ

The problem of economic dispatch with POZ can be formulated as

$$\min \sum_{j=1}^{N} F_j(P_j) = \min \sum_{j=1}^{N} (a_j P_j^2 + b_j P_j + c_j)$$
 (1)

Where  $a_j$ ,  $b_j$  and  $c_j$  are the cost coefficients of generator jand  $P_i$  is the power generated by the jth unit, subject to

(i) the power balance constraints

$$P_D = \sum_{j=1}^{N} P_j - P_L \tag{2}$$

Where  $P_d$  is the system load demand and  $P_L$  is the system loss which can be found through the use of B-matrix loss coefficients.

Corresponding Author: School of Electrical Sciences, VIT University (t barathi63@vahoo.com)

School of Electrical Sciences, VIT University, Tamilnadu,

School of Mechanical and Building Sciences, VITU.

(ii) Generating capacity constraints

$$P_{i,\min} \le P_i \le P_{i,\max}$$
 for  $j = 1, 2, 3...N$  (3)

where  $P_{j min}$  and  $P_{j,max}$  are the minimum and maximum possible powers generated by unit j.

(iii) For units with prohibited zones, there are additional constraints on the unit operating range

(iv) 
$$\begin{aligned} P_{j,\min} &\leq P_{j} \leq P_{j,1}^{1} \\ P_{j,k-1}^{u} &\leq P_{j} \leq P_{j,k}^{l} \ k = 2, \dots, Z_{j} \\ P_{j,Z_{j}}^{u} &\leq P_{j} \leq P_{j,\max} \end{aligned} \tag{4}$$

where  $P_{j,k}^l$  and  $P_{j,k}^u$  are lower and upper bounds of  $k_{th}$  the prohibited zones and  $Z_j$  is the number of prohibited zones of unit j.

# 2.2 Formulation of the problem of ED with PQCF

If multiple fuels are used each for a different range of operation, the cost function will become piecewise quadratic. The ED problem with PQCF will be defined as

$$\min \sum_{j=1}^{N} F_j(P_j) \tag{5}$$

$$F_{j}(P_{j}) = \begin{cases} a_{j1}P_{j}^{2} + b_{j1}P_{j} + c_{j1}, & \text{fuel } 1, P_{j,\min} \leq P_{j} \leq P_{j,1} \\ a_{j2}P_{j}^{2} + b_{j2}P_{j} + c_{j2}, & \text{fuel } 2, P_{j,\min} \leq P_{j} \leq P_{j,2} \\ & . \\ & . \\ & . \\ a_{jm}P_{j}^{2} + b_{jm}P_{j} + c_{jm}, & \text{fuelm } P_{j,m-1} \leq P_{j} \leq P_{j,\max} \end{cases}$$

Where  $a_{jm}$ ,  $b_{jm}$  and  $c_{jm}$  are coefficients of generator j for fuel type m subject to the constraints given by Eqs. (2) and (3).

## 2.3 Formulation of the CEED problem

In this case, reduction in emission is an additional objective besides cost. The emission can be expressed as a quadratic polynomial, similar to the cost curve as

$$F_2 = \sum_{j=1}^{N} d_j P_j^2 + e_j P_j + f_j$$
 (6)

where  $d_j, e_j \& f_j$  are the coefficients of the emission of unit j and  $F_2$  is the total emission of pollution of the N generating units.

The emission curve is directly related to the cost curve through the emission rate per MBTU, which is a constant factor for a given type of fuel.

The CEED problem is formulated by weighing the cost

given in Eq. (1) and the emission function in Eq. (6) according to their relative importance and the two weighted functions are added together to produce a final objective function as

$$\min f = \omega F_1 + (1 - \omega) F_2 \tag{7}$$

where  $F_I$  is the fuel cost function,  $F_2$  is the emission function and  $\omega$  is the weighting function in the range 0 and 1. When  $\omega = 0$ , only the emission objective is considered and when  $\omega = 1$ , only the cost objective is accounted for. The value of  $\omega$  can be varied to obtain a trade-off between fuel cost and emission cost. The CEED function is subject to the power balance constraint and capacity limits constraint given in Eqs. (2) and (3).

