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Effects of the Misspecification of Cointegrating Ranks in Seasonal Models

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Abstract

We investigate the effects of the misspecification of cointegrating(CI) ranks at other frequencies on the inference of seasonal models at the frequency of interest; our study includes tests for CI ranks and estimation of CI vectors. Earlier studies focused mostly on a single frequency corresponding to one seasonal root at a time, ignoring possible cointegration at the remaining frequencies. We investigate the effects of the misspecification, especially in finite samples, by adopting Gaussian reduced rank(GRR) estimation by Ahn and Reinsel (1994) that considers cointegration at all frequencies of seasonal unit roots simultaneously. It is observed that the identification of the seasonal CI rank at the frequency of interest is sensitive to the misprespecification of the CI ranks at other frequencies, mainly when the CI ranks at the remaining frequencies are underspecified.

Keywords: Gaussian reduced rank estimation, seasonal error correction model, asymptotically uncorrelated.

1. Introduction

Since Hylleberg *et al.* (1990), many approaches have been developed for the analysis of seasonal cointegration. Among others, Ahn and Reinsel (1994) and Ahn *et al.* (2004) used an iterative scheme by considering all the frequencies of seasonal unit roots simultaneously. Johansen and Schaumburg (1999)(hereafter JS) considered a switching algorithm based on partial regressions to avoid the complexity arising from simultaneous estimation. Cubadda (2001) considered the complex reduced-rank regression(CRRR) model in order to overcome the iterative algorithm of JS.

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Almost all seasonal cointegration procedures, except for the approaches of Ahn and Reinsel (1994) and Ahn *et al.* (2004), assume asymptotic uncorrelatedness among the nonstationary seasonal unit roots, even for a finite sample. Asymptotic uncorrelatedness implies that each seasonal unit root does not asymptotically affect inference with respect to different roots. In other words, the CI ranks at the non-focused roots, or the corresponding error correction terms in the seasonal error correction model(ECM), are ignored irrespective of whether they are full ranks or reduced ranks. The theoretical background is provided by corollary 7 of JS. However, we need to investigate the efficiency of asymptotic uncorrelatedness in a finite sample in order to identify the degree of interaction among the nonstationary roots in a finite sample. This efficiency can affect the performances of tests for CI ranks and the estimation of CI vectors.

For our purposes, we use Ahn and Reinsel's GRR approach, which differs from others in that it controls all nonstationary frequencies simultaneously and we investigate the sizes (*i.e.*, the rejection rates) of the CI rank test at the 5% nominal level using the GRR procedure when CI ranks at the non-focused frequencies are over- or underspecified. Note that if we use other approaches, it may be impossible to investigate the effect of the misspecifications because these approaches always assume that the CI ranks at the non-focused frequencies are full ranks.

If the asymptotic uncorrelatedness strictly holds in a finite sample, the performance; for example, the size of the CI rank test, would be the same irrespective of their over- or underspecification for the CI rank at the non-focused frequencies.

2. Seasonally Cointegrated Time Series and Its Estimations

Let $\mathbf{y}_t$ be an m-dimensional time series with non-stationary seasonal behavior and period s such that

$$\Phi(L)\mathbf{y}_t = \left(I_m - \sum_{j=1}^{p} \Phi_j L^j\right)\mathbf{y}_t = \Pi D_t + \varepsilon_t, \tag{2.1}$$

where ε_t are *i.i.d.* $N_m(0, \Omega)$ and D_t is a deterministic term that may contain a constant, a linear term, or seasonal dummies, with coefficient matrix Π. We assume that the initial values $\mathbf{y}_0, \dots, \mathbf{y}_{-p+1}$ are fixed and that the roots of the determinant $|\Phi(z)| = 0$ are on or outside the unit circle.

