Design of Robust Detector with Noise Variance Estimation Censoring Input Signals over AWGN

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ABSTRACT—As an alternative to the classic linear detector which only assumes noise variance, a new robust detector with noise variance estimation censoring input signals over AWGN is proposed. The results demonstrate that analytic detection probability matches the simulation results for the linear detector and that the new robust detector shows better performance than the linear detector when the number of samples increases.

Keywords—Robust detector, linear detector, parameter estimation, censoring.

I. Introduction

In this letter, two detectors are considered which use the Neyman-Pearson approach [1]. The first is the classic linear detector which only assumes noise variance. The new detector proposed here is based on the robust estimator (Huber-Strassen saddlepoint technique) [2] which incorporates noise variance estimation [3] censoring input signals over AWGN into its design [4]. The detection probability (P_d) performance of this detector is compared with that of the linear detector with fixed false alarm probability (P_f).

II. Mathematical Development for Detectors

The primary consideration is simple hypothesis testing in which there are two hypotheses, H_0 and H_1, corresponding to two Gaussian distributions, F_0 and F_1.

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$$\begin{cases} H_0 : Y \in F_0, \\ H_1 : Y \in F_1, \end{cases} \tag{1}$$

where $Y \in F_1$ denotes that Y has distribution F_1.

1. Analytic Calculation of P_d for Linear Detector

For analytic P_f,

$$P_f = (\sum_{i=1}^{n} x_i > \bar{T} \mid H_0 \text{ is true}), \tag{2}$$

where $\bar{T}$ is the analytic threshold of the linear detector, n is number of samples, and x_i has iid Gaussian noise distribution such that $x_i = N_i(S_j, \sigma^2)$ for $i = 1, \cdots, n$, and $j = 0, 1$. Accumulated iid Gaussian noise distribution with known signal strength $S_0 = 0$ and known noise variance σ^2 is

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} N_i(S_0, \sigma^2) = N(0, n\sigma^2). \tag{3}$$

For calculation of the threshold,

$$P_f = \int_{\bar{T}}^{\infty} \frac{1}{\sqrt{2\pi n\sigma^2}} \exp\left(-\frac{x^2}{2n\sigma^2}\right) dx, \tag{4}$$

$$\bar{T} = \sqrt{2n\sigma^2} \cdot \operatorname{erf}^{-1}(1 - 2P_f). \tag{5}$$

For analytic P_d,

$$P_d = (\sum_{i=1}^{n} x_i > \bar{T} \mid H_1 \text{ is true}), \tag{6}$$

with

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} N_i(S_1, \sigma^2) = N(nS_1, n\sigma^2). \tag{7}$$

Therefore,

$$P_d = \int_{\bar{T}}^{\infty} \frac{1}{\sqrt{2\pi n\sigma^2}} \exp\left(-\frac{(x - nS_1)^2}{2n\sigma^2}\right) dx \tag{8}$$

$$= 0.5 - 0.5 \cdot \mathrm{erf}((\overline{T} - nS_1)/\sqrt{2n\sigma^2}). \tag{9}$$

2. Design of Linear Detector

For the linear detector, under either statistical hypothesis, the operative iid Gaussian density f_j is of the form

$$f_j(x_1,\cdots,x_n) = \prod_{i=1}^{n} f_j(x_i) \tag{10}$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i - S_j)^2}{2\sigma^2}\right), \; j = 0,1, \tag{11}$$

with known S_j and σ^2.

The likelihood ratio $L(x_1,\cdots,x_n)$ for the linear detector is

$$L(x_1,\cdots,x_n) = \frac{f_1(x_1,\cdots,x_n)}{f_0(x_1,\cdots,x_n)} \gtrless T, \tag{12}$$

where T is the threshold.