# 2.4 Formulation of the MAED problem

Here, the objective is to determine the generation levels and interchange power between areas that minimize the system operation cost while satisfying a set of constraints as

$$\min \sum_{m=1}^{M} F_m = \min \sum_{m=1}^{M} \sum_{n=1}^{N_m} (a_{nm} P_{nm}^2 + b_{nm} P_{nm} + c_{mn})$$
 (8)

where  $N_m$  is the number of on-line units for area m in an M area system;  $a_{mn}$ ,  $b_{mn}$  and  $c_{mn}$  are the fuel cost coefficients and  $P_{mn}$  is the power output of generator n in area m subject to

(i)Area power balance constraint;

$$\sum_{n=1}^{N_m} P_{mn} + \sum_{j \in \beta_m} t_{jj} - \sum_{j \in \beta_m} t_{jk} - P_{Dm} = 0$$
(9)

(ii) Generation-limit-constraints:

$$P_{mn,\min} \le P_{mn} \le P_{mn,\max} \tag{10}$$

(iii) Tie-line limit constraints:

$$t_{jk,\min} \le t_{jk} \le t_{jk,\max} \tag{11}$$

where  $\beta_m$  is the set of tie-lines in area m;  $P_{Dm}$  is the load demand for area m,  $t_{jk,min}$   $t_{jk,max}$  are the tie-line minimum and maximum capacity limits; and  $t_{jk}$  is the economic tie transfer from area j to area k.

## 3. The Hybrid Differential Evolution Technique

Hybrid Differential Evolution [12] approach is a simple population based stochastic function method and has been extended from the original algorithm of differential evolution [10]. This method is used to solve unconstrained nonlinear, non-smooth, and non-differentiable optimization problems. The basic operations of HDE are:

Step 1: Representation and Initialization: HDE is a parallel direct search algorithm that utilizes  $N_P$  vectors of decision variables,  $\mathbf{x}$ , in the non-linear programming problem, i.e  $\mathbf{X}^G = \{\mathbf{x}_i^G, i=1,...., N_P\}$ , as a population in generation G. For convenience, the decision vector (chromosome),  $\mathbf{x}_i$ , is represented as  $(x_{Ii},...,x_{ji},...,x_{Ni})$ . Here, the decision variable (gene),  $x_{ji}$ , is directly coded as a real value within its corresponding lower-upper bounds. The initialization process generates  $N_P$  individuals  $\mathbf{x}_i$  randomly, and has to cover the entire search space uniformly in the form

$$\mathbf{x}_{i}^{0} = \mathbf{x}^{L} + \rho_{i} (\mathbf{x}^{U} - \mathbf{x}^{L}), i = 1, 2, \dots, N_{P}$$
 (12)

where  $\rho_i$  is a vector of random numbers in the range [0,1]. The N genes of each individual are the powers generated by each generator satisfying the inequality (generation limits) and equality (power balance) constraints and hence form a feasible solution.

**Step 2**: **Mutation**: Pairs of individual vectors from step 1 are chosen at random. A mutant individual is generated by

$$\mathbf{u}_{i}^{G+1} = \mathbf{x}_{p}^{G} + \rho_{m}(\mathbf{x}_{i}^{G} - \mathbf{x}_{k}^{G}), i = 1, 2 \dots, N_{P}$$
(13)

where random indices  $p,j,k \in \{1,2,.....N_P\}$  are integer values and are mutually different. Mutation factor  $\rho_m$  is a real-valued random number between zero and one.