Suppose for simplicity that the series $\mathbf{y}_t$ is observed on a quarterly basis, *i.e.*, $s = 4$. We know (Ahn and Reinsel, 1994; Ahn *et al.*, 2004) that if the series are cointegrated at frequencies 0, 1/2 and 1/4 (0, π and $\pi/2$, respectively), model (2.1) may be rewritten in the following ECM,

$$\begin{aligned}\Phi^*(L)\left(1-L^4\right)\mathbf{y}_t = {} & \Pi D_t + \alpha_{1R}\beta_{1R}'(1+L)\left(1+L^2\right)\mathbf{y}_{t-1} \\ & + \alpha_{2R}\beta_{2R}'(1-L)\left(1+L^2\right)\mathbf{y}_{t-1} + \left(\alpha_{3R}\beta_{3I}' + \alpha_{3I}\beta_{3R}'\right)\left(1-L^2\right)\mathbf{y}_{t-1} \\ & + \left(-\alpha_{3R}\beta_{3R}' + \alpha_{3I}\beta_{3I}'\right)\left(1-L^2\right)\mathbf{y}_{t-2} + \varepsilon_t,\end{aligned} \tag{2.2}$$

where $\alpha_{1R}\beta_{1R}' = -\Phi(1)/4$, $\alpha_{2R}\beta_{2R}' = \Phi(-1)/4$, $(\alpha_{3R} + i\alpha_{3R})(\beta_{3R} + i\beta_{3I})' = -\Phi(i)/2$; α_{jR}, α_{jI}, β_{jR} and β_{jI} are $m \times r_j$ real-valued matrices with a rank equal to r_j; $\beta_{jR} = [I_{r_j}, \beta_{0jR}']'$ and $\beta_{jI} = [O_{r_j}, \beta_{0jI}']'$. Further, I_{r_j} is an $r_j \times r_j$ identity matrix and O_{r_j} is an $r_j \times r_j$ zero matrix. Note that the subscripts $j = 1$, 2 and 3 denote the frequencies 0, 1/2 and 1/4, respectively.

Ahn and Reinsel's GRR estimation uses model (2.2) and the iterative Newton-Raphson method for finding the maximum likelihood estimators for the parameters. This procedure may be complicated and the computational time may be long because the αs and βs at all frequencies are included in the estimation. However, it has an advantage in that it can simultaneously impose reduced-rank(*i.e.*, cointegrated) structures at all different frequencies on the right side of model (2.2). Thus, the efficiency of the inference, such as tests for CI rank and estimation of CI vectors, may be improved in finite samples. The estimation requires the CI ranks at all frequencies to be simultaneously prespecified. It is not easy to find an optimal method for simultaneous prespecification. As an example of a simple method, we use the combination of the CI ranks resulting from the determination of the CI rank(by likelihood ratio test) on a single frequency at a time. However, we need to check the performance or investigate an optimal method.

The approaches of Lee (1992), Johansen and Schaumburg (1999) and Cubadda (2001) are different from the GRR estimation. On the basis of asymptotical uncorrelatedness among the terms on the right side of model (2.2), *i.e.*, $X_{t-1}^{(1)} = (1+L)(1+L^2)\mathbf{y}_{t-1}$, $X_{t-1}^{(2)} = (1-L)(1+L^2)\mathbf{y}_{t-1}$ and $X_{t-1}^{(3)} = (1-L^2)\mathbf{y}_{t-1}$, they focused on a single frequency at a time, *i.e.*, they ignored reduced-rank structures at other frequencies and regarded them as full rank. Note that the regressors, $X_t^{(i)}$, $i = 1, 2, 3$, are asymptotically uncorrelated, in the sense that

$$T^{-2}\sum_{t=1}^{T} X_t^{(i)} X_t^{\prime(j)} \xrightarrow{P} 0, \quad \text{for } i \neq j,$$

where $\xrightarrow{P}$ denotes the convergence in probability. These methods may be simpler than GRR; however, they may result in a loss of efficiency. Furthermore, when focusing on a single frequency, the best strategy is to regress out the other frequencies as full rank. There are other alternatives that need to be considered, for example, what would happen if we regress them out as zero rank? These issues may be interesting.

3. Monte Carlo Experiment

In this section, by adopting the GRR estimation, we conduct a Monte Carlo experiment to investigate the effects of the misspecification of CI ranks at other frequencies on tests for CI rank and estimation of CI vectors at a single frequency of interest.

