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TITIUS-BODE'S Relation and 55 Cancri

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Abstract

Two kinds of important issues on Titius-Bode's relation have been discussed up to now: one is if there is a simple mathematical relation between distances of natural bodies orbiting a central body, and the other is if there is any physical basis for such a relation. These may be tackled by answering a question whether Titius-Bode's relation is valid universally in exo-planetary systems. We have examined whether Titius-Bode's relation is also applicable to exo-planetary systems by statistically studying the distribution of the ratio of rotational periods of two planets in an exo-planetary system, 55 Cnc, by comparing it with that derived from Titius-Bode's relation. We find that the distribution of the ratio of rotational periods of randomly chosen two planets in the 55 Cnc system is apparently inconsistent with that derived from Titius-Bode's relation. The probability that two data sets are drawn from the same distribution function is 50 %. We also find that the Fourier power spectra show that the distribution of the semi-major axis of planets in the 55 Cnc system seems to be stretched. We conclude by pointing out that large numbers of planets should be examined to more convincingly explain the distribution of the distance of planetary formation regions.

Keywords: celestial mechanics, solar system, general, general

1. Introduction

Johann Daniel Titius von Wittenberg in 1766 attempted to represent planetary distances with an empirical relation, i.e., the semi-major axis a_m of a planet in AU given as

$$a_m = 0.4 + (0.3 \times 2^n) \tag{1}$$

for the values of n : $-\infty$ for Mercury, 0 for Venus, 1 for Earth, 2 for Mars, and so on (for further discussion see Nieto 1972). Johann Elert Bode in 1782 recognized, after discovery of Uranus in 1781 by Frederick William Herschel, that Titius' relation *predicted* the semi-major axis of Uranus. This announcement initiated a frantic search for *the lost planet* at the fifth position at various European observatories. The discovery of Ceres by Giuseppe Piazzi on the night of the 1st of January in 1801, with the semi-major axis predicted by Titius-Bode's relation, did make people believe this relation as a natural *law*. There have also been numerous attempts to apply variations of Titius-Bode's relation to the satellite systems of the Jovian planets (Nieto 1970, Pletser 1986, Rawal 1984, 1986, 1989, Li, Zhang, & Li 1995). Interestingly enough, relations similar to eq. (1) can be found for the satellite systems of the giant planets. It should be noted that there are *missing holes* to the satellite systems, corresponding to the asteroidal belt in the planetary system, which is indeed filled by rings and small satellites (e.g., Stone & Miner 1986).

On the other hand, there is a never-ending debate about the meaning of Titius-Bode's relation. While Titus-Bode's relation gives neat results for the eight first planets, including the asteroids, it breaks down for objects farther than Neptune. It has been also shown that this kind of representation may not be unique. Other expressions of the variety of Titius-Bode's relation have been proposed by several authors (Blagg 1913, Richardson 1945, Dermott 1968, Prentice 1977, Isaacman & Sagan 1977, Rawal 1978, Basano & Hughes 1979, Neuhäuser & Feitzinger 1986, Rawal 1984, 1986, 1989, Ragnarsson 1995, Ortiz et al. 2007). Basically, this relation is nothing but a geometric progression. Some authors do believe that there is no evidence for physical reasons, though several theories of the solar system formation have attempted to account for these distance relations (Dermott 1972, 1973, Dobó 1981, Louise 1982, Llibre & Pinõl 1987, Patton 1988, Graner & Dubrulle 1994a,b, Li, Zhang, & Li 1995, Ortiz et al. 2007). Instead, they have attempted to find a solution among random processes (Lecar 1973, Pletser 1988, Hayes & Tremaine 1998).

The question of whether the observed patterns have some physical basis or are due to chance may be addressed using a Monte Carlo approach. Lynch (2003), however, finds that the estimated probability of chance occurrence depends sensitively on the restrictions imposed on the population of orbits. The possibility of physical explanation for the observed distributions remains open. Meanwhile, interest in the Titius-Bode's relation has been heightened by the discovery of extrasolar planets. For instance, Vahia, Mahajani, & Rao (2003) have investigated the distribution of extrasolar system planets to search for periodicities in their distribution around their parent star. More recently, Poveda & Lara (2008) have investigated if the exo-planetary system, 55 Cancri, fits some form of the Titius-Bode relation.