**Step 3: Crossover Operation:** The crossover operation is performed to increase the local diversity of the population. This operation reproduces an offspring at the next generation. The newly mutant individual in  $\mathbf{u}_i^{G+1}$  in Eq. (13) and the current individual  $\mathbf{x}_j^G$  are chosen by a binomial distribution to perform the crossover operation. In this operation, each gene of the *i*th individual is reproduced from the mutant vectors  $\mathbf{u}_i^{G+1} = (u_{1i}^{G+1}, u_{2i}^{G+1}, \dots, u_{Ni}^{G+1})$  and the current individual  $\mathbf{x}_i^G = (x_{1i}^G, x_{2i}^G, \dots, x_{Ni}^G)$  as follows:

$$u_{ji}^{G+1} = \begin{cases} x_{ji}^{G}, & \text{if a random number} > C_R \\ u_{ji}^{G+1}, & \text{otherwise}; j=1,....,N, i=1,....,N_P \end{cases}$$
 (14)

Where the crossover factor  $C_R \in [0,1]$  is a constant and has to be set by the user.

**Step 4: Selection and Evaluation:** The offspring is compared with its parent and it replaces the parent if its fitness is higher. Otherwise, the parent is retained. Here the fitness function is the objective function of the various equations is Section 2. Two selection steps are performed

in this evaluation expression. The first step is a one-to-one competition, and the next step is to select the best individual in the population. These two steps are expressed in the forms

$$\hat{\mathbf{X}}_{i}^{G+1} = \operatorname{argmin}\{f(\mathbf{x}_{i}^{G}), f(\mathbf{u}_{i}^{G+1})\}, i=1,\dots,N_{p}$$
(15)

$$\hat{\mathbf{X}}_{b}^{G+1} = \operatorname{argmin}\{f(\mathbf{x}_{i}^{G+1}), i=1, \dots, N_{p}\}$$
 (16)

where argmin means the argument of the minimum. From Eqs. (15) and (16), the best individual,  $\mathbf{x}_b^{G+I}$ , can be kept at each generation.

Step 5: Accelerated Operation: An accelerated operation and a migration operation are used as a trade-off. The accelerated operation is used to speed up the convergence, whereas the migration operation is used to evade the local minima. If the best individual is no longer improved by mutation and crossover, the gradient of the objective function,  $(V_f)$ , obtained by finite difference is applied to push the best individual to a better point by the steepest descent method. The acceleration operation is therefore expressed as

$$\mathbf{x}_b^{N} = \mathbf{x}_b^{G+1} - \rho_a V_X f(\mathbf{x}) | \mathbf{x}_b^{G+1}$$
 (17)

where  $\mathbf{x}_b^N$  is the newest and best solution. The continuous gradient of the objective function,  $V_{\mathbf{X}}$   $f(\mathbf{x})$ , can be approximately calculated with a finite difference method. The step size,  $\rho_a \in [0,1]$ , is judiciously chosen for proper convergence. The objective function value,  $f(\mathbf{x}_b^N)$ , is then compared with  $f(\mathbf{x}_b^{G+I})$ . If the descent property is obeyed, i.e.,

$$f(\mathbf{x}_b^N) < f(\mathbf{x}_b^{G+1}) \tag{18}$$

the new individual,  $\mathbf{x}_b{}^N$ , is added into this population to replace the worst individual. On the other hand, if the descent property fails, the step size,  $\rho_a$ , is adjusted. The descent method is repeated to obtain  $\mathbf{x}_b{}^N$  until  $\rho_a \nabla_\mathbf{x} f$  becomes sufficiently small or an iteration limit is exceeded. Consequently, the best fitness,  $f(\mathbf{x}_b{}^N)$ , should be at least equal to or smaller than  $f(\mathbf{x}_b{}^{G+1})$ .

**Step 6: Migration Operation:** In order to greatly increase the exploration of the search space and decrease the selection pressure of a small population, a widespread search heuristic called migration is introduced to generate a newly diversified population of individuals. The newly migrant individuals are generated on the basis of the best individual,  $\mathbf{x}_b^{G+I} = (x_{Ib}^{G+I}, \dots, x_{Nb}^{G+I})$ , by using non-uniformly random choice. New genes of the *i*th individual are therefore generated by

number
$$(x_{ji}^{G+1}) = \begin{cases} x_{jb}^{G+1} + \rho(x_{j}^{L} - x_{jb}^{G+1}), & \text{if a random} \\ <(x_{jb}^{G+1} - x_{j}^{L})(x_{j}^{U} - x_{j}^{L}) & \text{(19)} \\ x_{jb}^{G+1} + \rho(x_{j}^{U} - x_{jb}^{G+1}), & \text{otherwise,} \\ j = 1, \dots, N \\ i = 1, \dots, N_{p-1} \end{cases}$$

Where  $\rho$  is a random number in the range [0, 1].

