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A Proposal for Improving the Perception of Differential Concept by Using a Well-Known Table Processor: MS Excel[1]

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In this study, an innovative computer support has been suggested to improve differential perception of the students. Research has been conducted on a calculus class which has 35 students. A semi-structured interview has been reported in the study. By this interview, it was tried to make differential concept more understandable by using Micro Soft Excel component of the well-known MS Office software. By this aim, students have been asked to integrate a simple function by using MS Excel. At the end of the study, it was observed that differential concept made more sense in students' minds than previous.

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MSC2000 Classification: 97C30, 97D40

1. INTRODUCTION

Most of the fundamental calculus concepts, like derivative and integral, are generally perceived as only procedural concepts that help us to take derivative of function or integrate a function. This situation can be met usually; if an instructor's educative purpose is to make his or her students get ready for a multiple choice exam, like national

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university entrance exams. But, we have a serious reality that, our students, who deserve to attend math or engineering department of an outstanding university, also do not be aware of what does differential mean, even if they could answer all kind of integral and derivative questions.

This kind of situation is one of the important handicaps, which cause to educate mathematical illiterate generation. We have to take note of the latest developments in using technology for mathematics education. If we can use appropriately, computer software provide us innovative teaching opportunities in mathematics.

Computer Algebra Systems like Maple and Mathlab, can be considered as latest innovative tools in Math teaching (Harris, 2000; Fuchs, 2002; 2003). By appropriate using of a CAS, meaningful improvements in conceptual understanding have been observed (Zehavi, 2006; Dubinsky & Schwingendorf, 2004). Furthermore, using a programming language has been also suggested for teaching math concepts (Dubinsky, 1995). Dubinsky has suggested that using a programming language like IELTS may help students to improve their perceptions on the most of the mathematical concepts. According to Dubinsky, constructing an algorithm for a math concept and applying this algorithm with a simple programming language can support students' understanding

2. PROBLEM SITUATION

Although, any carefully selected computer software can be helpful to make a math concept more meaningful in our students' mind, some obstacle may be lived while using that software. For example, if you intend to plan an application for your students with a CAS, you will have to organize a training session for the software.

Microsoft Office components are being taught in the first year of almost all universities and even in colleges. Therefore, without an extra software training session, this kind of software can be integrated to our math teaching environment. By this reason, we decided to use Excel while implementing our study.

3. METHODOLOGICAL FRAMEWORK OF THE RESEARCH

A qualitative research has been implemented in this study. A semi structured interview has been conducted in a calculus course.

3.1. Participant

Participants of the study were the 1^{st} grade calculus course students of an average university in the Aegean region of Turkey. Gender distribution of the class was approximately equal (19 boys and 16 girls). Participant had been already taught the calculus concepts, limit, differential and integral. But, it could be easily observed that their focal point was procedural properties of the concepts.

3.2. Data Collection and Analysis

The data were collected in the spring semester of the 2007 – 2008 education years. Framework of the research was constructed on two consequently questions. First was the following common conceptual calculus question;

"Can you explain the function of *dx* in the symbolization of $\int_a^b f(x)dx$."

After analyzing of the students' responses, second question, which was actually a problem, has been asked;

"Can you integrate a simple function like $f(x) = \sin(x)$ in the interval [1, 2] by using MS Excel?"

By this problem, we had intended to make a simple computer software help students. Students were asked to study on this problem in a discussion environment. Even almost every students of the class had joined to the discussion, only four students' talk (Students A, B, C and D), which can reflect the main idea, had been reported as an interview. By reporting the interview, it is tried to reveal the improvement of students' differential perception.

4. FINDINGS

Even if this class has been taught fundamental limit, derivative and integral concept, none of them would have made a perfect explanation for the first question. While most of them said;

"*dx* is an only a symbol which means we have to integrate the function according to *x* variable."

Only three students answered:

"*dx* means differential of the variable *x*."

but they could not explain the function of *dx* in the symbolization of $\int_a^b f(x)dx$.