For a Monte Carlo experiment, the data generating process considered is the bivariate quarterly VAR(4) process, $\mathbf{y}_t = \sum_{j=1}^{4} \Phi_j \mathbf{y}_{t-j} + \varepsilon_t$, which has the following error correction representation:

$$\begin{aligned}(1-L^4)\mathbf{y}_t = {} & \alpha_1 {\beta'}_1 \mathbf{u}_{t-1} + \alpha_2 {\beta'}_2 \mathbf{v}_{t-1} \\ & + \left(\alpha_3 {\beta'}_4 + \alpha_4 {\beta'}_3\right)\mathbf{w}_{t-1} + \left(-\alpha_3 {\beta'}_3 + \alpha_4 {\beta'}_4\right)\mathbf{w}_{t-2} + \varepsilon_t, \end{aligned} \tag{3.1}$$

where $\mathbf{u}_{t-1} = (1+L)(1+L^2)\mathbf{y}_{t-1}$, $\mathbf{v}_{t-1} = (1-L)(1+L^2)\mathbf{y}_{t-1}$, $\mathbf{w}_{t-1} = (1-L^2)\mathbf{y}_{t-1}$, $\alpha_1 = (a_{11}, a_{21})' = (0.6, 0.6)'$, $\alpha_2 = (a_{12}, a_{22})' = (-0.4, 0.6)'$, $\alpha_3 = (a_{13}, a_{23})' = (0.6, -0.6)'$, $\alpha_4 = (a_{14}, a_{24})' = (0.4, -0.8)'$, $\beta_1 = (1, b_1)' = (1, -0.7)'$, $\beta_2 = (1, b_2)' = (1, 0.3)'$, $\beta_3 = (1, b_3) = (1, 0.7)'$ and $\beta_4 = (0, b_4) = (0, -0.2)'$. The covariance matrix Ω of ε_t is taken to be

$$\Omega = \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix},$$

for $\rho = -0.5,\ 0,\ 0.5$ and $\sigma^2 = 0.5,\ 1,\ 2$. We note that the roots of the characteristic equation $\det\{\Phi(L)\} = 0$ are ± 1, $\pm i$, $0.9715 \pm 0.7328\, i$ and $-1.3508 \pm 0.3406\, i$ and $\mathbf{y}_t$ is seasonally cointegrated with the CI rank of one at frequencies 0, 1/2 and 1/4.

Table 3.1. Comparison of the rejection rates at the 5% level for hypotheses (3.2) with $f = 0$

ρ	σ^2	CI ranks $(r_0, r_{1/4}, r_{1/2})$								
		(1,0,0)	(1,0,1)	(1,0,2)	(1,1,0)	(1,1,1)	(1,1,2)	(1,2,0)	(1,2,1)	(1,2,2)
	0.5	0.249	0.233	0.236	0.028	0.025	0.025	0.025	0.023	0.025
−0.5	1.0	0.262	0.256	0.262	0.030	0.022	0.024	0.031	0.024	0.024
	2.0	0.273	0.276	0.280	0.035	0.025	0.025	0.038	0.026	0.026
	0.5	0.318	0.323	0.324	0.028	0.014	0.016	0.030	0.013	0.016
0.0	1.0	0.340	0.345	0.351	0.039	0.018	0.019	0.041	0.019	0.019
	2.0	0.376	0.374	0.373	0.047	0.020	0.020	0.047	0.022	0.023
	0.5	0.386	0.401	0.407	0.040	0.013	0.013	0.039	0.013	0.015
0.5	1.0	0.445	0.464	0.467	0.053	0.014	0.014	0.052	0.014	0.014
	2.0	0.494	0.516	0.526	0.061	0.014	0.014	0.059	0.014	0.014

For the effects on the tests for CI rank at a given frequency, we are interested in the following hypothesis:

$$H_0 : r_f \leq 1 \quad vs. \quad H_1 : r_f > 1, \qquad \text{for } f = 0, \ \frac{1}{2}, \ \frac{1}{4}. \tag{3.2}$$