In this paper, motivated by Poveda & Lara (2008), we study the distribution of the ratio of periods of two planets in the 55 Cancri system by comparing it with that derived from Titius-Bode's relation. According to Kepler's 3rd law and to a generalized Titius-Bode's relation, i.e., $a_n = \alpha\beta^n$, the orbital period P_n of a planet in an exo-planetary system can be given as $P_n = P_0(\beta^{3/2})^n$, where P_0 is $\alpha^{3/2}$. What this means is that one may check Titius-Bode's relation in terms of the period which is directly *measured* by observations, instead of the semi-major axis which is *secondary* information deduced from observations with assumptions, e.g., on mass. It should be stressed once again that we take the ratio of the rotational period. Hence we are dealing with a quantity that is almost independent of mass of the planetary system (stellar mass plus planet mass). The ratio of the orbital periods of two planets in an exo-planetary system is given as $P_n/P_m = (a_n^{3/2})/(a_m^{3/2}) = (\beta^{3/2})^{n-m}$. Thus, as long as exo-planetary systems obey Titius-Bode's relation, P_n/P_m computed with randomly drawn pairs should satisfy a specific distribution function. Thus, by comparing the distribution of the ratio of periods of two planets in a multiple planet system with that derived from Titius-Bode's relation one may attempt to examine if exo-planetary systems share the same form.

This paper begins with descriptions of data of the exo-planetary system and the distribution of the ratio we analyze in section 2. We present and discuss results obtained with the χ^2 test and the Fourier analysis in section 3. Finally, we summarize and conclude in section 4.

2. 55 CNC System and Distribution of Ratio of Orbital Period

The star 55 Cancri (= 55 Cnc = HD 75732 = HR3522 = HIP 43587) is a well-known nearby dwarf star, whose distance from the Sun is 13.4 pc. It is found to have 5 planets. It is very much similar to the Sun. Its spectral type is G8V, its mass is 1.03 $M_\odot$, its effective temperature is 5243 K, and its radius is 1.15 $R_\odot$ (e.g., Fischer et al. 2008). In Table 1, the observed parameters for

Table 1. Properties of the 55 Cancri system.

	Mass(M_J)	Period(days)	Semi-axis(AU)	Year of Discovery
55 Cnc b	0.824	$14.65 \pm 7 \times 10^{-4}$	$0.12 \pm 1.1 \times 10^{-6}$	1996
55 Cnc c	0.169	$44.34 \pm 7 \times 10^{-3}$	$0.24 \pm 4.5 \times 10^{-5}$	2002
55 Cnc d	3.835	5218 ± 230	5.77 ± 0.11	2002
55 Cnc e	0.034	$2.82 \pm 1 \times 10^{-4}$	$0.038 \pm 1.0 \times 10^{-6}$	2004
55 Cnc f	0.144	260 ± 1.1	0.78 ± 0.007	2007

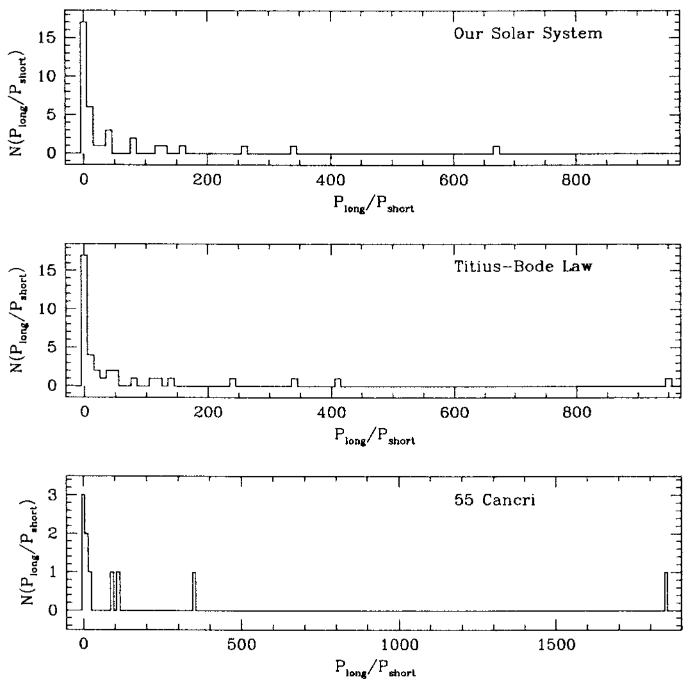

Figure 1. Distribution of the period ratios of two planets. In the top panel, the distribution from our solar system is shown. In the middle panel, the distribution of the ratio of calculated periods of two planets from Mercury to Neptune by Titius-Bode's relation is shown. In the bottom panel, we show the distribution from the 55 Cancri system. To obtain the distribution we randomly choose two planets and take a ratio such that the value is always greater than unity.

its planetary system are listed. Parameters of the planets are taken from the *Extrasolar Planets Encyclopedia*[1], in which recently discovered extrasolar planets are constantly included and updated by Jean Schneider. Note that errors in the orbital period and the semi-major axis are quoted from Poveda & Lara (2008).

