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Intelligent Sliding Mode Control for Robots Systems with Model Uncertainties

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Abstract : This paper proposes an intelligent sliding mode control method for robotic systems with the unknown bound of model uncertainties. In our control structure, the unknown bound of model uncertainties is used as the gain of the sliding controller. Then, we employ the function approximation technique to estimate the unknown nonlinear function including the width of boundary layer and the uncertainty bound of robotic systems. The adaptation laws for all parameters of the self-recurrent wavelet neural network and those for the reconstruction error compensator are derived from the Lyapunov stability theorem, which are used for an on-line control of robotic systems with model uncertainties and external disturbances. Accordingly, the proposed method can not only overcome the chattering phenomenon in the control effort but also have the robustness regardless of model uncertainties and external disturbances. Finally, simulation results for the five-link biped robot are included to illustrate the effectiveness of the proposed method.

Keywords : intelligent sliding mode control, function approximation technique, robotic systems, biped robots

I. Introduction

The sliding mode control (SMC) is known as one of simple and popular techniques for a robust control of robotic systems with uncertainties and external disturbances due to its simplicity, fast response and good performance [1-5]. Even if the conventional SMC (CSMC) has these advantages, it has two important drawbacks. First, the bounds of uncertainties and external disturbances of the plant must be known for solving control problems [6-8]. However, in real applications, since the variation of the system parameters is difficult to predict and the external disturbance changed according to the environment is also difficult to know, the switching gain of the sliding phase in the CSMC law cannot be computed accurately. In [6], the SMC algorithm with the ability to estimate the unknown bounds of parameters is developed for the trajectory control of robot manipulators. However, this method requires the regressor matrix of robotic systems which is difficult to compute for the complex robotic system such as a biped robot. Second, the CSMC suffers from chattering control input owing to its discontinuous switching control input and its delays in the sliding phase [7,8]. The chattering control input may excite high-frequency dynamics neglected in the modeling which degrades the performance of the system, and may even lead to instability. Therefore, many approaches have been reported to alleviate the chattering phenomenon in the control input [7-11]. Although the fixed boundary layer method [11] is used commonly for attenuating the chattering control input, the stability of the controller is

guaranteed only outside of the boundary layer and the tracking error is bounded by the width of boundary layer [9]. Owing to these disadvantages of the fixed boundary layer, the SMC design methods for a time-varying boundary layer and a time-varying gain are proposed [8,9]. However, these techniques require the uncertainty bound of nonlinear systems and the complex mathematical efforts.

Accordingly, in this paper, the intelligent SMC method is proposed to control robotic systems with model uncertainties. We first introduce the robot model whose uncertainties are separated, then the sliding controller with the intelligent gain and boundary is designed. In the proposed control system, the bound of the uncertainty assumed to be unknown is used as the gain of the sliding phase with the boundary layer. The function approximation technique using self-recurrent wavelet neural networks (SRWNN) [12] is used to approximate the unknown nonlinear function including the width of boundary layer and the uncertainty bound of robotic systems. Besides, the acceleration term in the sliding phase is added to accelerate the convergence of the operating point. From Lyapunov stability theorem, the adaptation laws for learning the weights of the SRWNN and those for the error compensator are induced out of consideration for the stability, robustness and performance of the proposed control system. In addition, we prove that all signals in a closed-loop adaptive control system are uniformly ultimately bounded. Finally, we simulate the five-link biped robot to show the effectiveness of the proposed SRWNN based SMC system. In addition, our control method is compared with the CSMC method.

This paper is organized as follows. In Section 2, we introduce the model of robot systems with uncertainties and present the problems of the CSMC through the design procedure of CSMC. Section 3 discusses the SRWNN based SMC strategy for solving the robust control problem of the robotic systems. Here, the stability, robustness, and performance of the proposed control system are analyzed via the Lyapunov stability theorem.

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Simulation results are discussed in Section 4. Finally, Section 5 gives some conclusions.

II. Problem Formulation

1. Model of robot systems with uncertainties

The nominal model of a robot system having n rigid joints can be expressed in the following Lagrange form [5]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) = \boldsymbol{\tau}, \quad (1)$$

where \mathbf{q} , $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}} \in R^n$ are the joint position, velocity, and acceleration respectively. $\mathbf{M}(\mathbf{q}) \in R^{n \times n}$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in R^n$ denotes the Coriolis and centripetal forces, $\mathbf{G}(\mathbf{q}) \in R^n$ is the gravity vector, $\mathbf{F}(\dot{\mathbf{q}}) \in R^n$ represents the friction term, and the control input torque is $\boldsymbol{\tau} \in R^n$.

However, due to the model uncertainty, the nominal values $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{G}(\mathbf{q})$, and $\mathbf{F}(\dot{\mathbf{q}})$ for a given robot may be different from the actual values. Therefore, we define the actual values of $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{G}(\mathbf{q})$, and $\mathbf{F}(\dot{\mathbf{q}})$ as $\bar{\mathbf{M}}(\mathbf{q})$, $\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})$, $\bar{\mathbf{G}}(\mathbf{q})$, and $\bar{\mathbf{F}}(\dot{\mathbf{q}})$, respectively. If the external disturbance τ_d influences the robot system, the actual dynamics of the nominal model (1) can be represented as

$$\bar{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) + \bar{\mathbf{G}}(\mathbf{q}) + \bar{\mathbf{F}}(\dot{\mathbf{q}}) + \tau_d = \boldsymbol{\tau}. \quad (2)$$

Assumption 1: Suppose that the nominal values $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{G}(\mathbf{q})$, and $\mathbf{F}(\dot{\mathbf{q}})$ are only the known functions for a given robot, but the actual values $\bar{\mathbf{M}}(\mathbf{q})$, $\bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})$, $\bar{\mathbf{G}}(\mathbf{q})$, $\bar{\mathbf{F}}(\dot{\mathbf{q}})$, and the external disturbance τ_d are the unknown functions.

