Simple Amplitude and Phase Predistortion for PAPR Reduction in OFDM Systems

Jeong-Sang Park, Jae-Kwon Lee, and Jin-Up Kim

ABSTRACT—One of the drawbacks in an OFDM system is the high peak-to-average power ratio (PAPR). Among a number of techniques to reduce the high PAPR, simple amplitude predistortion (SAP), a form of active constellation extension, has been proposed to effectively achieve the desired PAPR. In this letter, a novel scheme, simple amplitude and phase predistortion (SAPP), is proposed. In SAP the carriers' amplitude is utilized to combat the peak signal. Each amplitude is amplified according to its degree of contribution as a metric. In addition to amplitude, SAPP also utilizes the phase. Simulation results indicate that the proposed scheme provides better PAPR reduction than SAP.

Keywords—ACE, simple amplitude predistortion, simple amplitude and phase predistortion, OFDM, PAPR.

I. Introduction

Techniques to reduce peak-to-average power ratio (PAPR) must be low in complexity and cause no performance degradation, out-of-band radiation, or side information. Clipping is the simplest method to achieve this goal; however, it causes out-of-band radiation due to non-linear processing [1]. Phase rotation techniques search the optimum set of phase factors [2]; however, the search complexity of the optimum phase increases exponentially with the number of subblocks, and the phase factors in the receiver must be known. The active constellation extension (ACE) technique can reduce PAPR by extending a constellation point toward the outside of the original constellation [3]. Compared with the previously mentioned techniques, ACE induces no BER degradation or Tx-Rx handshake, and it requires no special processing unlike other techniques in Rx. However, it introduces a power increase and high complexity because of an iterative constellation extension process. In this letter, we propose simple amplitude and phase predistortion (SAPP) involving an ACE method with a systematic search [3], [4].

II. Simple Amplitude Predistortion

Interest in ACE reduction techniques has been growing over the past few years. However, the original ACE approach requires high complexity (brute force search) and out-of-band radiation. To mitigate the drawbacks of ACE, studies have been conducted on simple amplitude predistortion (SAP). This approach provides a systematic search algorithm and reduction of the PAPR with low complexity.

Cost function $f(n, k)$ is a function of θ_{nk}, the phase difference between an OFDM symbol and each N carrier. In other words, the phase correlation refers to the degree of contribution each of the N carriers makes to the peak. Hence, the phase correlation is based on the following function:

$$f(n,k) = -\cos(\theta_{nk}) = -\frac{\mathrm{Re}\{x_n \cdot X_k^* \cdot e^{-j2\pi nk/N}\}}{|x_n||X_k|}. \qquad (1)$$

When θ_{nk} equals π, the cost function has a maximum value because of the inverse relation between the peak OFDM symbol and each X_k symbol. The purpose of the cost function is to find the proper values k so that the metric of index k makes the peak OFDM signal. Generally, the metric that predistorts the constellation points is $\mu_k = \sum_{n=0}^{N-1} w(n) \cdot f(n,k)$, where $w(n)$ is called a weight function. This weight function plays an important role to give a high weight to the output signal of large magnitude. The weight function is defined as

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$w(n) = |x_n|^p$, where the proper value of p is empirically 6 [4].

$$\mu_k = \frac{-1}{|X_k|} \sum_{n \in S_K} |x_n|^{p-1} \operatorname{Re}\{x_n \cdot X_k^* \cdot e^{-j2\pi nk/N}\}, \tag{2}$$

where S_K is a subset whose magnitude exceeds a threshold value. Positive metric $\mu_k{}^+$ denotes the degree of contribution, and $\mu_k{}^-$ denotes the degree of interference. In this letter, all negative values $\mu_k{}^-$ are set to 0.

$$x'[n] = x[n] + \frac{1}{\sqrt{N}} \sum_{k \in S_L} d_k \cdot e^{j2\pi nk/JN}, \quad 0 \le n < JN, \tag{3}$$

$$d_k = \alpha \sqrt{\mu_k^+} \cdot X_k, \tag{4}$$

where α is a scaling factor. Once all of the metrics are computed, the constellation points are linearly shifted in the decreasing order of their metrics. The constellation points with the highest metrics are chosen as the elements of S_L. As shown in Fig. 1, some amplitudes are linearly distorted by distance $|d_k|$. It is necessary beforehand to obtain the proper parameters such as p, α, S_K, and S_L. Once the suboptimal parameters are determined, it is no longer necessary to find the parameters per every OFDM symbol.

