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Interval-Valued Fuzzy Minimal Structures and Interval-Valued Fuzzy Minimal Spaces

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Abstract

We introduce the concept of interval-valued minimal structure which is an extension of the interval-valued fuzzy topology. And we introduce and study the concepts of IVF m-continuous and several types of compactness on the interval-valued fuzzy m-spaces.

Key words : interval-valued fuzzy minimal spaces, interval-valued fuzzy m-open sets, interval-valued fuzzy m-closed sets, IVF m-continuous, IVF m-compact

1. Introduction and preliminaries

Zadeh [5] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concept of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3].

In [2], Alimohammady and Roohi introduced fuzzy minimal structures and fuzzy minimal spaces and some results are given. In this paper, we introduce the concept of interval valued fuzzy minimal structure (simply, IVF minimal structure) as a generalization of interval valued fuzzy topology introduced by Mondal and Samanta [5]. Also we introduce the concepts of IVF m-continuity and IVF m-open (IVF m-closed) map and we study some results about them. Finally we introduce and study IVF m-compactness, almost IVF m-compactness and nearly IVF m-compactness.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

1. $(\forall M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$.
2. $(\forall M, N \in D[0,1])(M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U)$.

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$.

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Let X be a nonempty set. A mapping $A : X \quad D[0,1]$ is called an interval-valued fuzzy set(simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [a,b]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all $x \ X$ is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. Denoted by IVF(X) the set of all IVF sets in X.

For every $A, B \in IVF(X)$, we define

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L$$

and

$$[A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L$$

and

$$[A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by

$$[A^c(x)]^L = 1 - [A(x)]^U \text{ and } [A^c(x)]^U = 1 - [A(x)]^L$$

for all $x \in X$.

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$(\forall x \in X)\ ([G(x)]^L = \sup_{i\in J}[A_i(x)]^L,$

$[G(x)]^U = \sup_{i\in J}[A_i(x)]^U),$

$(\forall x \in X)\ ([F(x)]^L = \inf_{i\in J}[A_i(x)]^L,$

$[F(x)]^U = \inf_{i\in J}[A_i(x)]^U),$

respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined as follows:

$$[f(A)(y)]^L = \begin{cases} \sup_{f(x)=y}[A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise ,} \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup_{f(x)=y}[A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise ,} \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined as follows:

$(\forall x \in X)([f^{-1}(B)(x)]^L = [B(f(x))]^L,$

$[f^{-1}(B)(x)]^U = [B(f(x))]^U).$

Definition 1.1 ([5]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies:

1. $\mathbf{0}, \mathbf{1} \in \tau$.
2. $A, B \in \tau \Rightarrow A \cap B \in \tau$.
3. For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i\in J}A_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set A is called an IVF closed set if the complement of A is an IVF open set. And (X, τ) is called an *interval-valued fuzzy topological space*.

In an IVF topological space (X, τ), for an IVF set A in X, the IVF closure and the IVF interior of A, denoted by $Cl(A)$ and $Int(A)$, respectively, are defined as

$Cl(A) = \cap\{B \in IVF(X) : B^c \in \tau \text{ and } A \subseteq B\},$

$Int(A) = \cup\{B \in IVF(X) : B \in \tau \text{ and } B \subseteq A\},$

respectively.

Let (X, τ) be an IVF topological space. An IVF set A in X is said to be *IVF compact* if every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of B has a finite IVF subcover. And an IVF set A in X is said to be *almost IVF compact* (resp., *nearly IVF compact*) if for every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of B, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i\in J_0}Cl(A_i)$ (resp., $A \subseteq \cup_{i\in J_0}Int(Cl(A_i))$).

Definition 1.2 ([4]). An IVF set A in an IVF topological space (X, τ) is called

1. an IVF semiopen set in X if there exists $B \in \tau$ such that $B \subseteq A \subseteq Cl(B)$);
2. an IVF preopen set in X if $A \subseteq Int(Cl(A))$;
3. an IVF α-open set in X if $A \subseteq Int(Cl(Int(A)))$).

And an IVF set A is called an IVF semiclosed (resp., IVF preclosed, IVF α-closed) set if the complement of A is an IVF semiopen (resp., IVF preopen, IVF α-open) set. Denoted by IVFSO(X) (resp., IVFPO(X), IVFα(X)) the set of all IVF semiopen (resp., IVF preopen, IVF α-open) sets. Denoted by IVFSC(X) (resp., IVFPC(X), IVFαC(X)) the set of all IVF semiclosed (resp., IVF preclosed, IVF α-closed) sets.