These kinds of answers easily mean that, the concept definition image and the concept image of basic differential do not match each other. Our students had already been reached following conclusion for the meaning of

$$\int_a^b f(x)dx$$

at the end of previous calculus course session.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

as n is the partition number of the interval $[a, b]$, i is the index number of partition, Δx_i is the ith partition' length and x_i is any x value in the ith partition.

Although students have this concept definition image, they could not give a satisfying response for the question. So, we can say that desired concept image is not constituted in students' brain.

4.1. Report of the Interview While Solving the Excel Problem

MS Excel can perform simple mathematical operations but can not perform the operations related with infinitesimals. If you want to integrate a function by using MS Excel you need to adapt conceptual principles to a language which is applicable by MS Excel. By helping our students to construct a solution for this problem, we expected our students can strength their differential concept image.

A brief report of this interview in class is below:

Student A: I know that Excel can calculate the sinus of an x value. But I think it can not integrate a function.

Teacher: Yes! You are right. To make Excel to integrate a function you have to consider what integration actually is.

Student B: Do you mean the pure definition of integral?

Teacher: Yes! But the problem is to find how to adapt the definition to MS Excel. Could you analyze the definition, in terms of its performable operations in Excel?

Student C: let's look at the definition again.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

Teacher: Who wants to explain this definition?

Student A: Here, we divide $[a, b]$ interval into n parts. Every partition's length is Δx_i. We need to find the sum of the product of $f(x_i)$ and Δx_i.

Teacher: We need to add something more! Every Δx_i can be equal and $f(x_i)$ is a value of xi from ith partition. And particularly we can choose the x_i as right end value or left end value of the partition.

Student B: OK! But I still could not understand how we will use the MS Excel because Excel can not perform a limit operation.

Student C: Yes! But Excel can calculate almost all function like trigonometric, logarithmic and exponential.

Teacher: We can try to solve limit operation problem, with an approximate solution. Could you please think about the relation between dx and Δx ?

Student D: Actually, integral basically means continuous sum. When we take limit of

$$\sum_{i=1}^{n} f(x_i)\Delta x_i ,$$

the length of Δx_i approaches to 0 and we call it dx.

Teacher: OK! Now, I want you to design a table in Excel like below; in the table-1, write a value which is the left end value of the interval [a, b] in the Cell A2. Then write an Excel formula, which calculate the image of a, in the Cell B2.

Then, we need to make decision for Δx. Instead of dx value, we will determine a Δx value approximate to 0. After decision, let's write the value $(a + \Delta x)$ in the Cell A3.

	A	B	C	D
1	x value	f(x) value	Δx value	f(x).Δx
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Table - 1

	A	B	C	D
1	x value	f(x) value	Δx	f(x).Δx
2	1	0,841471	0,2	0,168294
3	1,2	0,932039	0,2	0,186408
4	1,4	0,98545	0,2	0,19709
5	1,6	0,999574	0,2	0,199915
6	1,8	0,973848	0,2	0,19477
7	2	0,909297	0,2	0,181859
8				1,128336
9				

Table - 2

Student C: I think, I understand what we will try to do. I have just constructed the table-2. After filling only first two rows as in table, I have scrolled down all table, to the x value is 2 which is the right end value of the interval [1, 2].

The sum of the interval D2:D7, which is calculated in the yellow Cell, must be

$$\int_{1}^{2} \sin(x)dx$$

Teacher: OK! Let's check the result by using a Computer Algebra System (CAS) like Maple.

> **evalf**(int(sin(x), x = 1..2));

0.9564491424

Student A: I thing everything was correct. But, I could not understand why the result is incorrect.

Teacher: Yes, everything is being seen as correct. But, you did not make a care that $f(x)$ values are calculated by using left end of the intervals whose length is Δx. So, try to find the sum after deleting the D7 Cell (look table-3).