We must express the model uncertainty separately with the nominal model according to Assumption 1. The actual robot dynamics (2) can be rewritten in the following formulation using the nominal model [13]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) + \Xi(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}) = \boldsymbol{\tau}, \quad (3)$$

where

$$\begin{aligned} \Xi(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}) \equiv & -\mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1}(\mathbf{q})\{\boldsymbol{\tau} - \tau_d - \bar{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) - \bar{\mathbf{G}}(\mathbf{q}) - \bar{\mathbf{F}}(\dot{\mathbf{q}})\} \\ & + \{\boldsymbol{\tau} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}})\} \end{aligned} \quad (4)$$

denotes the total uncertainty of the robot system. The total uncertainty term $\Xi(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau})$ cannot be evaluated directly by Assumption 1.

2. Problem statement of the CSMC system

The actual model (3) of the robot system with the model uncertainty can be written as follows:

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})\{-\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}}) + \boldsymbol{\tau}\} + \Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d), \quad (5)$$

where $\Lambda(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}) \equiv -\mathbf{M}^{-1}(\mathbf{q})\Xi(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau})$ denotes the uncertainty term. Here, $\boldsymbol{\tau}$ is a function of \mathbf{q} , $\dot{\mathbf{q}}$ and $\mathbf{Q}_d = (\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)$ which denotes the reference position, velocity, and acceleration. Accordingly, we obtain

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})\{-\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}}) + \boldsymbol{\tau}\} + \Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d), \quad (6)$$

where the bounds of the components $\Lambda_i(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d)$ of the uncertainty vector $\Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d)$ are assumed to be given, that is,

$$|\Lambda_i(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d)| < \sigma_i, \quad (7)$$

where $i = 1, 2, \dots, n$, the components σ_i of the vector $\boldsymbol{\sigma}$ are positive constants. To design the CSMC system, the uncertainty bound σ_i must be known. Therefore, in this section, suppose that σ_i is a known positive constant. This is one of the problems of the CSMC which will be discussed in Remark 1.

The control problem is to find a control law so that the joint position \mathbf{q} can track any reference position \mathbf{q}_d . Define the time-varying sliding surface vector $\mathbf{S}(t)$ as follows [8]:

$$\mathbf{S}(t) = \dot{\mathbf{E}}(t) + \mathbf{P}_1\mathbf{E}(t) + \mathbf{P}_2 \int_0^t \mathbf{E}(\tau) d\tau, \quad (8)$$

where $\mathbf{E}(t) = \mathbf{q}(t) - \mathbf{q}_d(t)$ is the tracking error vector, $\mathbf{P}_1 = \text{diag}[p_{1,i}]$, and $\mathbf{P}_2 = \text{diag}[p_{2,i}]$. Here $p_{1,i} > 0$ and $p_{2,i} > 0$ ($i = 1, \dots, n$) are constants and $\text{diag}[\cdot]$ denotes a diagonal matrix. If the states are outside the sliding surface, to drive the states to the sliding surface, we need the sliding condition as follows [8]:

$$\begin{aligned} \mathbf{S}^T(t)\dot{\mathbf{S}}(t) &= \sum_{i=1}^n S_i(t)\dot{S}_i(t) \\ &\leq -\sum_{i=1}^n \mu_i |S_i(t)|, \quad \text{if } \mathbf{S}(t) \neq 0. \end{aligned} \quad (9)$$

where $S_i(t)$ is the i th component of $\mathbf{S}(t)$ and the constant μ_i is strictly positive. Differentiating (8) and substituting (6) into it, we can obtain

$$\begin{aligned} \dot{\mathbf{S}}(t) &= \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d + \mathbf{P}_1\dot{\mathbf{E}} + \mathbf{P}_2\mathbf{E} \\ &= \mathbf{M}^{-1}(\mathbf{q})\{-\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}}) + \boldsymbol{\tau}\} \\ &\quad + \Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d) - \ddot{\mathbf{q}}_d + \mathbf{P}_1\dot{\mathbf{E}} + \mathbf{P}_2\mathbf{E}. \end{aligned} \quad (10)$$

Then, the total CSMC law is assumed to take the following form:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_r + \boldsymbol{\tau}_s, \quad (11)$$

where

$$\boldsymbol{\tau}_r = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) + \mathbf{M}(\mathbf{q})(\ddot{\mathbf{q}}_d - \mathbf{P}_1\dot{\mathbf{E}} - \mathbf{P}_2\mathbf{E}), \quad (12)$$

$$\boldsymbol{\tau}_s = -\mathbf{M}(\mathbf{q})\mathbf{L}\text{sgn}(\mathbf{S}(t)) \quad (13)$$

in which $\mathbf{L} = \text{diag}[L_i]$; $L_i > 0$ are constants, and $\text{sgn}(\cdot)$ is a sign function. The reaching control input $\boldsymbol{\tau}_r$ is used to cancel the nonlinear term and to specify the desired system performance. And the sliding control input $\boldsymbol{\tau}_s$ is used to keep the controlled system dynamics in the sliding surface and to guarantee the convergence of the state trajectory. Substituting (10) into (9), and using (11) - (13), the sliding condition can be written as follows:

$$\begin{aligned} \mathbf{S}^T(t)\dot{\mathbf{S}}(t) &= \mathbf{S}^T(t)[\mathbf{M}^{-1}(\mathbf{q})\{-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}}) + \tau\} \\ &\quad + \Lambda(\mathbf{q},\dot{\mathbf{q}},\mathbf{Q}_d) - \ddot{\mathbf{q}}_d + \mathbf{P}_1\dot{\mathbf{E}} + \mathbf{P}_2\mathbf{E}] \\ &\leq \sum_{i=1}^n \sigma_i |S_i(t)| - L_i |S_i(t)|, \end{aligned}$$

so that, letting $L_i = \mu_i + \sigma_i$, the sliding condition (9) is satisfied throughout the whole control period and the stability of controlled system is guaranteed. However, the CSMC method has the following problems.

