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Large Hydromagnetic Axisymmetric Instability of a Streaming Gas Cylinder Surrounded by Bounded Fluid with Non Uniform Field

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Abstract. The magnetohydrodynamic axisymmetric instability of a streaming gas jet surrounded by bounded fluid with non-uniform field has been developed. The problem is formulated, solved and the boundary conditions are applied across the interfaces. The eigenvalue relation is derived and discussed analytically and the results are confirmed numerically. Some reported works are recovered as limiting cases from the present general results. The streaming has a destabilizing effect for all short and long wavelengths. The capillary force is stabilizing for short wavelengths but it is destabilizing for long wavelengths. The axial magnetic fields interior the gas and fluid media are stabilizing. The transverse field is destabilizing for all wavelengths. The radii ratio of the gas and fluid cylinders plays an important role for stabilizing the model and made it more realistic one than the full liquid jet or/and the ordinary hollow jet. The numerical analysis clarify the stable and unstable domains based on different values of the various parameters of the problem.

1. Introduction

The classical studies of the capillary instability of a gas cylinder submerged into a liquid are given by Chandrasekhar (1961), Drazin & Reid (1980) and Cheng (1985). Kendall (1986) performed experiments with modern equipment to check the breaking - up of that model. Moreover, he (1986) attracted the attention for the importance of the stability discussions of that model and its applications in many domains of science. Soon afterwards a lot of researchers treated with the stability of such model (see Radwan (1989), (1998), (2002) and (2005)) analytically and numerically upon utilizing appropriate basic equations and boundary conditions.

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One has to mention here that the most important mode is the axisymmetric one where in all foregoing studies it is found that the model is unstable only in this mode.

Concerning the hydrodynamic stability of a hollow jet endowed with surface tension we may refer to (Chauhan et.al.(2000), Chen & Lin (2002), Cousin & Dumouchel (1966), Lee & Wang (1986) and (1989), Mehring & Sirignano (2000), Parthasrathy & Chiang (1998), Shen & Li (1996), Shi et.al. (1999), Shukudov & Sisoev (1996) and Villermaux (1998)).

Here we extend the foregoing study upon discussing the stability of a gas jet surrounded by a bounded fluid pervaded by non-uniform magnetic field, acted by the combining effect of the pressure gradient, capillary and electromagnetic forces. This will be done for axisymmetric perturbations mode.

The phenomenon of the present model may be occurred in nature as a gas jet penetrated oil layers. Also in geological drilling when a gas escapes from below liquid layers.

2. Formulation of the problem

We consider a gas cylinder of radius R_0 streams through a bounded fluid of radius qR_0 $(1 < q < \infty)$ with velocity $\underline{u}_0 = (0, 0, U)$. The fluid is assumed to be incompressible, non-viscous and perfectly conducting.

The gas region is penetrated by the variable magnetic field

$$\underline{H}_0^{\mathrm{g}} = (0, \frac{\beta H_0 r}{R_0}, \alpha H_0), \tag{2.1}$$

while the fluid one by the uniform field $\underline{H}_0 = (0, 0, H_0)$ where H_0 is the intensity of the magnetic field in the fluid and α & β are parameters. The fluid matter is acted by the pressure gradient, capillary, inertia and electromagnetic forces. The gas jet is acted by the inertia and electromagnetic forces in addition to the force due to the gas natural constant pressure P_0^{g}. However, the fluid inertia force is assumed to be predominant over that of the gas. The components of $\underline{H}_0$ and $\underline{H}_0^{\mathrm{g}}$ are considered along the utilizing cylindrical coordinates (r, φ, z).

The hydromagnetic fundamental equations appropriate for studying the stability of the fluid model under consideration are the combination of the pure hydrodynamic equations and those of Maxwell concerning the electromagnetic theory. These equations may be given as follows.

