

# Non-linear Chirp UWB Ranging System with Narrowband Interference Suppression Abilities

Hanbing Shen, Weihua Zhang, and Kyung Sup Kwak

*ABSTRACT*—In this letter, we analyze an ultra-wideband ranging system based on non-linear chirp waveforms with the ability of narrowband interference suppression. A number of non-linear chirp waveforms are proposed and evaluated by simulation. The results verify that the proposed schemes can suppress the Narrowband interference to a certain degree.

*Keywords*—Non-linear chirp, ranging system, UWB, NBI.

## I. Introduction

Ultra-wideband (UWB) systems occupy ultra wide bandwidth that has been assigned to other existent narrowband (NB) wireless applications; thus, NB applications degrade UWB ranging system performance or even cause it to malfunction [1]. The method of using non-linear chirp waveforms to suppress NBI has been proposed in [2].

The UWB ranging system based on linear chirp waveform is discussed in [3]. In this letter, we propose a design method for non-linear chirp waveforms for the UWB ranging system with the ability to suppress narrowband interference (NBI). By simulation, we compare the performance of the linear chirp scheme and the proposed non-linear chirp schemes.

## II. Chirp in Ranging Systems

A chirp waveform is modeled in [4] as

$$w(t) = c(t) \cos[\Omega(t)], \quad (1)$$

Manuscript received Feb. 5, 2007; revised Mar. 31, 2007.

This work was supported by the MIC (Ministry of Information and Communication), S. Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Advancement), grant number IITA-2006-(C1090-0603-0019).

Hanbing Shen (phone: + 82 32 860 9189, email: shen\_ice@hotmail.com), Weihua Zhang (email: zhweihua2000@hotmail.com), and Kyung Sup Kwak (email: kskwak@inha.ac.kr) are with the Graduate School of IT&T, Inha University, Incheon, S. Korea.

where  $c(t)$  is the envelop of the chirp signal, which is zero outside a time duration  $T_s$ , and  $\Omega(t)$  is the phase function. The instantaneous frequency of  $w(t)$  is defined as

$$f_c(t) = \frac{1}{2\pi} \frac{d\Omega(t)}{dt}. \quad (2)$$

The chirp rate is the derivative of  $f_c(t)$ :

$$\mu(t) = \frac{df_c(t)}{dt} = \frac{1}{2\pi} \frac{d^2\Omega(t)}{dt^2}. \quad (3)$$

### 1. Linear Chirp

Linear chirp means that the instantaneous frequency  $f_c(t)$  varies linearly with time  $t$ :

$$f_c(t) = f_0 + \mu t, \quad (4)$$

where  $f_0$  is the center frequency. The phase function and the chirp rate are respectively given by

$$\begin{aligned} \Omega(t) &= 2\pi f_0 t + \pi \mu t^2, \\ \mu(t) &= \mu. \end{aligned} \quad (5)$$

Because the UWB spectrum band inevitably overlays some NB applications, NBI suppression is required.

### 2. Non-linear Chirp

The instantaneous frequency  $f_c(t)$  of a non-linear chirp waveform varies with a non-linear relationship over time  $t$ . In the proposed scheme, we force the instantaneous frequency to pass the central frequency of the NBI with the highest velocity. Thus, NBI will affect the proposed chirp scheme very little.

#### A. Sinusoidal Chirp Waveform

The instantaneous frequency of a sinusoidal chirp is given by

$$f_c(t) = f_i + a \cos(bt + c), \quad (6)$$

where parameters  $a$  and  $b$  adjust the chirp waveform,  $c$  is a real constant value, and  $f_i$  is the central frequency of the NBI. The proposed sinusoidal chirp waveform and the chirp rate are respectively given by

$$\begin{aligned} w(t) &= \cos(2\pi f_i t + 2\pi a \sin(bt + c) / b), \\ \mu(t) &= -ab \sin(bt + c). \end{aligned} \quad (7)$$