In each test, at a given frequency f, the CI ranks at other frequencies are set to be underspecified as 0, exact-specified as 1, or overspecified as 2. (We omit a report for the powers of the CI rank test: $H_0 : r_f = 0$ because almost all the powers were 100% irrespective of the misspecification of CI ranks at other frequencies.) For testing the hypotheses, the likelihood ratio test statistic is used:

$$\text{LR} = \ln\left(\frac{\max_{H_1} L}{\max_{H_0} L}\right) = -T \ln\left(\frac{\max_{H_1} |\hat{\Omega}|}{\max_{H_0} |\hat{\Omega}|}\right),$$

where L is the likelihood function based on the normality assumption and $\hat{\Omega}$ is the covariance matrix of the residuals resulting from the GRR estimation.

Samples of series with length $T = 100$, representing quarterly data over 25 years, are generated with 1,000 replications. Note that the quarter is one of the most frequently used seasonal periods; *e.g.*, gross domestic product(GDP), which is a very important macroeconomic variable, is observed quarterly and the series length $T = 100$ is a frequently used size. For example, Cubadda (2001) analyzed Italian quarterly time series over 25 years and Ahn *et al.* (2004) modeled monthly US Housing Starts over 10 years($T = 120$).

We use initial values that are set to zero, but discard the first 50 observations in order to eliminate dependence on the starting conditions. The estimated model is a VAR(5) with unrestricted seasonal deterministic terms. All the tests are based on the 5% asymptotic critical values that are obtained from JS and Lee and Siklos (1995).

The results of the simulations are summarized in Tables 3.1–3.6. Tables 3.1, 3.3 and 3.5 show the rejection rates of the tests for hypothesis (3.2) when $f = 0$, 1/2 and 1/4, respectively. The other tables measure the performances in terms of the means and the mean squared errors(MSEs) of the CI vectors in the selected cases. From the performances, we can investigate the effects of the misspecification of CI ranks at other frequencies on the estimation of seasonal CI vectors at the frequency of interest.

From Tables 3.1, 3.3 and 3.5, it is generally observed that irrespective of the value of ρ the larger the value of σ^2, the worse is the size distortion. Conditional on a given σ^2, the larger the value of ρ, the better is the size distortion. From Table 3.1, for frequency 0, we see that the sizes are worse when we underspecify the CI rank at frequency 1/4. In overspecifying the CI rank, the tests seem

Table 3.2. Means and mean squared errors for frequency 0 in the case of $\rho = 0.5$ and $\sigma^2 = 2$, based on 1,000 replications

CI ranks	$a_{11} = 0.6$		$a_{21} = 0.6$		$b_1 = -0.7$	
$(r_0, r_{1/4}, r_{1/2})$	Mean	MSE	Mean	MSE	Mean	MSE
(1,0,0)	−0.025	0.404	1.322	0.579	−0.671	0.002
(1,0,1)	−0.007	0.386	1.271	0.531	−0.666	0.003
(1,0,2)	−0.015	0.395	1.267	0.526	−0.666	0.003
(1,1,0)	0.492	0.031	0.552	0.067	−0.705	0.001
(1,1,1)	0.571	0.012	0.423	0.068	−0.709	0.001
(1,1,2)	0.567	0.013	0.414	0.072	−0.709	0.001
(1,2,0)	0.473	0.036	0.533	0.070	−0.704	0.001
(1,2,1)	0.554	0.014	0.400	0.076	−0.709	0.001
(1,2,2)	0.549	0.014	0.391	0.081	−0.709	0.001

Table 3.3. Comparison of the rejection rates at the 5% level for hypotheses (3.2) with $f = 1/2$