In the gas cylinder,

$$\nabla \cdot \underline{H}^{\mathrm{g}} = 0, \tag{2.2}$$

$$\nabla \times \underline{H}^{\mathrm{g}} = 0. \quad \text{(there is no current)} \tag{2.3}$$

Along the gas-fluid interface,

$$P_s = T(\nabla \cdot \underline{N}). \tag{2.4}$$

In the liquid region,

$$\rho\Big[\frac{\partial}{\partial t}+(\underline{u}\cdot\nabla)\Big]\underline{u} = -\nabla P+\mu(\underline{J}\times\underline{H}), \tag{2.5}$$

$$\underline{J} = \nabla\times\underline{H}, \tag{2.6}$$

$$\nabla\cdot\underline{H} = 0, \tag{2.7}$$

$$\nabla\cdot\underline{u} = 0, \tag{2.8}$$

$$\frac{\partial\underline{H}}{\partial t} = \nabla\times(\underline{u}\times\underline{H}). \tag{2.9}$$

Here $\underline{H}^{\mathrm{g}}$ and $\underline{H}$ are the magnetic field intensities in the gas and fluid regions, P_s the curvature pressure due to the capillary force, T the surface tension coefficient, $\underline{N}$ the outward unit vector normal to the gas-fluid interface and indicates as r (the radial cylindrical coordinate) does, μ the magnetic permeability coefficient and $\underline{J}$ is the current density; while $\rho, \underline{u}$ and P are the fluid mass density, velocity vector and kinetic pressure. Upon using some vector identities, equations (2.5) and (2.9) may be written in the form:

$$\frac{\partial\underline{u}}{\partial t}+(\underline{u}\cdot\nabla)\underline{u}-\frac{\mu}{\rho}(\underline{H}\cdot\nabla)\underline{H} = -\nabla\Pi, \tag{2.10}$$

$$\frac{\partial\underline{H}}{\partial t}+(\underline{u}\cdot\nabla)\underline{H} = (\underline{H}\cdot\nabla)\underline{u}, \tag{2.11}$$

with

$$\rho\Pi = P+\frac{\mu}{2}(\underline{H}\cdot\underline{H}) \tag{2.12}$$

represents the total hydromagnetic pressure which is the sum of the fluid kinetic pressure and magnetic pressure.
The basic equations (2.2)-(2.11) are solved, with $\underline{u}=(0,0,U)$, appropriate boundary conditions are applied at $r=R_0$ and consequently the unperturbed fluid kinetic pressure distribution P_0 is given by

$$P_0 = -\frac{T}{R_0}+P_0^{\mathrm{g}}+\frac{\mu H_0^2}{2}(\alpha^2+\beta^2-1). \tag{2.13}$$

Here P_0^{g} is the gas constant pressure in the initial state, $(-\frac{T}{R_0})$ the contribution of the capillary force, while the third term in the right side of equation (2.13) is being the net magnetic pressure due to the effect of the electromagnetic forces acting in the gas and fluid regions.

One has to refer here that in absence of the magnetic field as:

(i) $H_0=0$,

(ii) $\beta=0$ and $\alpha=1$ or

(iii) $\alpha = 0$ and $\beta = 1$,

the constant gas pressure P_0^{g} must be greater than $(-\frac{T}{R_0})$ in order that $P_0 > 0$, otherwise the model collapses and there will not be a gas pervades into the fluid region.
Keep in your mind that $\mu = 4\pi \times 10^{-7}$ Henry/m. So its contribution will be infinitesimally small.

In the general case, in order to $P_0 \geq 0$ the gas kinetic pressure P_0^{g} in the initial state must satisfy the restriction:

$$P_0^{\mathrm{g}} \geq \frac{T}{R_0} + \frac{\mu H_0^2}{2}(1 - \alpha^2 - \beta^2), \tag{2.14}$$

otherwise the model collapses and will be a homogeneous fluid medium.

3. Linearization and solutions

For small departure from the unperturbed state due to axisymmetric perturbation, every variable quantity $Q(r, z; t)$ could be expressed as:

$$Q(r, z; t) = Q_0(r) + \varepsilon_0 Q_1(r, z; t) + \cdots, \qquad |Q_1| << Q_0. \tag{3.1}$$

Here Q stands for $\underline{u}, P, \underline{H}^{\mathrm{g}}$ and $\underline{\mathrm{H}}$ where Q_0 denotes the unperturbed part and Q_1 is a small fluctuating part due to perturbation. From the view point of the expansion (3.1) and upon considering axisymmetric sinusoidal wave, the perturbed radial distance of the gas jet in the perturbed state could be expressed as

$$r = R_0 + \varepsilon_0 R_1 + \cdots \tag{3.2}$$

with

$$R_1 = exp[i(kz) + \sigma t]. \tag{3.3}$$

Here R_1 represents the elevation of the surface wave, σ is the growth rate, k (real number) is the longitudinal wave number and ε_0 $(= \varepsilon$ at $t = 0)$ is the initial amplitude of the perturbation.