The instantaneous frequency  $f_c(t)$  comparison is shown in Fig. 1. For the non-linear chirp, the maximum NBI suppression can be achieved at the instant when the absolute value of  $\mu(t)$  reaches its maximum value, that is,

$$t_0 = \arg \max_t |\mu(t)|. \quad (8)$$

We adjust  $f_c(t_0)$  to the central frequency of the NBI (5.3 GHz), which means  $f_c(t_0) = f_i$ . In the proposed design,  $f_{high} = 10.6$  GHz,  $f_{low} = 3.1$  GHz, and  $f_m = (f_{high} + f_{low})/2$  are the highest, lowest, and central frequencies of the UWB spectrum band, respectively. A sinusoidal curve with a phase range of  $\pi$  and a center frequency of  $f_i$  is adopted. To utilize the whole spectrum band, at least one of the two edge frequencies of the curve should be  $f_{high}$  or  $f_{low}$ . If  $f_i$  is not equal to  $f_m$ , the frequency range of the whole curve exceeds the UWB spectrum band. We truncate the curve to cover the frequency scope of 3.1 GHz to 10.6 GHz. Therefore, only the upper half and about one third of the lower half of the sinusoidal curve can be utilized. Parameters  $a$  and  $b$  can be generalized as follows:

$$\begin{aligned} a &= f_{high} - f_i \\ b &= \arccos((f_{low} - f_i) / (f_{high} - f_i)) / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= f_{high} - f_i \\ b &= \arccos((f_{low} - f_i) / (f_{high} - f_i)) / T_s \end{aligned}} \right\} \text{if } f_i \leq f_m, \\ a &= f_{low} - f_i \\ b &= \arccos((f_{high} - f_i) / (f_{low} - f_i)) / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= f_{low} - f_i \\ b &= \arccos((f_{high} - f_i) / (f_{low} - f_i)) / T_s \end{aligned}} \right\} \text{if } f_i > f_m. \quad (9)$$

With the same design principle, we can extend to the other three non-linear chirp waveforms.

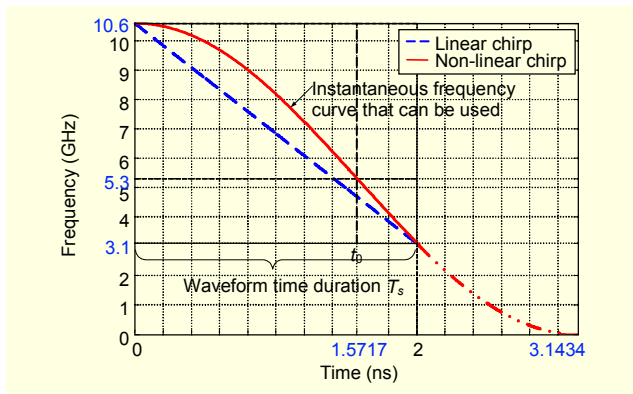


Fig. 1. Time-frequency relationship of proposed non-linear chirp waveform.

### B. Hyperbolic Tangent (tanh) Chirp Waveform

The instantaneous frequency of a hyperbolic tangent (tanh) chirp waveform is given by

$$f_c(t) = f_i + a \tanh(bt + c). \quad (10)$$

The waveform and the chirp rate are respectively given by

$$\begin{aligned} w(t) &= \cos(2\pi f_i t + 2\pi a \log[\cosh(bt + c) / b]), \\ \mu(t) &= ab \operatorname{sech}^2(bt + c), \\ a &= (f_{high} - f_i) / \tanh(c) \\ b &= [\operatorname{arctanh}((f_i - f_{low}) / (f_{high} - f_i) \tanh(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{high} - f_i) / \tanh(c) \\ b &= [\operatorname{arctanh}((f_i - f_{low}) / (f_{high} - f_i) \tanh(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i \leq f_m, \\ a &= (f_{low} - f_i) / \tanh(c) \\ b &= [\operatorname{arctanh}((f_{high} - f_i) / (f_i - f_{low}) \tanh(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{low} - f_i) / \tanh(c) \\ b &= [\operatorname{arctanh}((f_{high} - f_i) / (f_i - f_{low}) \tanh(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i > f_m. \quad (11)$$