	A	B	C	D
1	x value	f(x) value	Δx	f(x).Δx
2	1	0,841471	0,2	0,168294
3	1,2	0,932039	0,2	0,186408
4	1,4	0,98545	0,2	0,19709
5	1,6	0,999574	0,2	0,199915
6	1,8	0,973848	0,2	0,19477
7	2	0,909297	0,2	
8				0,946476

Table - 3

	A	B	C	D
1	x value	f(x) value	Δx	f(x).Δx
2	1	0,841471	0,1	0,084147
3	1,1	0,891207	0,1	0,089121
4	1,2	0,932039	0,1	0,093204
5	1,3	0,963558	0,1	0,096356
6	1,4	0,98545	0,1	0,098545
7	1,5	0,997495	0,1	0,099749
8	1,6	0,999574	0,1	0,099957
9	1,7	0,991665	0,1	0,099166
10	1,8	0,973848	0,1	0,097385
11	1,9	0,9463	0,1	0,09463
12	2	0,909297	0,1	
13				0,952261

Table - 4

	A	B	C	D
1	x value	f(x) value	Δx	f(x).Δx
2	1	0,841471	0,05	0,042074
3	1,05	0,867423	0,05	0,043371
4	1,1	0,891207	0,05	0,04456
5	1,15	0,912764	0,05	0,045638
6	1,2	0,932039	0,05	0,046602
7	1,25	0,948985	0,05	0,047449
8	1,3	0,963558	0,05	0,048178
9	1,35	0,975723	0,05	0,048786
10	1,4	0,98545	0,05	0,049272
11	1,45	0,992713	0,05	0,049636
12	1,5	0,997495	0,05	0,049875
13	1,55	0,999784	0,05	0,049989
14	1,6	0,999574	0,05	0,049979
15	1,65	0,996865	0,05	0,049843
16	1,7	0,991665	0,05	0,049583
17	1,75	0,983986	0,05	0,049199
18	1,8	0,973848	0,05	0,048692
19	1,85	0,961275	0,05	0,048064
20	1,9	0,9463	0,05	0,047315
21	1,95	0,92896	0,05	0,046448
22	2	0,909297	0,05	
23				0,954554

Table - 5

Student B: Yeah! Now the result is very close to the real solution.

Student D: This is very normal. Because we have just used Δx as 0,2 instead of dx which approaches to 0.

Teacher: Try same procedure by taking Δx as 0,1.

Student A: While Δx approaches to 0 the sum is getting closer to original value as in table-4.

Let's try for $\Delta x = 0{,}05$ in table-5.

Student B: I am sure that if we choose the Δx closer to 0 the sum value we obtained will be closer to original integral value. But, we will need a very big table. I have just exactly realized the meaning of differential of x.

Consequently, we have reached the pure meaning of the differential concept with our students. By the way, we also had done an interesting application of integrating process. Our students have found a chance of studying a mathematician from the math history. We have only supplied a technological opportunity to fasten the discovery process.

In a study, which is conducted by using Maple, students had lived an abstraction process while making a two variable function (Sugeng, 1999). In our study, we had tried to make our students experience same process by using more simple software.

5. RESULT AND DISCUSSION

In this research, an example of how a simple table processor can be used to improve students conceptual understanding on a calculus concept, has been presented. At the end of the study, it is observed that the differential perception made more sense in students' minds.

Many researchers have used a CAS to make some Math concepts more tangible. Most of these studies have shoved a positive effect of computers to conceptual understanding (Donald, 1997; Embse, 2001). One of the important common claim of these researches is providing opportunity to students develop their own solution methods on computers. That is, using computers, as producing ready solutions, can not be useful to improve their comprehension. When we consider on this point, computer based teaching environment, proposed in this study, may be accepted an innovative and effective one.

Furthermore, an algorithm, which makes the theoretical meaning of differential is applicable in Excel, has been developed. By this way, working of the basic mean of differential has been made more clear and comprehensible for the students as Dubinsky had already been said (Dubinsky, 1995).

As a conclusion it can be said that, we should take into consideration all kind of opportunity of using computer to help our students to construct their own conceptual understandings.

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