Based on the foregoing expansions, the MHD basic equations (2.2)-(2.12) in the perturbed state are given by:

$$\nabla \cdot \underline{H}_1^{\mathrm{g}} = 0, \tag{3.4}$$

$$\nabla \times \underline{H}_1^{\mathrm{g}} = 0, \tag{3.5}$$

$$P_{1s} = \frac{T}{R_0^2}\left[R_1 + R_0^2 \frac{\partial^2 R_1}{\partial z^2}\right], \tag{3.6}$$

$$\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0 \cdot \nabla)\underline{u}_1 - \frac{\mu}{\rho}(\underline{H}_0 \cdot \nabla)\underline{H}_1 = -\nabla \Pi_1, \tag{3.7}$$

$$\rho \Pi_1 = P_1 + \mu(\underline{H}_0 \cdot \underline{H}_1), \tag{3.8}$$

$$\nabla \cdot \underline{H}_1 = 0, \tag{3.9}$$

$$\nabla \cdot \underline{u}_1 = 0, \tag{3.10}$$

$$\frac{\partial \underline{H}_1}{\partial t} + (\underline{u}_0 \cdot \nabla)\underline{H}_1 = (\underline{H}_0 \cdot \nabla)\underline{u}_1. \tag{3.11}$$

By the aid of the series expansion (3.1) and the space-time dependence (3.2), based on the linear perturbation technique, the variables $Q_1(r, z; t)$ could be expressed as:

$$Q_1(r, z; t) = Q^*(r)\ exp[i(kz) + \sigma t]. \tag{3.12}$$

This means that every perturbed quantity could be expressed as an amplitude function of r times the space-time dependence $exp[i(kz) + \sigma t]$. Consequently, the basic equations (3.4)-(3.11) in the perturbed state are analyzed and solved. Apart from the infinite (singular) solution, we obtain:

$$\Pi_1 = \big(AI_0(kr) + BK_0(kr)\big)R_1, \tag{3.13}$$

$$\underline{H}_1^{g} = C\nabla\big(I_0(kr)R_1\big), \tag{3.14}$$

$$\underline{u}_1 = \frac{-(\sigma + ikU)}{\rho\big((\sigma + ikU)^2 + \Omega_A^2\big)}\ \nabla\Pi_1, \tag{3.15}$$

$$\underline{H}_1 = \frac{-H_0}{\rho\big((\sigma + ikU)^2 + \Omega_A^2\big)}\ \nabla\frac{\partial \Pi_1}{\partial z}, \tag{3.16}$$

$$\Pi_1 = \frac{P_1}{\rho} + \frac{\mu}{\rho}(\underline{H}_0 \cdot \underline{H}_1), \tag{3.17}$$

$$P_{1s} = \frac{T}{R_0^2}(1 - x^2)R_1. \tag{3.18}$$

The cylindrical functions $I_0(kr)$ and $K_0(kr)$ are the modified Bessel functions of the first and second kind of order zero with $\Omega_A = \sqrt{\frac{\mu H_0^2 k^2}{\rho}}$ is the Alfven wave frequency defined in terms of H_0 while A, B and C are constants of integration to be determined, where $x(= kR_0)$ is the dimensionless longitudinal wavenumber.

4. Eigenvalue relation

The solution of the basic equations (2.2)-(2.11) in the unperturbed state given by equation (2.13) and in the perturbation one given by equations (3.13)-(3.18) must satisfy appropriate boundary conditions at $r = R_0$ and $r = qR_0$. Under the present circumstances these boundary conditions are given as follows.