Remark 1: (Major drawbacks of the CSMC design)

1) In real robot systems, since the parameter variations (internal uncertainties) are difficult to measure, and the exact value of the external disturbance is also difficult to know in advance, we cannot apply the CSMC technique to the robust control of robotic systems with unknown bounds of the uncertainties and disturbances.

2) In the CSMC technique, the sign function (sgn(·)) of the sliding control input τ_s results in the chattering phenomenon, exciting unstable system dynamics and easy damage of mechanism, in the control effort. It is common that the boundary layer method [11] is used for attenuating the chattering control input. Indeed, the control signal is smoother than the original one without using a boundary layer. However, the boundary layer method has a drawback that the width of boundary layer is difficult to choose optimally. A thin boundary layer for obtaining extreme tracking accuracy has risks exciting a high-frequency control input and the chattering control input. Also, a thick boundary layer results in the large steady-state error and does not ensure the convergence of the state trajectory of the system to the sliding surface.

III. Intelligent SMC System for Robotic System

In order to solve problems of the CSMC technique in Remark 1, we now introduce the design method of SMC system based on the SRWNN for robotic systems. See [12] for the details structure of the SRWNN.

To design the intelligent SMC system shown in Fig. 1, suppose that the uncertainty bounds σ_i in (7) are unknown. First, based on the facts that the nominal values are only known, we propose the reaching control input τ_r as follows:

$$\tau_r = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) + \mathbf{M}(\mathbf{q})\{\ddot{\mathbf{q}}_d - \mathbf{P}_1\dot{\mathbf{E}} - \mathbf{P}_2\mathbf{E}\}, \quad (14)$$

Then, the sliding control input τ_s is presented to guarantee the convergence of the state trajectory and to eliminate the chattering phenomena as follows:

$$\tau_s = -\mathbf{M}(\mathbf{q})(\Upsilon + \mathbf{K})\mathbf{S}(t),$$

where $\Upsilon = \text{diag}[\sigma_i / (|S_i(t)| + R_i)]$; $\sigma_i / (|S_i(t)| + R_i) > 0$ ($i = 1, 2, \dots, n$). Here, the uncertainty bound σ_i is used as the gain of the sliding control input. Also, R_i , which is the small positive constant, denotes the width of the boundary layer. The second term $-\mathbf{M}(\mathbf{q})\mathbf{K}\mathbf{S}(t)$ is used to accelerate the convergence of the operating point; $\mathbf{K} = \text{diag}[K_i]$ and $K_i > 0$ is a constant.

According to the powerful approximation ability [12] and

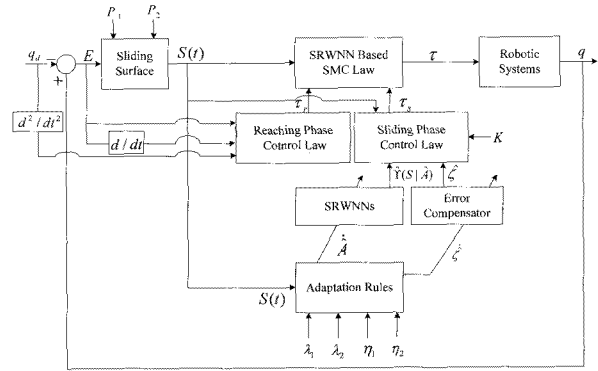


그림 1. 제안된 제어 시스템의 구성도.

Fig. 1. Block diagram of the proposed control system.

Assumption 1, the SRWNN system $\hat{\Upsilon}$ will be employed to approximate the nonlinear term Υ to a sufficient degree of accuracy. Thus, the nonlinear term Υ including the unknown uncertainty bound and the width of the boundary layer can be described by the optimal SRWNN plus a reconstruction error vector ε over a compact set K_s as follows:

$$\begin{aligned} \Upsilon &= \hat{\Upsilon}(\mathbf{S} | \mathbf{A}^*) + \varepsilon \\ &= \hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}}) + [\hat{\Upsilon}(\mathbf{S} | \mathbf{A}^*) - \hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}})] + \varepsilon, \end{aligned} \quad (15)$$

where the sliding surface $\mathbf{S} \in K_s$ is used as the SRWNN input, $\hat{\mathbf{A}} = \text{diag}[\hat{A}_i]$; \hat{A}_i are the estimated values of the weighting vector \mathbf{A} of the SRWNN and \mathbf{A}^* is the optimal weighting diagonal matrix defined as

$$\mathbf{A}^* = \arg \min_{\hat{\mathbf{A}}} \left[\sup_{\mathbf{S} \in K_s} \|\Upsilon(\mathbf{S}) - \hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}})\| \right],$$

Assumption 2: Assume that the diagonal elements of the optimal weight matrix are bounded as $\|\mathbf{A}^*\| \leq A_{M,i}$.