2. Interval-valued fuzzy minimal structures and IVF minimal-spaces

Definition 2.1. A family $\mathcal{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

In this case, $(X, \mathcal{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathcal{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Example 2.2. Let (X, τ) be an IVF topological space. Then τ, IVFSO(X), IVFSC(X), IVFPO(X) and IVFPC(X) are all interval-valued fuzzy minimal spaces.

Definition 2.3. Let $(X, \mathcal{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure and the IVF minimal-interior of A, denoted by $mCl(A)$ and $mInt(A)$, respectively, are defined as

$mCl(A) = \cap\{B \in IVF(X) : B^c \in \mathcal{M} \text{ and } A \subseteq B\},$

$mInt(A) = \cup\{B \in IVF(X) : B \in \mathcal{M} \text{ and } B \subseteq A\},$

respectively.

Theorem 2.4. Let $(X, \mathcal{M})$ be an IVF minimal space and A, B in IVF(X).

1. $mInt(A) \subseteq A$ and if A is an IVF m-open set, then $mInt(A) = A$.
2. $A \subseteq mCl(A)$ and if A is an IVF m-closed set, then $mCl(A) = A$.
3. If $A \subseteq B$, then $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$.
4. $mInt(A) \cap mInt(B) = mInt(A \cap B)$ and $mCl(A) \cup mCl(B) = mCl(A \cup B)$.
5. $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(A)) = mCl(A)$.
6. $\mathbf{1} - mCl(A) = mInt(\mathbf{1} - A)$ and $\mathbf{1} - mInt(A) = mCl(\mathbf{1} - A)$.

Proof. (1), (2) and (3) are obvious.

(4) For A, B in IVF(X), since $mCl(A) \subseteq mCl(A \cup B)$ and $mCl(B) \subseteq mCl(A \cup B)$, we get $mCl(A) \cup mCl(B) \subseteq mCl(A \cup B)$. For ${K_1}^c, {K_2}^c \in \mathcal{M}$, we note that ${K_1}^c \cap {K_2}^c$ does not always belong to $\mathcal{M}$. Thus we get the following:

$mCl(A) \cup mCl(B) = (\cap\{K_1 \in IVF(X) : {K_1}^c \in \mathcal{M}$ and $A \subseteq K_1\}) \cup (\cap\{K_2 \in IVF(X) : {K_2}^c \in \mathcal{M}$ and $B \subseteq K_2\}) = \cap\{K_1 \cup K_2 \in IVF(X) : {K_1}^c, {K_2}^c \in \mathcal{M}$ and $A \subseteq K_1, B \subseteq K_2\} \supseteq \{K \in IVF(X) : K^c \in \mathcal{M}$ and $A \cup B \subseteq K\} = mCl(A \cup B)$.

Hence we get $mCl(A) \cup mCl(B) = mCl(A \cup B)$.
Similarly, $mInt(A) \cap mInt(B) = mInt(A \cap B)$.

(5) By (1) and (2), it is obvious.

(6) Obvious. □

Example 2.5. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows:

$$A(a) = [0.1, 0.6], A(b) = [0.2, 0.5]$$

and

$$B(a) = [0.2, 0.5], B(b) = [0.3, 0.4].$$

Consider $\mathcal{M} = \{\emptyset, A, B, X\}$ as an IVF m-structure on X. Let C be an IVF set defined as follows: $C(a) = [0.2, 0.6]$ and $C(b) = [0.3, 0.5]$; then since $C = A \cup B$, $mInt(C) = C$ but C is not IVF m-open.

Definition 2.6. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is said to be *interval-valued fuzzy minimal continuous* (simply, IVF m-continuous) function if for every $A \in \mathcal{M}_Y$, $f^{-1}(A)$ is in $\mathcal{M}_X$.

Theorem 2.7. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$.

1. f is IVF m-continuous.
2. $f^{-1}(B)$ is an IVF m-closed set, for each IVF m-closed set B in Y.
3. $f(mCl(A)) \subseteq mCl(f(A))$ for $A \in IVF(X)$.
4. $mCl(f^{-1}(B)) \subseteq f^{-1}(mCl(B))$ for $B \in IVF(Y)$.
5. $f^{-1}(mInt(B)) \subseteq mInt(f^{-1}(B))$ for $B \in IVF(Y)$.

Then (1) ⇔ (2) ⇒ (3) ⇔ (4) ⇔ (5).

Proof. (1) ⇔ (2) Obvious.