### C. Hyperbolic Arc Sine (asinh) Chirp Waveform

The instantaneous frequency of a hyperbolic arc sine (asinh) chirp waveform is given by

$$f_c(t) = f_i + a \operatorname{arcsinh}(bt + c). \quad (12)$$

The waveform and the chirp rate are respectively given by

$$\begin{aligned} w(t) &= \cos\left(2\pi f_i t + 2\pi a \left[ (bt + c) \operatorname{arcsinh}(bt + c) - \sqrt{1 + (bt + c)^2} \right] / b\right), \\ \mu(t) &= ab / \sqrt{1 + (bt + c)^2}, \\ a &= (f_{high} - f_i) / \operatorname{arcsinh}(c) \\ b &= [\sinh((f_i - f_{low}) / (f_{high} - f_i) \operatorname{arcsinh}(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{high} - f_i) / \operatorname{arcsinh}(c) \\ b &= [\sinh((f_i - f_{low}) / (f_{high} - f_i) \operatorname{arcsinh}(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i \leq f_m, \\ a &= (f_{low} - f_i) / \operatorname{arcsinh}(c) \\ b &= [\sinh((f_{high} - f_i) / (f_i - f_{low}) \operatorname{arcsinh}(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{low} - f_i) / \operatorname{arcsinh}(c) \\ b &= [\sinh((f_{high} - f_i) / (f_i - f_{low}) \operatorname{arcsinh}(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i > f_m. \quad (13)$$

### D. Arc Tangent (atan) Chirp Waveform

The instantaneous frequency of an arc tangent (atan) chirp waveform is given by

$$f_c(t) = f_i + a \arctan(bt + c). \quad (14)$$

The waveform and the chirp rate are respectively given by

$$\begin{aligned} w(t) &= \cos\left(2\pi f_i t + a\pi \left[ \log(1 + (bt + c)^2) + 2(bt + c) \arctan(bt + c) \right] / b\right), \\ \mu(t) &= ab / (1 + (bt + c)^2), \\ a &= (f_{high} - f_i) / \arctan(c) \\ b &= [\tan((f_{low} - f_i) / (f_{high} - f_i) \arctan(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{high} - f_i) / \arctan(c) \\ b &= [\tan((f_{low} - f_i) / (f_{high} - f_i) \arctan(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i \leq f_m, \\ a &= (f_{low} - f_i) / \arctan(c) \\ b &= [\tan((f_{high} - f_i) / (f_{low} - f_i) \arctan(c)) + c] / T_s \end{aligned} \left. \vphantom{\begin{aligned} a &= (f_{low} - f_i) / \arctan(c) \\ b &= [\tan((f_{high} - f_i) / (f_{low} - f_i) \arctan(c)) + c] / T_s \end{aligned}} \right\} \text{if } f_i > f_m. \quad (15)$$

The instantaneous frequency and chirp rate are shown in Fig. 2. At the corresponding instants when the instantaneous frequency curves cross the NBI central frequency of 5.3 GHz in Fig. 2(a), the slopes of the curves, which are the chirp rates shown in Fig. 2(b), achieve the highest absolute values. Therefore, the NBI contributes the least effect on the proposed schemes. The non-linear atan scheme has the highest  $|\mu(t)|$  among all the schemes.

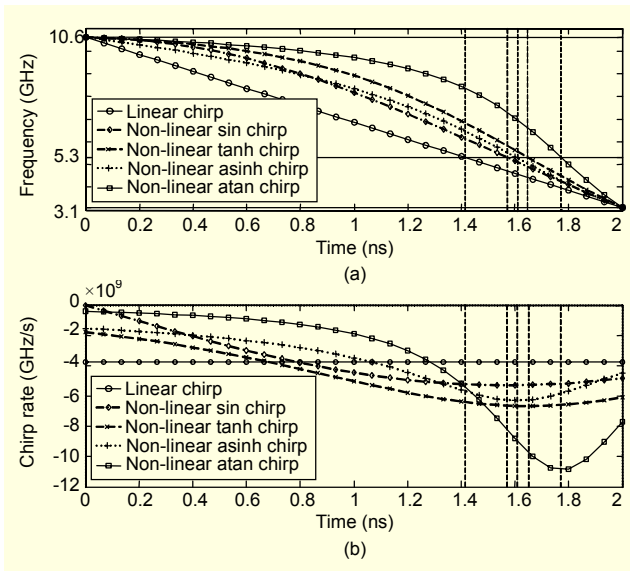


Fig. 2. Comparison of (a) instantaneous frequency; (b) chirp rate ( $f_c=5.3$  GHz).