Note that the bounded values $A_{M,i}$ are not required to implement the controller proposed in this paper. These values are used only for the stability analysis of the intelligent control system. Then, taking the Taylor series expansion of $\hat{\Upsilon}(\mathbf{S} | \mathbf{A}^*)$ around $\hat{\mathbf{A}}$, it can be obtained that [14]

$$\hat{\Upsilon}(\mathbf{S} | \mathbf{A}^*) - \hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}}) = \tilde{\mathbf{A}}^T \Theta_{\mathbf{A}} + \mathbf{H}(\mathbf{A}^*, \hat{\mathbf{A}}), \quad (16)$$

where $\tilde{\mathbf{A}} = \mathbf{A}^* - \hat{\mathbf{A}}$, $\Theta_{\mathbf{A}} = \text{diag} \left[\frac{\partial \hat{\Upsilon}(\mathbf{S} | \mathbf{A}^*)}{\partial A_i} \right]$, and \mathbf{H} is a high-order term. Substituting (16) into (15), we obtain

$$\Upsilon = \hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}}) + \tilde{\mathbf{A}}^T \Theta_{\mathbf{A}} + \alpha, \quad (17)$$

$$|\alpha_i| \leq \zeta_i, \quad (18)$$

where $\alpha = \mathbf{H}(\mathbf{A}^*, \hat{\mathbf{A}}) + \varepsilon = \text{diag}[\alpha_i]$. ζ_i is a positive constant. Therefore, the sliding control law is redefined as

$$\tau_s = -\mathbf{M}(\mathbf{q}) \left(\hat{\Upsilon}(\mathbf{S} | \hat{\mathbf{A}}) + \hat{\zeta} + \mathbf{K} \right) \mathbf{S}, \quad (19)$$

where $\hat{\Upsilon} = \text{diag}[\hat{\Upsilon}_i]$, the second term $-\mathbf{M}(\mathbf{q})\hat{\zeta}\mathbf{S}$ is added to compensate the approximation error ε and the high-order term \mathbf{H} of the SRWNN; $\hat{\zeta} = \text{diag}[\hat{\zeta}_i]$ ($i=1,2,\dots,n$) is the estimated diagonal matrix of ζ . Accordingly, from (14) and (19), we propose the total control input as follows:

$$\begin{aligned} \tau &= \tau_r + \tau_s \\ &= \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) \\ &\quad + \mathbf{M}(\mathbf{q})\left\{\ddot{\mathbf{q}}_d - \mathbf{P}_1\dot{\mathbf{E}} - \mathbf{P}_2\mathbf{E} - \left(\hat{\Upsilon}(\mathbf{S}|\hat{\mathbf{A}}) + \hat{\zeta} + \mathbf{K}\right)\mathbf{S}\right\}. \end{aligned} \quad (20)$$

Theorem 1: Assume that the robot system (6) with unknown model uncertainty is controlled by the intelligent SMC law (20). If the proposed control system satisfies Assumptions 1-2 and the adjustable parameters of the SRWNN are tuned by the following adaptation rules:

$$\dot{\hat{A}}_i = \lambda_{1,i}\Theta_{A,i}S_i^2 - \eta_1\lambda_{1,i}\hat{A}_i, \quad (21)$$

$$\dot{\hat{\zeta}}_i = \lambda_{2,i}S_i^2 - \eta_2\lambda_{2,i}\hat{\zeta}_i, \quad (22)$$

where $i=1,2,\dots,n$, $\lambda_{1,i}$ and $\lambda_{2,i}$ are positive tuning gains, η_1 and η_2 are positive constants, and $\Theta_{A,i}$ is the i -th diagonal element of Θ_A , then there exist $\lambda_{l,i}$, η_l , ($l=1,2$) and \mathbf{K} such that the errors of states and adjustable weights of the closed-loop system are uniformly ultimately bounded and may be kept arbitrarily small.

Proof: Let the Lyapunov function candidate be given by

$$V = \frac{1}{2}\mathbf{S}^T\mathbf{S} + \frac{1}{2}\text{tr}(\tilde{\mathbf{A}}^T\lambda_1^{-1}\tilde{\mathbf{A}}) + \frac{1}{2}\text{tr}(\tilde{\zeta}^T\lambda_2^{-1}\tilde{\zeta}), \quad (23)$$

where $\tilde{\zeta} = \zeta - \hat{\zeta}$, $\lambda_j = \text{diag}[\lambda_{j,i}]$; $\lambda_{j,i} > 0$ ($j=1,2, i=1,\dots,n$) is a constant, $\text{tr}(\cdot)$ denotes the trace of a matrix. Differentiating the Lyapunov function (23) and using (20),

$$\begin{aligned} \dot{V} &= \mathbf{S}^T\dot{\mathbf{S}} - \text{tr}(\tilde{\mathbf{A}}^T\lambda_1^{-1}\dot{\tilde{\mathbf{A}}}) - \text{tr}(\tilde{\zeta}^T\lambda_2^{-1}\dot{\tilde{\zeta}}) \\ &= \mathbf{S}^T[\mathbf{M}^{-1}(\mathbf{q})\{-\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q}) - \mathbf{F}(\dot{\mathbf{q}}) + \tau\} + \Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d) \\ &\quad - \ddot{\mathbf{q}}_d + \mathbf{P}_1\dot{\mathbf{E}} + \mathbf{P}_2\mathbf{E}] - \text{tr}(\tilde{\mathbf{A}}^T\lambda_1^{-1}\dot{\tilde{\mathbf{A}}}) - \text{tr}(\tilde{\zeta}^T\lambda_2^{-1}\dot{\tilde{\zeta}}) \\ &= \mathbf{S}^T\left\{\Lambda(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{Q}_d) - \left(\hat{\Upsilon} + \hat{\zeta} + \mathbf{K}\right)\mathbf{S}\right\} \\ &\quad - \text{tr}(\tilde{\mathbf{A}}^T\lambda_1^{-1}\dot{\tilde{\mathbf{A}}}) - \text{tr}(\tilde{\zeta}^T\lambda_2^{-1}\dot{\tilde{\zeta}}) \\ &\leq \sum_{i=1}^n \left[\sigma_i |S_i| - \left(\hat{\Upsilon}_i + \hat{\zeta}_i + K_i\right)S_i^2 \right. \\ &\quad \left. - \frac{1}{\lambda_{1,i}}\tilde{A}_i^T\dot{\tilde{A}}_i - \frac{1}{\lambda_{2,i}}\tilde{\zeta}_i^T(t)\dot{\tilde{\zeta}}_i(t) \right] \end{aligned} \quad (24)$$