In Figure 1, we show the distribution of the ratio of periods of two planets. In the top panel, the distribution of the ratio of observed periods of two planets in our solar system is shown. To obtain the distribution we randomly choose two planets and take a ratio such that the value is always greater than unity. If the value is equal to unity we discard the pair and select another pair of planets. It

[1] http://exoplanet.eu/catalog-all.php

Table 2. Results of the χ^2 test. The first and second columns indicate the data set from which the distribution is derived. Values of χ^2, the degrees of freedom, df, and the probability, P, indicating two data sets are drawn from the same distribution function are listed in the third, fourth, and fifth columns, respectively.

		χ^2	df	P(%)
Titus-Bode	Solar System	12.3	18	83
Titus-Bode	55 Cnc	16.3	17	50

should be noted that we do not consider Pluto (*one* of large known bodies in the Kuiper-Edgeworth belt) as a proper planet. On the other hand, it appears reasonable to include Ceres as a representative of the putative former planet. Both in the classical Titius-Bode's relation and in almost all of its modifications, the mean asteroid-belt distance is included in the regular planetary distances. Thus, an exception of the Pluto to the observed distribution alone is allowed. The middle panel shows the distribution of the ratio of *calculated* periods of two planets from Mercury to Neptune by Titius-Bode's relation. Again, the distribution is obtained from randomly chosen pairs of planets. In the bottom panel, we show the distribution of the ratio of observed periods of two planets in the 55 Cnc system. Note that the size of bins are all same.

3. Results and Discussion

We perform a statistical test to see if obtained distributions are *consistent* with each other. In Table 2, we show the χ^2 values with the degrees of freedom, df, between two binned distributions of the rotational period and probability that two data sets are drawn from the same distribution function. In the first row results from the distribution of the rotational period deduced from Titius-Bode's relation and that observed in our solar system are shown. The χ^2 value is 12.3. The probability that the sum of the squares of ν random normal variables will be greater than χ^2 is 83 %. As one may expect, the distribution derived from Titius-Bode's relation rounded-off at Uranus agrees more satisfactorily with the distribution observed in our Solar system. The probability that distributions from the Solar system and from Titius-Bode's relation are drawn the single population is 96 %. In the second row the value of χ^2 and probability resulted from Titius-Bode's relation and the observed exo-planetary system, 55 Cnc, are shown. The χ^2value is 16.3. The probability is 50 %. The value is somewhat lower and small enough to indicate two distributions are significantly different. Hence, we tentatively conclude that the traditional Titius-Bode's relation cannot be applied to the planetary system of the 55 Cnc. Our conclusion should be compared with what Poveda & Lara (2008) claimed. They found that a simple exponential Titius-Bode relation reproduces very well the five observed major semi-axis, provided they assign the orbital $n = 6$ to the largest semi-major axis. It is, however, fair to point out that there is another claim that their calculations contain mistakes (Kotliarov 2008).

In Figure 2, we show the Fourier power spectra to search for any regularity, which gives a hint where such discrepancy may come from. We have found that there is an ample power at low frequency $\leq 2 \times 10^{-3}$. What we see interesting in the power spectrum is that there are double-peak structures at frequency of ~ 0.025 for our solar system and at ~ 0.01 for 55 Cnc system. Apparently, there is a scaling factor of 2.5. In fact, the scaling factor can be also seen in the highest power at low frequency range. That is, peaks appear at frequency of ~ 0.001 for our solar system and at ~ 0.0004 for 55 Cnc system. What it means is that the distribution of period ratio resulted from the 55 Cnc system has a more sparse structure. In other words, the ratios of P_{long}/P_{short} tend to have large

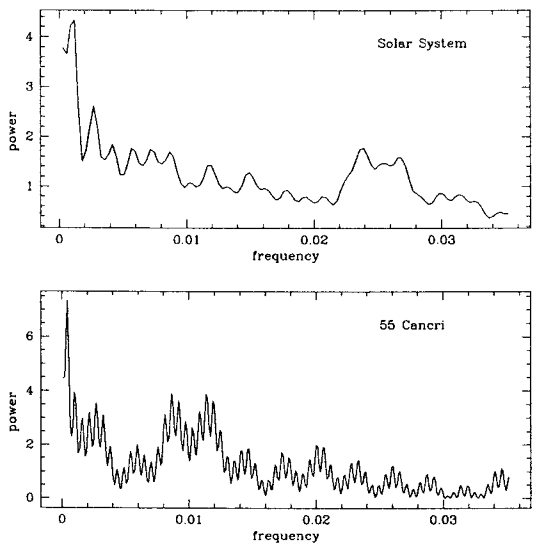

Figure 2. Fourier power spectra of the distribution of period ratios. In the upper panel and in the lower panel we show the power spectrum of our solar system and of 55 Cnc system, respectively. It is interesting to note that there are double-peak structures at frequency of $\sim$ 0.025 for our solar system and at $\sim$ 0.01 for 55 Cnc system. Apparently, there is a scaling factor of 2.5. In fact, the scaling factor can be also seen in the highest power at low frequency range. That is, peaks appear at frequency of $\sim$ 0.001 for our solar system and at $\sim$ 0.0004 for 55 Cnc system.

values, compared with our solar system. It can be seen in Figure 1, too. According to Kepler's third law, it tells us that the distribution of the semi-major axis of planets in the 55 Cnc system seems to be stretched, so to speak. This finding is very interesting if one reminds that the host star is very much similar to the Sun, e.g., $M = 1.03M_{\odot}$. We speculate its cause either that Titius-Bode's relation cannot be applied to exo-planet systems or that there is another factor to control planetary spacing other than the mass.