The migration operation of HDE is performed only if a measure of population diversity does not match the desired tolerance. Hence we use a measure,  $\rho_x$ , defined as follows.

$$\rho_{m} = \left(\sum_{\substack{i=1\\i\neq b}}^{N_{p}} \sum_{\substack{j=1\\j\neq b}}^{N} \eta_{ji}\right) / \left(N(N_{p} - 1)\right) < \varepsilon_{1}$$
 (20)

Where

$$\eta_{ji} = \begin{cases} 1, & \text{if } |(u_{ji} - u_{jb})/u_{jb}| > \varepsilon_2 \\ 0, & \text{otherwise} \end{cases}$$
 (21)

where  $\varepsilon_1$  and  $\varepsilon_2$  are the desired tolerance for the group diversity and gene diversity with respect to the best

individual. In this case,  $\eta_{ji}$  is defined as an index of gene diversity. Its value is zero if the *j*th gene of the *i*th individual closely clusters with the *j*th gene of the best individual.

The migration operation is performed only if the degree of population diversity is smaller than the desired tolerance  $\mathcal{E}_1$ . From (20) it is inferred that the degree of population diversity is between zero and one. A value of zero implies that all genes cluster around the best individual. Conversely, a value of 1 indicates that the current candidate individuals are a completely diversified population. The desired tolerance for population diversity is accordingly assigned within this region. Zero tolerance implies that the migration is switched off whereas a tolerance of 1 implies that the migration operation is performed at every generation.

#### 4. Simulation Results

To test the effectiveness of the HDE algorithm, four different types of economic dispatch problems are considered. These are: (i) ED with prohibited operating zones, (ii) ED with multiple fuel options, (iii) CEED, and (iv) multi-area ED with tie-line constraints. The results obtained for all these problems are compared with the Particle Swarm Optimization Approach. The software was written using the Matlab 6.5 platform. All four types

of ED problems were solved using both HDE method and PSO approach in the same Matlab 6.5 platform for the purpose of comparison.

For all four examples considered in this paper, the desired tolerances for the population diversity,  $\mathcal{E}_1$ =0.001, and the desired tolerance for the gene diversity,  $\mathcal{E}_2$ =0.02, are used.

## 4.1 Example 1. Prohibited Zones

This example problem is based on a 15-unit practical power system with four of the units having up to three operating zones from [1]. The system load demand is 2650 MW. Table 1 shows the results obtained by the HDE. These results are compared with those obtained using PSO and CEP methods [1]. The results are comparable. The exact value of fuel cost obtained using dynamic programming is 32,506 \$/hr. [1]. The HDE method took 1.7s and the PSO method took 1.97s to converge to the optimal solution. Therefore, there is a significant reduction in computation time. The parameters used in HDE approach are: Crossover factor  $C_R$ =0.7, Population size = 50, Iterations=200.

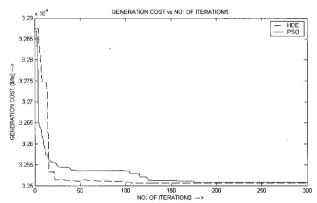
Table 1. Simulation results of ED with POZ of Example 1