Using $\Upsilon_i^* + \varepsilon_i = \sigma_i/|S_i| + R_i$, σ_i in (24) can be given by

$$\begin{aligned} \sigma_i &= (\Upsilon_i^* + \varepsilon_i)(|S_i| + R_i) \\ &= \Upsilon_i^*|S_i| + \varepsilon_i|S_i| + (\Upsilon_i^* + \varepsilon_i)R_i. \end{aligned} \quad (25)$$

Then, in the CSMC, since the narrow boundary layer R_i is chosen to satisfy the stability of the control system (i.e., $R_i \rightarrow 0$), the term $[\Upsilon_i^* + \varepsilon_i]R_i$ in (25) can be ignored. Thus, using (16),

(24) is approximated as

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^n \left[-K_i S_i^2 + \zeta_i \left(S_i^2 - \frac{1}{\lambda_{2,i}} \dot{\zeta}_i \right) \right. \\ &\quad \left. + \tilde{A}_i^T \left(\Theta_{A,i} S_i^2 - \frac{1}{\lambda_{1,i}} \dot{\tilde{A}}_i \right) \right] \end{aligned} \quad (26)$$

Then, substituting the adaptation laws (21) and (22) via the σ -modification into (26), we can obtain

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^n \left[-K_i S_i^2 + \eta_1 \tilde{A}_i^T \dot{\tilde{A}}_i + \eta_2 \tilde{\zeta}_i^T \dot{\tilde{\zeta}}_i \right] \\ &\leq \sum_{i=1}^n \left[-K_i S_i^2 - \frac{1}{2} \eta_1 \|\tilde{A}_i\|^2 - \frac{1}{2} \eta_2 |\tilde{\zeta}_i|^2 + C \right], \end{aligned} \quad (27)$$

where $C = \frac{1}{2}\eta_1 A_{M,i}^2 + \frac{1}{2}\eta_2 \zeta_i^2$. Then, let us choose the constant ρ satisfying the following condition:

$$0 < \rho < \min \left[K_i, \frac{\eta_1 \lambda_{1,m}}{2}, \frac{\eta_2 \lambda_{2,m}}{2} \right]$$

where $\lambda_{1,m}$ and $\lambda_{2,m}$ are the minimum eigenvalues of λ_1 and λ_2 , respectively. Therefore, (27) can be obtained as $\dot{V} \leq -2\rho V + C$. This relation implies that $\dot{V} < 0$ when $V > (C/2\rho)$. Accordingly, the tracking error \mathbf{S} , the weight estimation error $\tilde{\mathbf{A}}$, and the compensation error $\tilde{\zeta}$ are uniformly ultimately bounded in the following compact set D :

$$D = \left\{ \mathbf{S}, \tilde{\mathbf{A}}, \tilde{\zeta} \left\| \|\mathbf{S}\|^2 + \frac{1}{\max[\lambda_{1,M}, \lambda_{2,M}]} \left(\|\tilde{\mathbf{A}}\|_F^2 + \|\tilde{\zeta}\|_F^2 \right) \leq \frac{C}{\rho} \right\}, \right.$$

where $\|\cdot\|_F$ denotes the Frobenius norm, $\lambda_{1,M}$ and $\lambda_{2,M}$ are the maximum eigenvalues of λ_1 and λ_2 , respectively. In addition, the compact set D can be kept arbitrarily small by adjusting \mathbf{K} , λ_l , and η_l , ($l=1,2$). That is, the sliding surface error \mathbf{S} can be made arbitrarily small. This completes the proof.

Remark 2: 1) In Theorem 1, $\Theta_{A,i}$ can be computed using the chine rule and it is shown in [12]. 2) In the adaptation laws (21) and (22), σ -modification technique [15] is used to prevent the parameter drift. Also, we can use a smooth projection operator method [16], and e -modification method [17] in place of σ -modification technique.

Remark 3: It is common that the boundary layer method [11] is used for attenuating the chattering control input. This is difficult to achieve optimal trade-off between the control bandwidth and the tracking precision. That is, a thin boundary layer for obtaining extreme tracking accuracy has risks exciting a high-frequency control input and the chattering control input. Also, a thick boundary layer results in the large steady-state error and does not ensure the convergence of the state trajectory of the system to the sliding surface. In addition, the upper bounds of the internal uncertainties and external disturbances are required to compute the gains of the sliding controllers [1-5].

However, in the proposed intelligent SMC method, the bounds of the total uncertainties including both the internal uncertainties and external disturbances are used as the gains of the sliding controllers. These gains (bounds of the total uncertainties) and the width of the boundary layer are approximated by using SRWNNs. Therefore, our control system can tune the sliding gains according to the size of uncertainties and no upper bounds of the uncertainties and external disturbances are needed. Besides, the proposed intelligent SMC method can remove the chattering phenomena in the control inputs without the choice of the boundary layer.

IV. Computer Simulations

In this Section, the five-link biped robot with model uncertainties is simulated. The proposed control system is compared with the CSMC system designed in Section 2.2 to illustrate the robustness and the chattering elimination ability in the control effort of the proposed control system. The complete motion of the biped robot can be explained by a single support phase, a double support phase, double impact, switching and transformation [3]. Thus, there is a need to switch the dynamic equations and controllers during the iterative computation of the

표 1. 5족 이족 로봇의 파라미터들.