4. Summary and Conclusion

The distance distribution in our planetary system has been an uncomfortable issue, like a hot potato. The question we wish to tackle in this paper is whether Titius-Bode's relation is valid universally in exo-planetary systems. It is because, if it is the case Titius-Bode's relation may tell us about not only the relation itself but also formation and evolution of the planetary systems. For this purpose, we statistically study the distribution of the ratio of rotational periods of two planets in an exo-planetary system, 55 Cnc, by comparing it with that derived from Titius-Bode's relation. We find that the distribution of the ratio of rotational periods of two planets in the 55 Cnc system is apparently inconsistent with that derived from Titius-Bode's relation. We take the Fourier power spectrum to see if there is any difference and similarity, showing that the distribution of the semi-major axis of planets in the 55 Cnc system seems to be extended. We also find that the power spectra show some self-similarity, of which origin should be studied. We need to further study what causes such a scaling factor, if there were, in the future work.

We conclude by pointing out that without other examples than our own planetary system the general applicability of this form can be questioned since there is no convincing physical explanation for the values of the constants and the expression has some well-known deficiencies for our planetary system(failure beyond Uranus). The question of whether other solar systems might show similar patterns in the distance of planetary formation regions will only be resolved, due to the statistical significance, when large numbers of planets be found around many stars.

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References

Basano, L. & Hughes, D. W. 1979, Il Nuovo Cimento, Vol.2C, 505
Blagg, M. A. 1913, MNRAS, 73, 414
Dermott, S. F. 1968, MNRAS, 141, 363
Dermott, S. F. 1972, in On the Origin of the Solar System-Nice Symposium, ed. H. Reeves (Paris: CNRS), p.320
Dermott, S. F. 1973, Nature, 244, 18
Dobó, A. 1981, Astron. Nachr., Bd.302, H.2
Fischer, D. A., Marcy, G. W., Butler, R. P., Vogt, S. S., Laughlin, G., Henry, G. W., Abouav, D., Peek, K. M. G., Wright, J. T., Johnson, J. A., McCarthy, C., & Isaacson, H. 2008, ApJ, 675, 790
Graner, F. & Dubrulle, B. 1994a, A&A, 282, 262
Graner, F. & Dubrulle, B. 1994b, A&A, 282, 269
Hayes, W. & Tremaine, S. 1998, Icarus, 135, 549
Isaacman, R. & Sagan, C. 1977, Icarus, 31, 510
Kotliarov, I. 2008, astro-ph/0806.3532
Lecar, M. 1973, Nature, 242, 318
Li, X. Q., Zhang, H., & Li, Q. B. 1995, A&A, 304, 617
Llibre, J. & Pinõl, C. 1987, AJ, 93, 1272
Louise, R. 1982, M&P, 26, 93
Lynch, P. 2003, MNRAS, 341, 1174
Neuhäuser, R. & Feitzinger, J. V. 1986, A&A, 170, 174
Nieto, M. M. 1970, A&A, 8, 105
Nieto, M. M. 1972, The Titius-Bode Law of Planetary Distances: Its History and Theory (Oxford: Pergamon Press)
Ortiz, J. L., Moreno, F., Molina, A., Sanz, P. S., & Gutierrez, P. J. 2007, MNRAS, 379, 1222
Patton, J. M. 1988, CeMec, 44, 365
Pletser, V. 1986, Earth, Moon, and Planets, 36, 209
Pletser, V. 1988, Earth, Moon, and Planets, 42, 1
Poveda, A. & Lara, P. 2008, astro-ph/0803.2240
Prentice, A. J. R. 1977, in The Origin of the Solar System, ed. S. F. Dermott (New York: Wiley)
Ragnarsson, S.-I. 1995, A&A, 301, 609
Rawal, J. J. 1978, BASI, 6, 92
Rawal, J. J. 1984, Earth, Moon, and Planets, 31, 175
Rawal, J. J. 1986, Earth, Moon, and Planets, 34, 93
Rawal, J. J. 1989, Earth, Moon, and Planets, 44, 265
Richardson, D. E. 1945, Astron., 53, 14
Stone, E. C. & Miner, E. D. 1986, Science, 233, 39
Vahia, M. N., Mahajani, P., & Rao, A. R. 2003, BASI, 31, 37