(i) The normal component of the velocity vector $\underline{u}$ of the fluid must be compatible with the velocity of the gas - fluid interface at $r = R_0$, i.e.,

$$u_{1r} = \frac{\partial r}{\partial t} \quad \text{at} \quad r = R_0. \tag{4.1}$$

(ii) The normal component of the velocity $\underline{u}$ must vanish across the wall surrounded the fluid at $r = qR_0$, i.e.,

$$u_{1r} = 0 \quad \text{at} \quad r = qR_0. \tag{4.2}$$

Upon applying these conditions and solving the resulting equations for A and B, we finally obtain:

$$A = -B\frac{K_0'(y)}{I_0'(y)} \tag{4.3}$$

$$B = \frac{\big((\sigma+ikU)^2+\Omega_A^2\big)[I_0'(y)]}{k[I_0'(x)K_0'(y)-I_0'(y)K_0'(x)]}, \tag{4.4}$$

where

$$x = kR_0 \tag{4.5}$$

$$y = qx, \qquad 1<q<\infty \tag{4.6}$$

are the longitudinal dimensionless wave numbers.

(iii) The normal component of the magnetic field across the gas-fluid interface must be continuous at $r = R_0$.
This condition reads:

$$\underline{N}_0\cdot\underline{H}_1+\underline{N}_1\cdot\underline{H}_0=\underline{N}_0\cdot\underline{H}_1^{\mathrm{g}}+\underline{N}_1\cdot\underline{H}_0^{\mathrm{g}}, \tag{4.7}$$

where

$$\underline{N}_0=(1,0,0) \tag{4.8}$$

and

$$\underline{N}_1=(0,0,-ik)R_1. \tag{4.9}$$

The condition (4.7) yields:

$$C=\frac{iH_0\alpha}{I_0'(x)}. \tag{4.10}$$

Now, we have to apply the balance of pressure. This condition states that the normal component of the total stress tensor must be continuous across the gas-fluid interface at $r = R_0$.
This compatibility condition reads:

$$\Big[P_1+\mu(\underline{H}_0\cdot\underline{H}_1)+R_1\frac{\partial P_0}{\partial r}\Big]-\Big[\mu(\underline{H}_0\cdot\underline{H}_1)+\frac{\mu}{2}R_1\frac{\partial}{\partial r}(\underline{H}_0\cdot\underline{H}_0)\Big]^{\mathrm{g}}=P_{1s}. \tag{4.11}$$

By substituting for $P_1, R_1, P_{1s}, \underline{H}_0, \underline{H}_0^{\mathrm{g}}, \underline{H}_1$ and $\underline{H}_1^{\mathrm{g}}$ in equation (43), we get

$$(\sigma+ikU)^2 = (\sigma+ikU)_T^2+(\sigma+ikU)_{H_0}^2, \tag{4.12}$$

$$(\sigma+ikU)_T^2 = -\frac{T}{\rho R_0^3}(1-x^2)F_0(x,y), \tag{4.13}$$

$$(\sigma+ikU)_{H_0}^2 = \frac{\mu H_0^2}{\rho R_0^2}\Big\{-x^2+\Big[-\beta^2+\alpha^2\frac{xI_0(x)}{I_0'(x)}\Big]F_0(x,y)\Big\}, \tag{4.14}$$

where $F_0(x, y)$ is defined by:

$$F_0(x, y) = x \frac{L^0_{x,y}}{L^0_y} \tag{4.15}$$

with

$$L^0_{x,y} = I_0'(x)K_0'(y) - I_0'(y)K_0'(x), \tag{4.16}$$

$$L^0_y = I_0(x)K_0'(y) - I_0'(y)K_0(x). \tag{4.17}$$

5. Limiting cases

Equation (4.12) is the eigenvalue relation of the present model of a gas jet pervaded into bounded fluid of radius qR_0 ($1 < q < \infty$, R_0 is the radius of the gas jet) with variable magnetic field, see equation (2.1).

The relation (4.12) relates the growth rate σ or rather the oscillation frequency ω as $\sigma(= i\omega)$ is imaginary with the natural quantity $\left(\frac{T}{\rho R_0^3}\right)^{\frac{-1}{2}}$ as well as $\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{\frac{-1}{2}}$ as a unit of time, the modified Bessel functions I_0 & K_0 and their derivatives of different arguments, the longitudinal wave numbers x and y, the magnetic field parameters α and β in the gas region and with the parameters ρ, μ, R_0, T and H_0 of the problem.