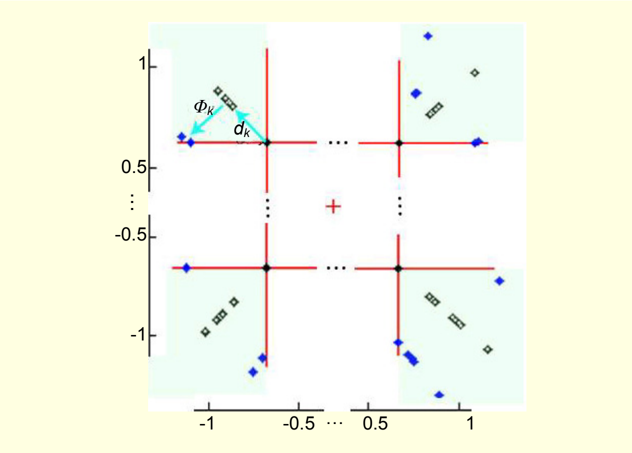

Fig. 1. QPSK constellation of SAP (circle) and SAPP (dot) with ± π/18 constraint.

III. Proposed Scheme

We have manipulated the carriers' amplitude as a positive metric $\mu_k{}^+$ to reduce the PAPR. However, to date, the phase of the carrier has not been studied. If not only amplitude but also its phase is utilized, we can expect better performance. First, the cost function is considered in a similar fashion. The cost function $f(n,k) = -\cos(\theta_{nk})$ can be maximized by rotating its phase such that

$$\begin{aligned} X_k &= \operatorname{Re}\{X_k\} + \operatorname{Im}\{X_k\} \cdot \mathrm{j}, \\ \operatorname{Re}\{X_{k,\phi_k}\} &= \operatorname{Re}\{X_k\} \cdot \cos\phi_k - \operatorname{Im}\{X_k\} \cdot \sin\phi_k \\ \operatorname{Im}\{X_{k,\phi_k}\} &= \operatorname{Re}\{X_k\} \cdot \sin\phi_k + \operatorname{Im}\{X_k\} \cdot \cos\phi_k \\ X_{k,\phi_k} &= \operatorname{Re}\{X_{k,\phi_k}\} + \operatorname{Im}\{X_{k,\phi_k}\} \cdot \mathrm{j}, \end{aligned} \tag{5}$$

where ϕ_k should be constrained so as to maintain a minimum distance which determines the BER performance. Hence, the cost function is defined as

$$\cos\theta_{nk} = \frac{\operatorname{Re}\{x_n \cdot X_{k,\phi_k}^* \cdot e^{-j2\pi nk/N}\}}{|x_n||X_k|}, \quad -\frac{\pi}{18} \le \phi_k \le \frac{\pi}{18}. \tag{6}$$

The final metric is represented by

$$(\mu_k, \phi_k) = \arg\{\max_{\mu_k}[-\sum_{n \in S_K} |x_n|^p \cdot \cos\theta_{nk}]\}, \tag{7}$$

$$d_k = \alpha \sqrt{\mu_k^+} \cdot X_k. \tag{8}$$

Therefore, the constellation points scale upward by $|d_k|$ and then rotate their phases by ϕ_k. The cardinality of S_L must be around 15 to 30 to obtain the suboptimal PAPR. Because the proposed method does not fully utilize the whole phase to allow for low computation, finding the suboptimum phase is simple. In this letter, the maximum phase is constrained by $\pm\pi/18$ and the phase step is $\pi/36$.

IV. Simulation Results

Table 1 lists all of the important parameters. In this simulation, we assume QPSK, an oversampling rate ($J = 4$), and no guard interval for convenience. The desired clipping level (6 dB) above the average power is used. For a given clipping level, both weight factor ($p = 6$) and α (40, 60) can be empirically determined such that the OFDM symbols have the desired PAPR.

Figure 2 presents the degree of contribution of SAP and SAPP. The high contribution of $\mu_k{}^+$ indicates the high degree of the inverse phase between the k-th carrier and the OFDM symbol. In Fig. 2, the maximum number of modified metrics utilized for the PAPR reduction is around 25 to 35, statically in SAP. However, in SAPP, the maximum number is changed up to 100 and its contribution is usually greater than that of SAP. This explains why the proposed algorithm finds a better PAPR by limited phase rotation.

Figure 3 shows the PAPR at CCDF=10^{-4} according to the number of modified carriers. In general, the PAPR of SAPP is roughly 0.3 to 0.5 dB lower than that of SAP. The PAPR is suboptimal at about $L = 15$ and $\alpha = 60$ (or, $L = 30$ and $\alpha = 40$),

Table 1. Simulation parameters.