Equation (12) is simplified because S_0 is 0 here and σ^2 of noise is a constant

$$L(x_1,\cdots,x_n) = \exp\left(\frac{2S_1\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}S_1^2}{2\sigma^2}\right) \gtrless T, \tag{13}$$

$$L_L(x_1,\cdots,x_n) = \sum_{i=1}^{n} x_i \gtrless \frac{2\sigma^2\ln(T) + \sum_{i=1}^{n}S_1^2}{2S_1} = \widetilde{T}. \tag{14}$$

With this L_L, a new threshold is calculated with known P_f.

$$P_d = F_1(L_L(X_1 + S_1,\cdots,X_n + S_1) > \widetilde{T}). \tag{15}$$

3. Design of Robust Detector with Variance Estimation

For design of the robust detector under either statistical hypothesis, the operative iid Gaussian density f_j is of the form

$$f_j(x_1,\cdots,x_n;\theta_j) = \prod_{i=1}^{n} f_j(x_i;\theta_j) \tag{16}$$

$$= (2\pi\theta_j)^{-n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i - S_j)^2}{2\theta_j}\right), \; j = 0,1, \tag{17}$$

with known S_j, where θ_j is a noise variance.

To find maximum likelihood variance estimator, the following equation must be satisfied:

$$0 = \frac{\partial \ln(f_j)}{\partial \theta_j} = \frac{-n\theta_j + \sum_{i=1}^{n}(x_i - S_j)^2}{2\theta_j^2}, \tag{18}$$

$$\hat{\theta}_j = \frac{1}{n}\sum_{i=1}^{n}(x_i - S_j)^2, \quad j = 0,1. \tag{19}$$

The robust detector is formed first by taking a ratio of the joint densities of H_0 and H_1 and replacing the unknown parameter θ_j by its estimator censoring input signals, which yields

$$L(x_1,\cdots,x_n) = \frac{f_1(x_1,\cdots,x_n;\check{\theta}_1)}{f_0(x_1,\cdots,x_n;\check{\theta}_0)} \gtrless T. \tag{20}$$

In order to acquire more flexibility in the robustness, the detector is modified by introducing [4]

$$\check{\theta}_j = \frac{1}{n}\sum_{i=1}^{n} h_R(x_i - S_j), \quad j = 0,1, \tag{21}$$

where

$$h_R(x) = \begin{cases} k^2, & |x| \ge k, \\ x^2, & |x| < k, \end{cases} \tag{22}$$

and k is a censoring level.

Signal S_0 is 0 and then (21) is inserted into (20). Thus,

$$L(x_1,\cdots,x_n) = \left\{ \frac{\sum_{i=1}^{n} h_R(x_i)}{\sum_{i=1}^{n} h_R(x_i - S_1)} \cdot \exp\left(\frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} h_R(x_i)} - \frac{\sum_{i=1}^{n}(x_i - S_1)^2}{\sum_{i=1}^{n} h_R(x_i - S_1)} \right) \right\}^{n/2} \gtrless T, \tag{23}$$

$$L_R(x_1,\cdots,x_n) = (L(x_1,\cdots,x_n))^{2/n} \gtrless T^{2/n} = \check{T}. \tag{24}$$

With this L_R, $\check{T}$ is calculated with known P_f, then

$$P_d = F_1(L_R(X_1 + S_1,\cdots,X_n + S_1) > \check{T}). \tag{25}$$

III. Simulation and Results

The σ^2 of Gaussian noise distribution and S_0 are always set to 1 and 0 here, respectively. After that, S_1 is changed according to E_b/N_o. False alarm probabilities are constrained to $\{10^{-2}, 10^{-3}, 10^{-4}\}$; then, new thresholds are calculated for each n and each P_f at S_1=1 (E_b/N_o = –3.02 dB). For L_L and L_R, P_d is computed when E_b/N_o varies.

Table 1. Thresholds for linear detector.