Since the present problem is general one, some published works could be obtained from the present result as limiting cases under appropriate choices.
If we assume that $\alpha = 0, \beta = 0, H_0 = 0, U = 0$, and $q \to \infty$, then the relation (4.12) yields:

$$\sigma^2 = \frac{T}{\rho R_0^3}(1 - x^2)\, \frac{xK_1(x)}{K_0(x)}, \qquad K_1(x) = -K_0'(x). \tag{5.1}$$

This relation is indicated by Chandrasekhar (1961) in comparing the stability analysis of a hollow jet with that of full liquid jet.

If we assume $\alpha = 0, \beta = 0, H_0 = 0$, and $q \to \infty$, the dispersion relation (4.12) reduces to:

$$(\sigma + ikU)^2 = -\frac{T}{\rho R_0^3}(1 - x^2)\, \frac{xK_0'(x)}{K_0(x)}. \tag{5.2}$$

That coincides with the relation derived by Radwan (1989). One has to infer here that the results due to the discussions of the relations (5.1) and (5.2) are for some extent are in good agreement with the experimental results of Kendall (1986).

If we suppose that $\beta = 0$ and $q \to \infty$, we have

$$(\sigma + ikU)^2 = -\frac{T}{\rho R_0^3}(1 - x^2)\, \frac{xK_0'(x)}{K_0(x)} + \frac{\mu H_0^2}{\rho_0 R_0^2}\Big\{ - x^2 + \alpha^2 x^2\, \frac{I_0(x)K_0'(x)}{I_0'(x)K_0(x)}\Big\} \tag{5.3}$$

which is the MHD stability criterion of a hollow jet, endowed with surface tension and pervaded by axial magnetic fields, derived before by Radwan (1988) as $m = 0$ there and $U = 0$ here. See also Radwan's result (1998) (equation (6.9) there).

If we neglect here the effect of the capillary force on the stability of the present model of a hollow jet (and put $U = 0$ here), the relation (4.12) degenerates to Radwan's results (2002) in discussing the magneto dynamic stability of a bounded hollow cylinder pervaded by varying magnetic field.

If $T = 0$ and at the same time $\alpha = 0$ and $\beta = 0$, we have

$$\sigma^2 = -\frac{\mu H_0^2}{\rho R_0^2}\, x^2, \tag{5.4}$$

which indicates that the present model is always stable for all short and long wavelengths.

6. Stability discussions

The eigenvalue relation (4.12) is a linear combination of the relations of a gas jet bounded by radially finite fluid subjected by electromagnetic forces only and that one endowed with surface tension only. The relation (4.12) is a quadratic equation in σ. The model is unstable or otherwise based on the sign of σ^2:

(i) If σ^2 is negative, this means that σ is imaginary say $\sigma = i\omega$ $(i = \sqrt{-1})$, then the model will be ordinary stable.

(ii) If σ^2 is positive, this means that σ is real, then the model is ordinary unstable.

(iii) If $\sigma^2 = 0$, this means that there is no growth rate of the perturbation, then the model is neutral (marginally) stable.

In order to discuss the stability of the present model analytically, we need studying and writing down about the behavior and characters of the modified Bessel functions and their derivatives.

Consider the recurrence relations (see Abramowitz and Stegun (1970))

$$2I'_m(x) = I_{m-1}(x) + I_{m+1}(x), \tag{6.1}$$

$$2K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \tag{6.2}$$

and for each non-zero real value of x, that $I_m(x)$ is positive definite and monotonic increasing while $K_m(x)$ is monotonic decreasing but never negative (m is the order of the Bessel function here). So we have

$$I'_0(x) > 0 \quad \text{and} \quad K'_0(x) < 0 \tag{6.3}$$

also since $x < y$, we have

$$K_0(x) > K_0(y), \tag{6.4}$$

$$I_0(x) < I_0(y), \tag{6.5}$$

$$-|K'_0(x)| > -|K'_0(y)|. \tag{6.6}$$

It follows, for $y \neq 0$ and $x \neq 0$, that:

$$I_0(x)K_0'(y) - I_0'(y)K_0(x) < 0, \tag{6.7}$$

$$I_0'(x)K_0'(y) - I_0'(y)K_0'(x) > 0, \tag{6.8}$$

and consequently

$$F_0(x,y) = x\,\frac{L^0_{x,y}}{L^0_y} < 0. \tag{6.9}$$

Now, let us return to our task of discussing the stability of the present model. If we neglect the effect of the magnetic field i.e., the permeability coefficient is zero or rather there are no pervading magnetic field ($H_0 = 0$), the relation (4.12) reduces to the relation (4.13).