Parameter	Specifications
Number of subcarriers (N)	128
Weight factor (p)	6
α	40, 60
Cardinality of S_L	0 - 40
Phase rotation constraint (ϕ_k)	$-\pi/18$ - $\pi/18$
Phase step	$\pi/36$

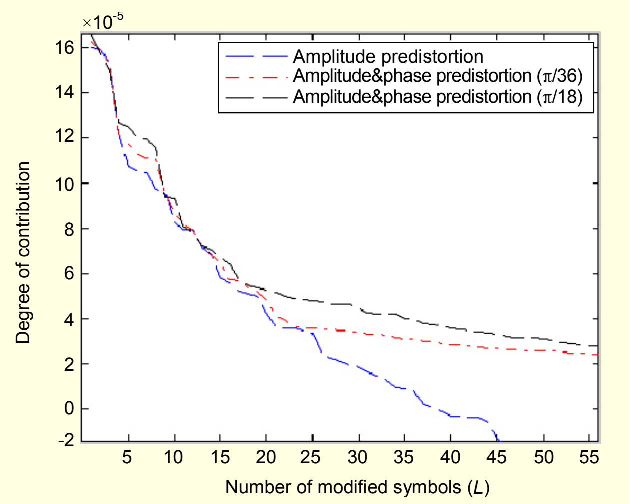

Fig. 2. Contribution of proposed and original scheme.

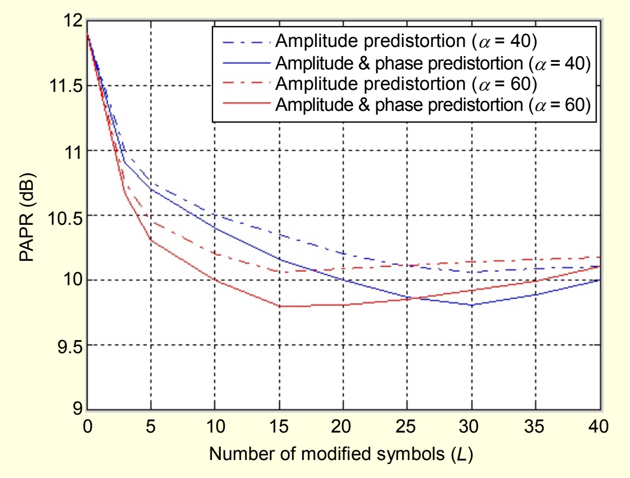

Fig. 3. PAPR at CCDF=10^{-4} vs. number of modified carriers.

and the OFDM signal power increases only by 0.58 dB.

Figure 4 presents the CCDF of OFDM in $\alpha = 60$ ($L = 15$). The PAPR of the SAP technique is reduced by about 1.6 dB, while it is reduced by about 2.1 dB with SAPP. In comparison with SAP, SAPP shows about 0.5 dB improvement in the same power increment of 0.58 dB.

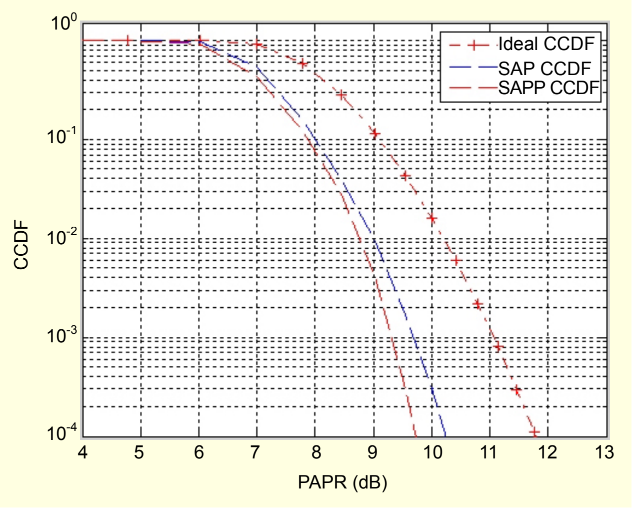

Fig. 4. CCDF of proposed and original scheme (1 iteration).

V. Conclusion

We proposed simple amplitude and phase predistortion, a new scheme integrating ACE with a systematic search. The proposed scheme requires no side information and no additional signal processing in Rx. It induces no BER degradation or out-of-band radiation. Compared with SAP, the proposed scheme has higher complexity; however, the PAPR of the proposed scheme shows a significant improvement at the same cost of power increase. In conclusion, SAPP provides better PAPR reduction performance at the cost of the phase search.

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