Table 1. Parameters for the five-link biped robots.

	m_i (Kg)		l_i (m)	d_i (m)	I_i Kg m^2
	Nominal	Actual	Nominal	Nominal	
LINK 1 (Right Leg)	2.23	4.51 $\times \cos(t)$	0.332	0.189	3.30×10^{-2}
LINK 2 (Right high)	5.28	6.82 $\times \sin(t)$	0.302	0.236	3.30×10^{-2}
LINK3 (Torso)	14.79	15.55	0.486	0.282	3.30×10^{-2}
LINK 4 (Left Thigh)	5.28	6.73 $\times \sin(t)$	0.302	0.236	3.30×10^{-2}
LINK 5 (Left Leg)	2.23	7.84	0.332	0.189	3.30×10^{-2}

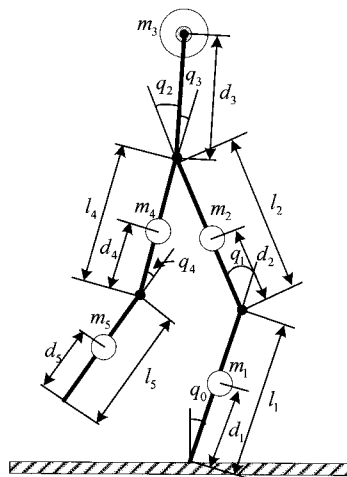


그림 2. 5절 이족 로봇.
Fig. 2. The five-link biped robots.

simulation program. However, this method causes the complex programming problems [18]. Accordingly, in this subsection, our control system is applied to the stable walking control of the planar five-link biped robot with only the single support phase. The dynamic model and the reference trajectory of the biped robot proposed in [2,18] are used in this simulation. Since q_0 of the biped robot model shown in Fig. 2 is the noncontrollable joint (i.e., $\tau_1 = 0$) [18], the control law is redefined as follows:

$$\tau = \mathbf{M}(\mathbf{q})\mathbf{U} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}),$$

where \mathbf{U} is the 5×1 vectors with components

$$u_1 = -\frac{1}{M_{11}} \left[\sum_{j=1}^4 M_{1,j+1} \cdot u_{j+1} + C_1 + G_1 + F_1 \right],$$

$$u_{j+1} = \ddot{q}_{d,j} - P_{1,j} \dot{E}_j - P_{2,j} E_j - \left(\hat{Y}_j(S_j | \hat{B}_j) + \hat{\zeta}_j + K_j \right) S_j(t).$$

Here, M_{1L} ($L=1,2,\dots,5$) denote the components of the first row of matrix $\mathbf{M}(\mathbf{q})$. C_1 , G_1 , and F_1 are the first components of vectors $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{G}(\mathbf{q})$, and $\mathbf{F}(\dot{\mathbf{q}})$, respectively. The initial positions are set to $q_1(0) = q_2(0) = q_3(0) = q_4(0) = 0$ and the link masses m_i s of the biped robot are assumed to be uncertain. Especially, m_1 , m_2 , and m_4 are assumed to have the time-varying uncertainties and m_5 is assumed to have about 300% uncertainty of its nominal value. The parameters of the five-link biped are listed in Table 1. In addition, it is assumed that the external disturbance $\tau_d = [0.5\sin(3t) \ 0.5\cos(3t) \ 0.5\sin(3t) \ 0.5\sin(3t) \ 0.5\cos(3t)]^T$ influences the biped robot. In both the CSMC method and the proposed method, the common parameters \mathbf{P}_1 and \mathbf{P}_2 for controlling from q_1 to q_4 are chosen as

$$\mathbf{P}_1 = \text{diag} [150 \ 150 \ 350 \ 350],$$

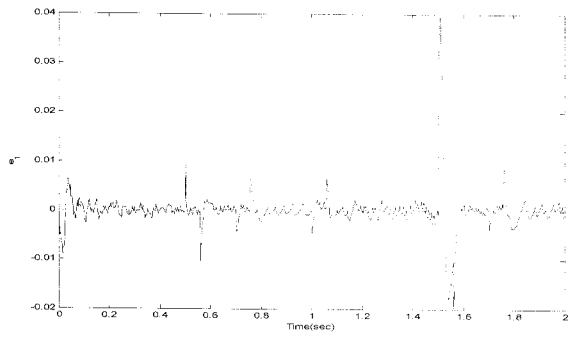
$$\mathbf{P}_2 = \text{diag} [10 \ 10 \ 10 \ 10].$$

The additional parameters for the CSMC method are chosen as $\sigma_1 = \sigma_2 = 300$, $\sigma_3 = \sigma_4 = 600$ and $\mathbf{L} = \text{diag} [0.001 \ 0.001 \ 0.001 \ 0.001]$. The additional design parameters for the intelligent SMC system are chosen as $\lambda_{1,i} = 0.02$, $\lambda_{2,i} = 0.8$, ($i=1,\dots,4$) and $\eta_1 = \eta_2 = 0.01$. In addition, to examine the control performance examine the control performance on the effect of the acceleration term, the acceleration gains are chosen in

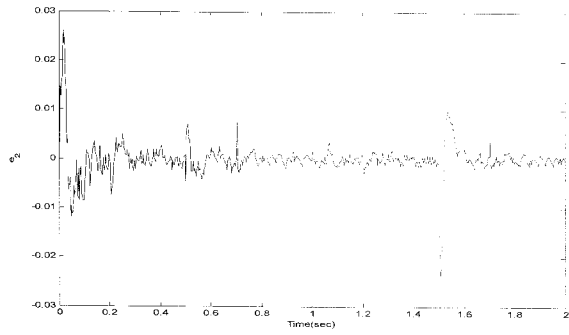
표 2. 가속 이득에 따른 제어 성능.

Table 2. Control performance on the acceleration gain.