(2) ⇒ (3) For $A \in IVF(X)$,

$f^{-1}(mCl(f(A))) = f^{-1}(\cap\{F \in IVF(X) : f(A) \subseteq F$ and F is an IVF m-closed set $\}) = \cap\{f^{-1}(F) \in IVF(X) : A \subseteq f^{-1}(F)$ and F is an IVF m-closed set $\} \supseteq \cap\{K \in IVF(X) : A \subseteq K$ and K is an IVF m-closed set $\} = mCl(A)$.

Hence $f(mCl(A)) \subseteq mCl(f(A))$.

(3) ⇔ (4) Let $B \in IVF(Y)$. From (3) it follows that

$$f(mCl(f^{-1}(B))) \subseteq mCl(f(f^{-1}(B))) \subseteq mCl(B).$$

Hence we get (4).
Similarly, we get (4) ⇒ (3).

(4) ⇔ (5) From Theorem 2.4(6), it is obvious. □

Example 2.8. Let $X = \{a, b\}$. Let A, B and C be IVF sets defined as in Example 2.5. Consider $\mathcal{M} = \{\emptyset, A, B, X\}$ and $\mathcal{N} = \{\emptyset, A, B, C, X\}$ as IVF m-structures on X. Let $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ be a function defined as follows: $f(x) = x$ for each $x \in X$. Then f satisfies the condition (5) in Theorem 2.7, but it is not IVF m-continuous because $f^{-1}(C)$ is not IVF m-open for an IVF m-open set C.

Corollary 2.9. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following equivalent:

1. $f(A) \subseteq mCl(f(A))$ for each IVF m-closed set A in X.
2. $f^{-1}(B) = mCl(f^{-1}(B))$ for each IVF m-closed set in Y.
3. $f^{-1}(B) = mInt(f^{-1}(B))$ for each IVF m-open set in Y.

Definition 2.10. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is called *interval-valued fuzzy minimal open* (simply, IVF m-open) map if for every $A \in \mathcal{M}_X$, $f(A)$ is in $\mathcal{M}_Y$.

Theorem 2.11. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$.

1. f is IVF m-open.
2. $f(mInt(A)) \subseteq mInt(f(A))$ for $A \in IVF(X)$.
3. $mInt(f^{-1}(B)) \subseteq f^{-1}(mInt(B))$ for $B \in IVF(Y)$.

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $A \in IVF(X)$,

$f(mInt(A)) = f(\cup\{B \in IVF(X) : B \subseteq A, B$ is an IVF m-open set $\}) = \cup\{f(B) \in IVF(X) : f(B) \subseteq f(A), f(B)$ is an IVF m-open set $\} \subseteq \cup\{U \in IVF(X) : U \subseteq f(A), U$ is an IVF m-open set $\} = mInt(f(A))$.

Hence $f(mInt(A)) \subseteq mInt(f(A))$.

(2) $\Leftrightarrow$ (3) For $B \in IVF(Y)$, from (3) it follows that

$$f(mInt(f^{-1}(B))) \subseteq mInt(f(f^{-1}(B))) \subseteq mInt(B).$$

Hence we get (3). Similarly, we get (3) $\Rightarrow$ (2). □

Corollary 2.12. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If f is IVF m-open, then $f(A) = mInt(f(A))$ for every IVF m-open set$A$ in X.

Proof. From Theorem 2.4 and Theorem 2.11, it is obvious. □

Definition 2.13. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is called *interval-valued fuzzy minimal closed* (simply, IVF m-closed) map if for every IVF m-closed set A in X, $f(A)$ is an IVF m-closed set in Y.

Theorem 2.14. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$.

1. f is IVF m-closed.
2. $mCl(f(A)) \subseteq (f(mCl(A)))$ for $A \in IVF(X)$.
3. $f^{-1}(mCl(B)) \subseteq mCl(f^{-1}(B))$ for $B \in IVF(Y)$.

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. See Theorem 2.11. □

Corollary 2.15. Let $f : X \to Y$ be a function on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If f is IVF m-closed, then $f(A) = mCl(f(A))$ for every IVF m-closed set$A$ in X.

Proof. From Theorem 2.4 and Theorem 2.14, it is obvious. □

3. Several types of IVF m-compactness

Definition 3.1. Let $(X, \mathcal{M}_X)$ be an IVF minimal space and $\mathcal{A} = \{A_i : i \in J\}$. $\mathcal{A}$ is called an *interval-valued fuzzy cover* (simply, IVF cover) if $\cup\{A_i : i \in J\} = \mathbf{1}$, also $\mathcal{B}$ is called an *IVF cover* of an IVF B in X if $B \subseteq \cup\{A_i : i \in J\}$. It is an *IVF m-open cover* if each A_i is an IVF m-open set. An IVF subcover of $\mathcal{A}$ is a subfamily of it which also is an IVF cover.