ρ	σ^2	CI ranks $(r_0, r_{1/4}, r_{1/2})$								
		(0,0,1)	(0,1,1)	(0,2,1)	(1,0,1)	(1,1,1)	(1,2,1)	(2,0,1)	(2,1,1)	(2,2,1)
	0.5	0.084	0.024	0.029	0.037	0.050	0.055	0.035	0.050	0.055
−0.5	1.0	0.084	0.037	0.043	0.039	0.058	0.059	0.040	0.059	0.059
	2.0	0.081	0.062	0.070	0.035	0.060	0.061	0.034	0.059	0.061
	0.5	0.086	0.026	0.029	0.035	0.056	0.059	0.040	0.057	0.062
0.0	1.0	0.083	0.035	0.038	0.040	0.060	0.065	0.042	0.060	0.065
	2.0	0.088	0.053	0.056	0.041	0.064	0.066	0.040	0.063	0.066
	0.5	0.082	0.020	0.025	0.045	0.056	0.058	0.051	0.055	0.057
0.5	1.0	0.075	0.025	0.033	0.040	0.060	0.065	0.044	0.060	0.063
	2.0	0.079	0.043	0.047	0.043	0.064	0.063	0.046	0.063	0.063

Table 3.4. Means and mean squared errors for frequency 1/2 in the case of $\rho = 0.5$ and $\sigma^2 = 2$, based on 1,000 replications

CI ranks	$a_{12} = -0.4$		$a_{22} = 0.6$		$b_2 = 0.3$	
$(r_0, r_{1/4}, r_{1/2})$	Mean	MSE	Mean	MSE	Mean	MSE
(0,0,1)	−0.3060	0.0182	0.7821	0.0624	0.2938	0.0002
(0,1,1)	−0.2391	0.0541	0.6164	0.0566	0.2989	0.0004
(0,2,1)	−0.2521	0.0462	0.6031	0.0534	0.2977	0.0009
(1,0,1)	−0.3186	0.0148	0.6789	0.0196	0.2931	0.0002
(1,1,1)	−0.3621	0.0107	0.5145	0.0443	0.3014	0.0000
(1,2,1)	−0.3603	0.0102	0.5160	0.0430	0.3014	0.0001
(2,0,1)	−0.3099	0.0148	0.6701	0.0181	0.2938	0.0001
(2,1,1)	−0.3600	0.0108	0.5038	0.0463	0.3014	0.0001
(2,2,1)	−0.3581	0.0104	0.5055	0.0449	0.3014	0.0001

to be conservative in the sense that almost all sizes never exceed 5%. This is consistent with the performances seen in Table 3.2.

From Table 3.3, for frequency 1/2, it is difficult to observe any clear regularity. However, we observe that the sizes are better when all the CI ranks at frequencies 0 and 1/4 are exact or overspecified. In general, when one of the CI ranks is underspecified, the tests seems to be conservative except for the case $(r_0, r_{1/4}, r_{1/2}) = (0, 1, 2)$. Similarly in Table 3.4, the performances seem to be better when all the CI ranks at frequencies 0 and 1/4 are exact or overspecified, especially for the cases of a_{11} and b_2.

From Table 3.5, for the case of frequency 1/4, it is observed that the underspecification of the CI rank at frequency 0 makes the sizes worse, irrespective of ρ and σ^2. This phenomenon may be

Table 3.5. Comparison of the rejection rates at the 5% level for hypotheses (3.2) with $f = 1/4$

ρ	σ^2	CI ranks $(r_0, r_{1/4}, r_{1/2})$								
		(0,1,0)	(0,1,1)	(0,1,2)	(1,1,0)	(1,1,1)	(1,1,2)	(2,1,0)	(2,1,1)	(2,1,2)
	0.5	0.093	0.058	0.066	0.019	0.020	0.017	0.018	0.019	0.017
−0.5	1.0	0.132	0.100	0.105	0.020	0.018	0.019	0.020	0.017	0.018
	2.0	0.167	0.138	0.143	0.019	0.016	0.016	0.020	0.016	0.016
	0.5	0.083	0.048	0.055	0.024	0.025	0.024	0.024	0.025	0.024
0.0	1.0	0.112	0.091	0.094	0.027	0.029	0.030	0.027	0.029	0.031
	2.0	0.137	0.113	0.116	0.030	0.024	0.026	0.030	0.025	0.026
	0.5	0.139	0.094	0.095	0.073	0.075	0.079	0.074	0.076	0.080
0.5	1.0	0.165	0.128	0.134	0.075	0.084	0.086	0.076	0.084	0.083
	2.0	0.199	0.164	0.167	0.078	0.077	0.077	0.079	0.079	0.078