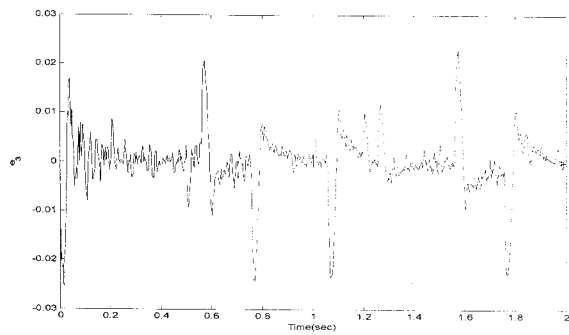
	e_1	e_2	e_3	e_4
$K_1 = K_2 = 5$ $K_3 = K_4 = 20$	0.0050	0.0037	0.0049	0.0022
$K_1 = K_2 = 8$ $K_3 = K_4 = 60$	0.0048	0.0036	0.0045	0.0020
$K_1 = K_2 = 12$ $K_3 = K_4 = 120$	0.0046	0.0034	0.0040	0.0018



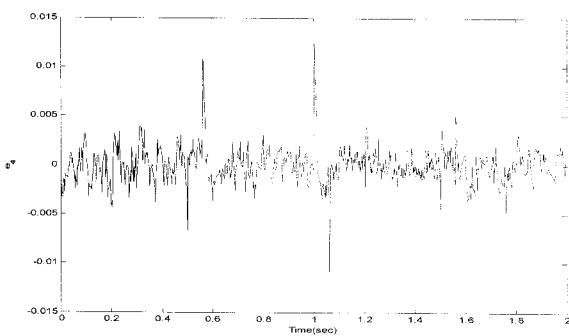
(a) e_1



(b) e_2



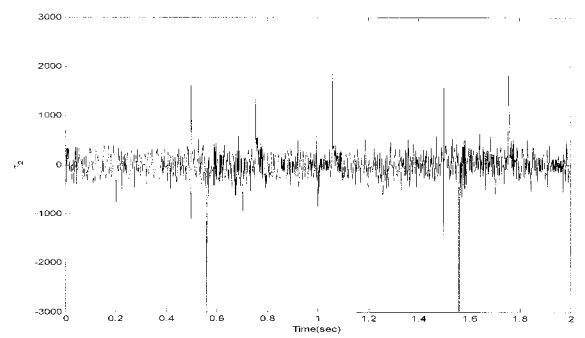
(c) e_3



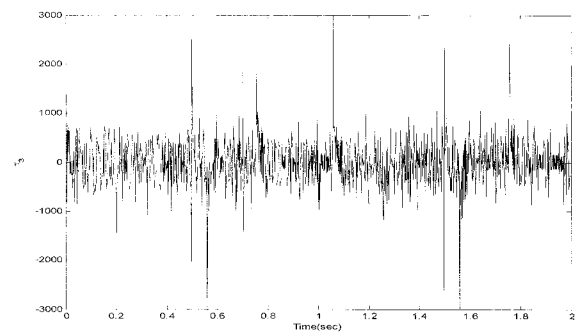
(d) e_4

그림 3. CSMC 시스템의 추종 오차.
Fig. 3. Tracking errors of CSMC system.

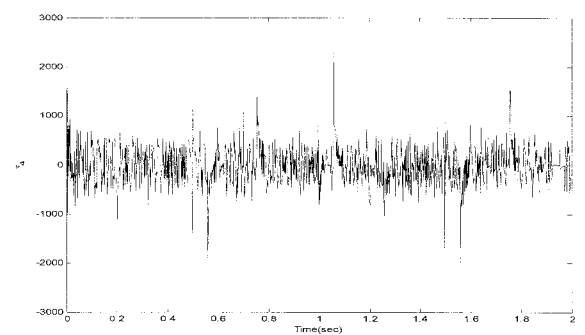
Table 2. In Table 2, note that as the acceleration gains increase, the control performance is improved. The simulation results of the CSMC system and the proposed control system for the biped robot are compared in Figs. 3-6. These figures and Table 2 reveal that the SRWNN based SMC system has the good tracking performance and the smooth control input even under the time-varying model uncertainties and external disturbances.



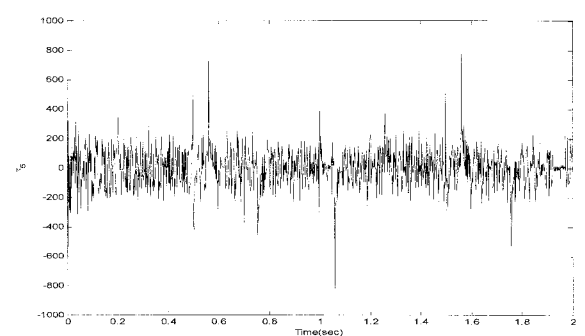
(a) τ_2



(b) τ_3



(c) τ_4

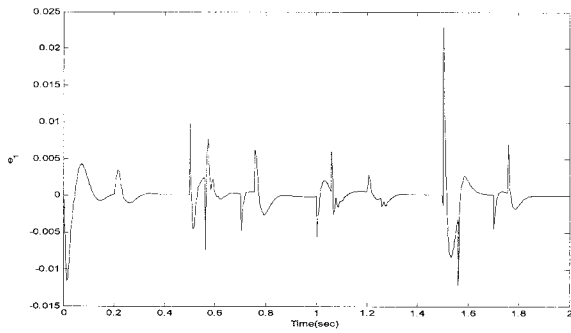


(d) τ_5

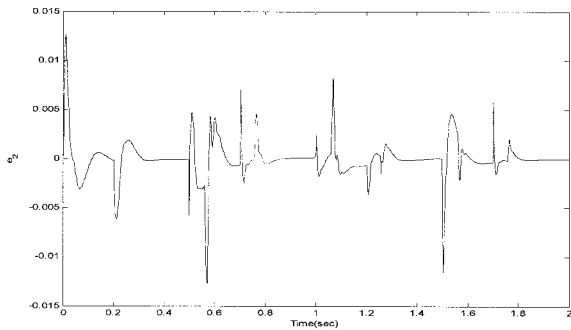
그림 4. CSMC 시스템의 제어 입력.
Fig. 4. Control inputs of CSMC system.