Output	Optimization Techniques			
in MW	HDE	PSO[7]	CEP[7]	
G1	455.00	455.00	449.946	
G2	454.998	455.000	450.000	
G3	130.000	130.000	130.000	
G4	130.000	130.000	130.000	
G5	259.658	260.000	260.000	
G6	459.998	460.000	460.000	
G7	15.000	015.000	015.000	
G8	60.000	060.000	060.000	
G9	25.000	025.204	025.000	
G10	20.252	020.001	020.001	
G11	60.085	069.804	020.000	
G12	75.021	065.000	055.004	
G13	25.000	025.000	025.000	
G14	15.000	015.000	015.000	
G15	464.988	464.991	465.000	
Total cost (\$/hr)	32506.226	32506.297	32507.553	
Iterations	48	89	396	
CPU time(s)	1.703	1.969	3.875	

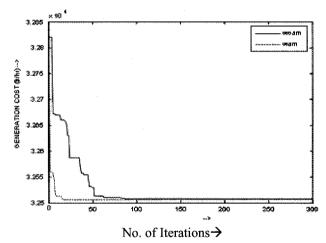
The convergence behavior of HDE and PSO methods are shown in Fig.1. From the figure it can be observed that initially the PSO method appears to converge faster but after less than 20 iterations HDE performs better. The reason for

this can be attributed to the initial exploratory behavior of HDE, which slows down the convergence initially in search of possibilities for avoiding local minima.

Fig. 2 shows the comparison of the two graphs – one without accelerated and migration operations (woam) and the other with accelerated and migration operations (wam). The accelerated operation has accelerated the convergence. However, since faster convergence leads to local minimum, the migration operation has been applied to prevent this. These effects on convergence can be clearly observed in this figure.



**Fig. 1.** Convergence Characteristics of HDE and PSO for the ED with POZ of Example 1



**Fig. 2.** Convergence Characteristics of HDE Showing the Effects of Acceleration and Migration factors for Example 1

woam: without acceleration and migration wam: with acceleration and migration

#### 4.2 Example 2. Piecewise Quadratic Cost Functions

This example considers ten generating units each with three types of fuel and a load demand of 2700 MW [9]. The results obtained by the HDE are compared against the PSO method in Table 2. From the Table it is observed that the solutions are almost identical. The HDE method took 0.74s and the PSO method 2.82s to converge to the optimal

solution which means that the HDE method is converging faster than the PSO method. The parameters used are: Crossover factor  $C_R$ =0.1, Population size = 25, Iterations=300.

The convergence characteristics of the HDE and PSO methods are shown in Fig. 3. From this figure it can also be inferred that HDE is definitely a faster and more superior method for solving this type of problem.

Table 2. Results of the PQCF Problem of Example 2

	Optimization Techniques		
Output in MW	HDE	PSO[7]	CEP[7]
P1	214.802	221.559	217.467
P2	210.656	210.318	211.650
Р3	280.303	279.650	281.58
P4	241.166	240.182	239.696
P5	279.765	278.170	279.394
P6	238.700	238.670	240.513
P7	282.424	278.767	287.681
P8	238.814	239.566	239.661
P9	440.000	440.000	428.704
P10	273.367	273.119	273.650
Tot.cost (\$/hr)	623.985	624.037	623.819
Iterations	18	50	236
CPU time (s)	0.74	2.82	1.52

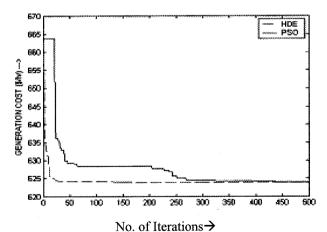


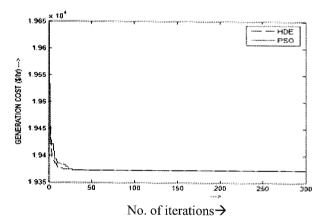
Fig. 3. Convergence of the HDE and PSO for the PQCF Problem of Example 2

#### 4.3 Combined Economic Emission Dispatch

This example has six generating units. Minimization of NOx emission is also considered as an objective. The fuel cost co-efficients and emission co-efficients are given in [20]. The results obtained using HDE are compared with those obtained using PSO and CEP in Table 3. It is seen from the Table that HDE and PSO converge to almost the same total cost of 19372 \$/h, but HDE took the least time