Definition 3.2. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. An IVF set A in X is said to be *IVF m-compact* if every IVF m-open cover $\mathcal{A} = \{A_i : i \in J\}$ of B has a finite IVF subcover.

Theorem 3.3. Let $f : (X, \mathcal{M}_X) \to (Y, \mathcal{M}_Y)$ be an IVF m-continuous function on two IVF minimal spaces. If A is an IVF m-compact set, then $f(A)$ is also an IVF m-compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF m-open cover of $f(A)$ in Y. Then since f is an IVF m-continuous function, $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By IVF m-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} f^{-1}(B_i)$. Hence $f(A) \subseteq \cup_{i \in J_0} B_i$. It completes the proof. □

Definition 3.4. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. An IVF set A in X is said to be *almost IVF m-compact* if for every IVF m-open cover $\mathcal{A} = \{A_i : i \in J\}$ of B, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(A_i)$.

Theorem 3.5. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. If an IVF set A in X is IVF m-compact, then it is also almost IVF m-compact.

Proof. Obvious. □

In Theorem 3.5, the converse is not always true as shown the next example.

Example 3.6. Let $X = \{a, b\}$. For each $n \in N$ let A_n be an IVF set defined as follows:

$$A_n(a) = [\frac{n}{1+n}, 1], A_n(b) = [1, 1].$$

Let B be an IVF set defined as follows:

$$B(a) = [\frac{1}{3}, \frac{3}{5}], B(b) = [0, 1].$$

Consider $\mathcal{M} = \{\emptyset, A_n, B, X\}$ as an IVF m-structure on X. Let $\mathcal{A} = \{A_n : n \in N\}$ be an IVF m-open cover of X. Then there does not exist a finite subcover of $\mathcal{A}$. Thus X is not IVF m-compact. But since $\mathbf{1}$ is the only IVF m-closed set including $A_n \in \mathcal{A}$ for $n \in N$, X is almost IVF m-compact.

Theorem 3.7. Let $f : (X, \mathcal{M}_X) \to (Y, \mathcal{M}_Y)$ be an IVF m-continuous function on two IVF minimal spaces. If A is an almost IVF m-compact set, then $f(A)$ is also an almost IVF m-compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF m-open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By almost IVF m-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(f^{-1}(B_i))$. From Theorem 2.7 (4), it follows

$$\cup_{i \in J_0} mCl(f^{-1}(B_i)) \subseteq \cup_{i \in J_0} f^{-1}(mCl(B_i))$$
$$= f^{-1}(\cup_{i \in J_0} mCl(B_i)).$$

Hence $f(A) \subseteq \cup_{i \in J_0} mCl(B_i)$. It completes the proof. □

Definition 3.8. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. An IVF set A in X is said to be *nearly IVF m-compact* if for every IVF m-open cover $\mathcal{A} = \{A_i : i \in J\}$ of B, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mInt(mCl(A_i))$.

Theorem 3.9. Let $(X, \mathcal{M}_X)$ be an IVF minimal space. If an IVF set A in X is IVF m-compact, then it is also nearly IVF m-compact.

Proof. For any IVF m-open set U in X, from Theorem 2.4, it follows $A = mInt(A) \subseteq mInt(mCl(A))$. Thus we get the result. □

Theorem 3.10. Let $f : (X, \mathcal{M}_X) \to (Y, \mathcal{M}_Y)$ be an IVF m-continuous and IVF m-open function on two IVF minimal spaces. If A is a nearly IVF m-compact set, then $f(A)$ is also a nearly IVF m-compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF m-open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By nearly IVF m-compactness, there exists $J_0 = \{J_1, J_2, \cdots J_n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mInt(mCl(f^{-1}(B_i)))$. From Theorem 2.7 and Theorem 2.12, it follows:

$$f(A) \subseteq \cup_{i \in J_0} f(mInt(mCl(f^{-1}(B_i))))$$
$$\subseteq \cup_{i \in J_0} mInt(f(mCl(f^{-1}(B_i))))$$
$$\subseteq \cup_{i \in J_0} mInt(f(f^{-1}(mCl(B_i))))$$
$$\subseteq \cup_{i \in J_0} mInt(mCl(B_i)).$$

Hence the proof is completed. □

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