	$\tilde{T}$ (analytic)			$\tilde{T}$ (simulated)		
$P_f =$	10^{-2}	10^{-3}	10^{-4}	10^{-2}	10^{-3}	10^{-4}
n = 5	5.20	-	-	5.21	-	-
n = 10	7.36	9.77	-	7.33	9.75	-
n = 15	9.01	11.97	14.40	9.00	12.03	14.51
n = 20	10.40	13.82	16.63	10.40	13.83	16.41
n = 25	-	15.45	18.59	-	15.30	18.09
n = 30	-	-	20.37	-	-	20.09

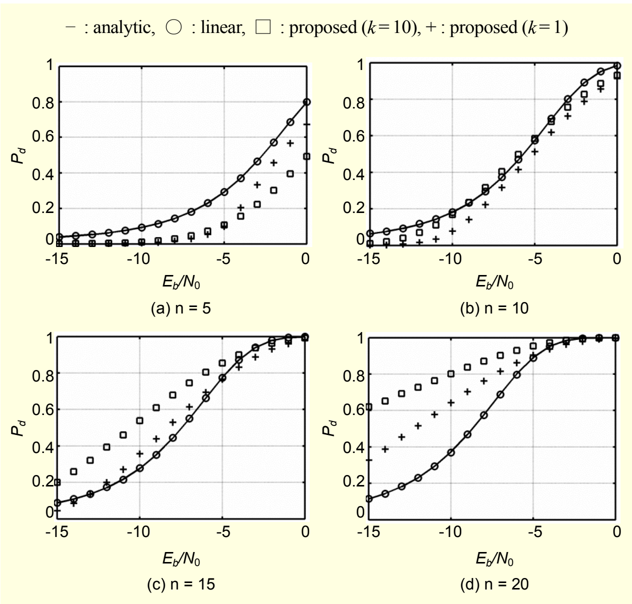

Fig. 1. Performance comparison at $P_f = 10^{-2}$ in AWGN.

Table 1 shows that the thresholds of analytic calculation and simulation are almost same. The difference between the detectors is how they find the σ^2 of noise distribution. The linear detector assumes $\sigma^2 = 1$ and then calculates the threshold and P_d, but the robust detector calculates the threshold and P_d together with estimated noise variance using (21).

In Figs. 1 and 2, analytic P_d of the linear detector matches the simulated P_d of the linear detector in every figure. In each figure, P_d of the new robust detector improves upon that of the classic linear detector when the number of samples increases. Also, it is interesting that a lower censoring level shows better performance than a higher censoring level in a small number of samples.

IV. Extensions and Future Work

Extensions may be possible to a wider arena, including correlated and non-Gaussian (such a Laplace) noise. This could be initiated through the use of a generalized Gaussian model. This work might feature an imperfectly known correlation matrix and the unbiased method as introduced by Varma [5]. Such future work might better address the demanding and sometimes severe problems faced by the practitioner.

V. Conclusion

For nominal iid Gaussian noise distribution, the linear detector and the robust detector based on variance estimation with censoring input signals over AWGN are considered.

The results indicated the robust detector showed better performance than the classic linear detector when the number of samples increased.

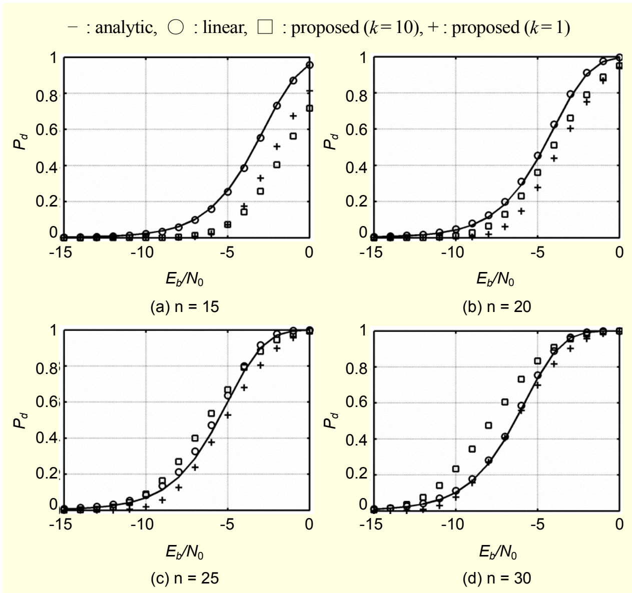

Fig. 2. Performance comparison at $P_f = 10^{-4}$ in AWGN .

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