Table 3.6. Means and mean squared errors for frequency 1/4 in the case of $\rho = 0.5$ and $\sigma^2 = 2$, based on 1,000 replications

CI ranks	$a_{13} = 0.6$		$a_{23} = -0.6$		$a_{14} = 0.4$		$a_{24} = -0.8$	
$(r_0, r_{1/4}, r_{1/2})$	Mean	MSE	Mean	MSE	Mean	MSE	Mean	MSE
(0,1,0)	0.0189	0.3471	−0.4488	0.0760	0.5756	0.0417	−1.4164	0.4307
(0,1,1)	0.1996	0.1959	−0.8406	0.1279	0.3849	0.0387	−0.9836	0.0999
(0,1,2)	0.2148	0.1820	−0.8293	0.1181	0.3746	0.0372	−0.9874	0.0972
(1,1,0)	0.2504	0.1319	−0.1150	0.2547	0.6760	0.0903	−1.3010	0.2889
(1,1,1)	0.5597	0.0163	−0.5433	0.0464	0.4080	0.0154	−0.9431	0.0713
(1,1,2)	0.5642	0.0152	−0.5392	0.0417	0.4042	0.0151	−0.9454	0.0678
(2,1,0)	0.2369	0.1418	−0.0851	0.2840	0.6654	0.0847	−1.2775	0.2653
(2,1,1)	0.5531	0.0172	−0.5096	0.0515	0.4055	0.0153	−0.9311	0.0684
(2,1,2)	0.5575	0.0161	−0.5059	0.0470	0.4018	0.0150	−0.9334	0.0650
CI ranks	$b_3 = 0.7$		$b_4 = -0.2$					
$(r_0, r_{1/4}, r_{1/2})$	Mean	MSE	Mean	MSE				
(0,1,0)	0.6934	0.0011	−0.1875	0.0009				
(0,1,1)	0.6837	0.0013	−0.1881	0.0006				
(0,1,2)	0.6840	0.0013	−0.1884	0.0006				
(1,1,0)	0.6979	0.0009	−0.2062	0.0008				
(1,1,1)	0.6945	0.0006	−0.2044	0.0005				
(1,1,2)	0.6948	0.0006	−0.2046	0.0005				
(2,1,0)	0.6976	0.0010	−0.2054	0.0008				
(2,1,1)	0.6942	0.0007	−0.2042	0.0005				
(2,1,2)	0.6945	0.0006	−0.2044	0.0005				

clearer when we compare the size of $(r_0, r_{1/4}, r_{1/2}) = (0, 1, 2)$ with that of $(2,1,2)$. From Table 3.6, the performances, measured in terms of the MSEs, are not coincident with the results of the sizes because the performances showed no improvement in cases where the CI rank at frequency 0 is underspecified. Cases where the CI rank at all frequencies is exact or overspecified appear to have better performances.

Therefore, the validity of the finite uncorrelatedness is very poor in spite of the asymptotic uncorrelatedness. The interesting observation is that the results of the cases for $f = 0$ and $1/4$ are symmetric, in that when the CI rank at one frequency is underspecified, the inference for the other frequency is worse. The commonly accepted fact that the frequencies $f = 0$ and $1/2$ are symmetric, is not validated in our simulation for the finite samples.

The results of our simulation provide guidelines for prespecifying the CI ranks at the frequencies we are not interested in when we conduct the inference using GRR procedures. If we arbitrarily prespecify the CI ranks based on the asymptotic uncorrelatedness, there is an ample possibility of the occurrences of a misspecification of the model.

4. Conclusion

The finite sample properties of the asymptotic uncorrelatedness assumed in most seasonal cointegration analyses have not been studied. We investigate the efficiency of the asymptotic uncorrelatedness in a finite sample in order to identify the degree of interaction among nonstationary roots in finite samples. For the investigation, we consider the performances of CI rank tests using GRR procedure when CI ranks at the non-focused frequencies are over- or underspecified. We observe that the validity of the finite uncorrelatedness has some drawbacks(*i.e.*, the rate of convergence to the asymptotic uncorrelatedness becomes very slow) when CI rank at frequency 0 is underspecified.

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