**Table 3.** Results of the CEED Problem of Example 3

	Optimization Techniques			
	HDE	PSO[7]	CEP[7]	
P1 (MW)	77.386	77.238	77.274	
P2 (MW)	49.632	49.935	49.639	
P3 (MW)	51.229	48.797	48.535	
P4 (MW)	102.800	103.767	103.525	
P5 (MW)	259.139	259.911	260.695	
P6 (MW)	190.966	191.259	191.233	
Line loss (MW)	31.152	30.906	30.901	
Emission cost (kg/hr)	524.764	527.167	524.49	
Fuel cost (\$/hr) Total	38219.740	38216.522	38216.470	
cost (\$/hr)	19372.252	19371.844	19369.842	
Iterations CPU	09	129	217	
time (s)	0.473	1.563	17.64	



**Fig. 4.** Convergence of HDE and PSO for the CEED Problem of Example 3

of 0.473s as against 1.563s for PSO and 17.64s for CEP. The parameters used are: Crossover factor  $C_R$ =0.4, Population size = 40, Iterations=300.

The convergence behavior of the HDE and the PSO methods are shown in Figure 4. From the figure it can be inferred that even though HDE is faster, PSO gives better results in terms of cost. A compromise has to be made between a quick good solution and a slower better solution depending on the objective of analysis.

# 4.4 Multi-area Economic Dispatch:

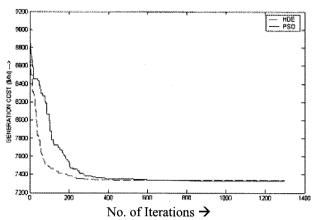
This example is a four area system interconnected by six tie lines. The cost co-efficients, generation limits, and tie line flow limits are given in [21]. The results obtained by

the HDE are compared against PSO and CEP in Table 4. It is seen from the Table that all the three methods converge to almost the same total cost of 7337 \$/h, but the HDE took the least time of 7.32s as against 8.156s for PSO and 11.49s for CEP. The parameters used are: Crossover factor  $C_R$ =0.4, Population size = 40, Iterations=1300.

Figure 5 shows the convergence behavior of the HDE and PSO methods. The HDE converged faster than the PSO. This is consistent with the three previous examples considered in this paper.