### III. Performance Evaluation

We set a ranging system to evaluate the ranging error rate of the linear and non-linear chirp waveforms. The proposed ranging system transmits trains of chirp waveforms and receives the reflected signals. The received signals are correlated with a local reference signal to achieve transmission time delay  $\Delta t$ . The measured distance is  $d_m = (\Delta t c)/2$ , where  $c$  is the velocity of light. If the difference from  $d_m$  to the actual distance  $d$  exceeds 0.3 m, a false measurement occurs. We evaluate ranging error rates (RERs) by simulation. The RER is the number of false distance measurements over the total number of distance measurements.

The simulation scenario is as follows. The chirp duration is 2 ns. The interference source is from an 802.11a WLAN device with BPSK modulation. The central frequency of the NBI is 5.3 GHz and the bandwidth is 20 MHz. The channel noise is additive white Gaussian noise (AWGN).

The results are shown in Figs. 3 and 4. Figure 3 shows the performance of RER vs. SIR at an  $E_b/N_0$  of 14 dB. Figure 4 shows the performance of RER vs.  $E_b/N_0$  at an SIR of -20 dB.

The simulation results demonstrate that the proposed non-linear chirp schemes have lower RERs than the linear scheme. Also,

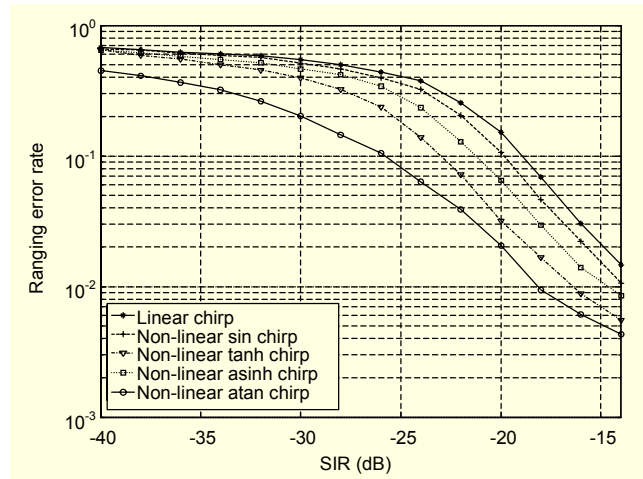


Fig. 3. Ranging error rate vs. SIR.  $E_b/N_0=14$  dB.

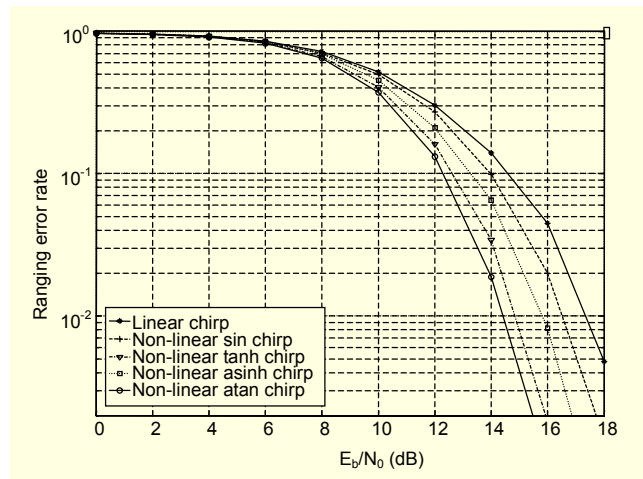


Fig. 4. Ranging error rate vs.  $E_b/N_0$ , SIR= -20 dB.

the atan chirp has the best performance among all the waveforms; the sinusoidal chirp can achieve a lower RER than the linear scheme, though it has the worst performance among the four proposed non-linear chirp waveforms.

### References

- [1] X. Chu and R.D. Murch, "The Effect of NBI on UWB Time-Hopping Systems," *IEEE Trans. on Wireless Commu.*, vol. 3, no. 5, Sep. 2004, pp. 1431-1436.
- [2] H. Shen, W. Zhang, X. An, and K.S. Kwak, "DS-PAM UWB System Using Non-linear Chirp Waveform," *ETRI Journal*, vol. 29, no. 3, June 2007, pp. 322-328.
- [3] K. Doi, T. Matsumura, K. Mizutani, and R. Kohno, "Ultra-Wideband Ranging System Using Improved Chirp Waveform," *RAWCON. 2003*, pp. 207-210.
- [4] D.P. Morgan, *Surface Wave Devices for Signal Processing*, Elsevier, Amsterdam, 1985.