V. Conclusions

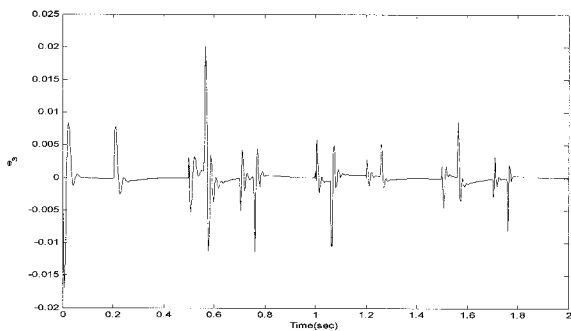
The intelligent SMC method for robotic systems with model uncertainties has been developed. The SRWNN having the simple structure has been used to approximate the nonlinear term including the sliding gain and the width of the boundary layer. The adaptation laws for training weights of SRWNNs and error compensators have been induced from the Lyapunov stability



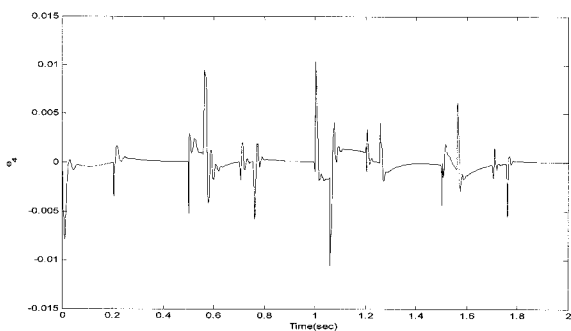
(a) e_1



(b) e_2



(c) e_3

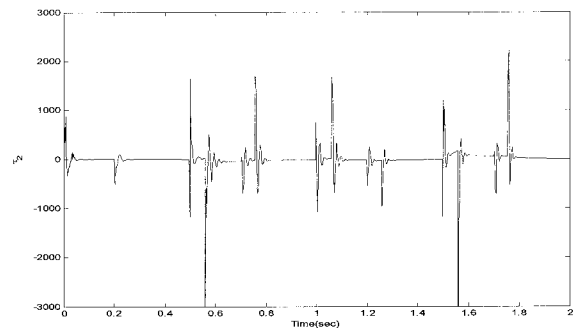


(d) e_4

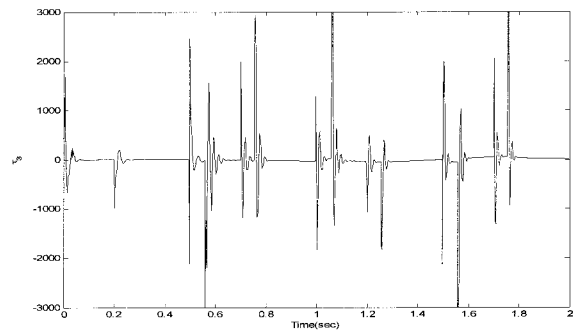
그림 5. 제안된 제어 시스템의 추종 오차.

Fig. 5. Tracking errors of the proposed control system.

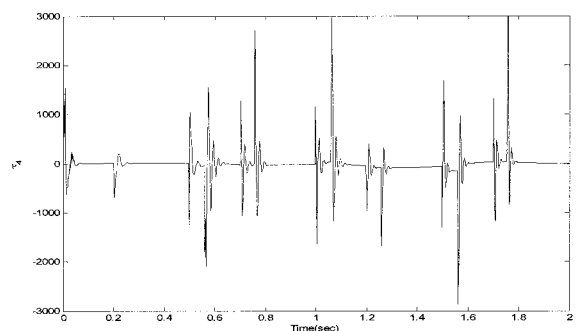
theory, which have been applied to guarantee the uniformly ultimately boundedness of all signals in the proposed control system. Besides, the acceleration term in the sliding phase has been added to accelerate the convergence to the inside of the sliding surface. Finally, the simulations have been performed to show that proposed control system could be applied to the robust control of robotic systems without the knowledge of the



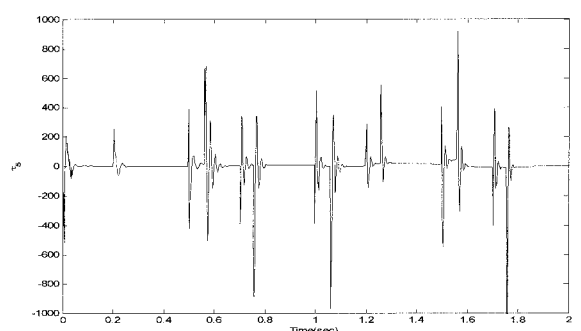
(a) τ_2



(b) τ_3



(c) τ_3



(d) τ_4

그림 6. 제안된 제어 시스템의 제어 입력.

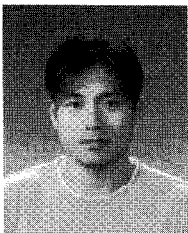
Fig. 6. Control inputs of the proposed control system.

uncertainty bound and overcome the chattering phenomena in the control efforts.

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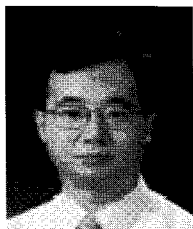
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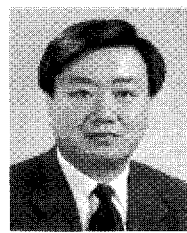
networks theories, and their applications to robotic, flight, chaos, and time delay systems.



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