Table 4. Results of the MAED Problem of Example 4

		Optimization Techniques		
		HDE	PSO[7]	CEP[7]
Area 1	P1 (MW)	149.994	150.000	150.000
Load	P2 (MW)	99.993	100.000	100.000
400(MW	) P3 (MW)	66.401	68.826	068.826
	P4 (MW)	99.997	99.985	099.985
AREA2	P5 (MW)	56.886	56.613	056.373
LOAD P6 (MW)		96.894	95.474	093.519
200(MW	) P7 (MW)	41.447	41.617	042.546
	P8 (MW)	73.071	72.356	072.647
AREA 3	P9 (MW)	50.004	50.000	050.000
LOAD	P10(MW)	36.272	35.973	036.399
50(MW)	P11(MW)	38.508	38.210	038.323
	P12(MW)	37.145	37.162	036.903
AREA 4	P13(MW)	149.997	150.000	150.000
LOAD	P14(MW)	100.000	100.000	100.000
300(MW	) P15(MW)	57.175	57.830	56.648
P16(MW)		96.217	97.349	95.349
AREA				L
AR	EA	Tie I	ine Values	(MW)
AR From	EA To	Tie I	Line Values	(MW)
		Tie I	Line Values	(MW)
From	То			
From 1	To 2	0.001	00.000	00.00
From 1 1	To 2 3	0.001 18.097	00.000 22.588	00.00 19.587
From 1 1 1 1	To 2 3 4	0.001 18.097 0.000	00.000 22.588 00.000	00.00 19.587 00.000
From  1  1  1  2	To 2 3 4 1	0.001 18.097 0.000 0.000	00.000 22.588 00.000 00.000	00.00 19.587 00.000 00.018
From  1 1 1 2 2	To 2 3 4 1 3	0.001 18.097 0.000 0.000 1.712	00.000 22.588 00.000 00.000 66.064	00.00 19.587 00.000 00.018 68.861
From  1 1 2 2 2 2	To 2 3 4 1 3 4	0.001 18.097 0.000 0.000 1.712 0.000	00.000 22.588 00.000 00.000 66.064 00.000	00.00 19.587 00.000 00.018 68.861 00.000
From  1 1 2 2 2 3	To 2 3 4 1 3 4 1	0.001 18.097 0.000 0.000 1.712 0.000 0.000	00.000 22.588 00.000 00.000 66.064 00.000 00.000	00.00 19.587 00.000 00.018 68.861 00.000 00.000
From  1 1 2 2 2 3 3 3	To  2 3 4 1 3 4 1 2	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000 69.975	00.000 22.588 00.000 00.000 66.064 00.000 00.000	00.00 19.587 00.000 00.018 68.861 00.000 00.000
From  1 1 2 2 2 3 3 3 3	To  2 3 4 1 3 4 1 2 4	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000	00.000 22.588 00.000 00.000 66.064 00.000 00.000 00.000 05.176 00.004	00.00 19.587 00.000 00.018 68.861 00.000 00.000 00.000
From  1 1 2 2 2 3 3 3 4 4 4	To  2 3 4 1 3 4 1 2 4 1 2 3	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000 69.975	00.000 22.588 00.000 00.000 66.064 00.000 00.000 00.000 00.000 05.176	00.00 19.587 00.000 00.018 68.861 00.000 00.000 00.000 00.000 00.758
From  1 1 2 2 2 3 3 3 4 4 4	To  2 3 4 1 3 4 1 2 4 1 2	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000 69.975 1.676	00.000 22.588 00.000 00.000 66.064 00.000 00.000 00.000 05.176 00.004	00.00 19.587 00.000 00.018 68.861 00.000 00.000 00.000 00.000 00.758 01.789
From  1 1 2 2 2 3 3 3 4 4 4 Total c	To  2 3 4 1 3 4 1 2 4 1 2 3	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000 0.000 69.975 1.676 100.000	00.000 22.588 00.000 00.000 66.064 00.000 00.000 00.000 05.176 00.004 100.00	00.00 19.587 00.000 00.018 68.861 00.000 00.000 00.000 00.000 00.758 01.789 99.927
From  1 1 2 2 2 3 3 3 4 4 4 Total c No. of	To  2 3 4 1 3 4 1 2 4 1 2 3 cost (\$/hr)	0.001 18.097 0.000 0.000 1.712 0.000 0.000 0.000 69.975 1.676 100.000 7337.13	00.000 22.588 00.000 00.000 66.064 00.000 00.000 00.000 05.176 00.004 100.00 7336.9	00.00 19.587 00.000 00.018 68.861 00.000 00.000 00.000 00.758 01.789 99.927 7337.75



**Fig. 5.** Convergence of the HDE and PSO for MAED Problem of Example 4

#### 5. Conclusion

Application of the HDE method to four different kinds of ED problems is demonstrated in this paper. The solutions and solution times for the four sample problems typical to each kind bring out the advantages of the HDE method. It is observed that the HDE technique is capable of finding the near global or global solutions of non-linear, non-smooth, and non-differentiable objective functions. The HDE method is computationally faster at finding the same solution. This can be observed from all the graphs plotted here. From the tabulated results for all the examples, it can be concluded that the HDE Technique is more favorable compared to the PSO and the CEP methods, in terms of the optimal solutions, the number of iterations, and CPU time.

The importance of introducing acceleration and migration factors can be seen by comparing figures 1 and 2 of the ED problem with POZ. Here, the inclusion of the acceleration factor has reduced the convergence time while the introduction of the migration factor has improved the optimal result by avoiding the local optima. Similar effects are also observed in the remaining examples. The results reported here and the observations made from them confirm that HDE is a better technique to solving all kinds of ED problems with a variety of constraints.

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