The discussions of the capillary eigenvalue relation (4.13) may be carried out by the aid of the relations (6.1)-(6.9).
In view of the recurrence relations (6.1), (6.2) and the inequalities (6.3)-(6.9), the sign of σ^2 is depending on the sign of the quantity $(1 - x^2)$. Therefore, we have the following:

$$\frac{\sigma^2}{[T/(\rho R_0^3)]} > 0 \quad \text{as} \quad 0 < x < 1, \tag{6.10}$$

$$\frac{\sigma^2}{[T/(\rho R_0^3)]} < 0 \quad \text{as} \quad 1 < x < \infty, \tag{6.11}$$

$$\frac{\sigma^2}{[T/(\rho R_0^3)]} = 0 \quad \text{as} \quad x = 1, \tag{6.12}$$

as we consider

$$U = 0, \tag{6.13}$$

we have the following three cases:

(i) The model is capillary unstable as $0 \leq x < 1$.

(ii) The model is marginally stable as $x = 1$.

(iii) It is ordinary stable as long as $1 < x < \infty$.

These results are similar to those of the full liquid cylinder and to those of ordinary hollow jet [a gas jet surrounded by unbounded liquid]. However, the maximum mode of instability of an ordinary hollow jet is much larger than that of a full liquid cylinder in vacuum. While the maximum modes of instability of the present bounded hollow jet corresponding to different values of $q = 2, 5, 10, 100, \cdots$ are among the maximum modes of the foregoing cases: e.g. as $100 < q < \infty$ the unstable curve is for some extent coincides with that of purely hollow jet. But whatever the smallest value of $1 < q \leq 1.001$, we see that the unstable curve is much lower than that of purely hollow jet and never to be near to the unstable curve of the full liquid jet.

The streaming has strong destabilizing effect for all long and short wavelengths. Therefore, the streaming has the effect of increasing and longating the unstable domain $0 \leq x < 1$ and simultaneously decreasing the stable domain $1 \leq x < \infty$ and change the critical value $x_c = 1$ to several values depends on the value of the speed of the fluid.

As we neglect the capillary force effect and considering that the model is acting upon the inertia, pressure gradient and electromagnetic forces; the magnetodynamic dispersion relation is obtained from (4.12) as $T = 0$ in the form of equation (4.14) together with equations (4.15)-(4.17).

The effect of the magnetic field in the fluid cylindrical region of radius qR_0 is represented by the term $(-x^2)$ followed by $\frac{\mu H_0^2}{\rho R_0^2}$. It has always stabilizing effects on the present model of bounded hollow jet or even if this model is unbounded. This effect is valid not only for short wavelengths but also for long wavelengths.

The effect of the magnetic fields in the gas region is represented by the terms including α and β following $\frac{\mu H_0^2}{\rho R_0^2}$ in equation (4.14).
The effect of the longitudinal magnetic field in the gas jet is represented by terms including α:

$$\frac{\mu H_0^2}{\rho R_0^2}\Big(\alpha^2 \ \frac{x I_0(x)}{I_0'(x)} \ F_0(x,y)\Big), \tag{6.14}$$

where $F_0(x,y)$ is still defined by equations (4.15)-(4.17).
In view of the inequalities (6.7)-(6.9), taking into account that $I_0(x)$ and its derivative are positive, we see that the longitudinal magnetic field penetrated in the gas jet is stabilizing for all short and long wavelengths.
The effect of the transverse magnetic field in the gas jet is represented by the term including β:

$$\Big(\frac{\mu H_0^2}{\rho R_0^2}\Big)\Big(-\beta^2 \ F_0(x,y)\Big), \tag{6.15}$$

that has a destabilizing effect for all wavelengths.

Therefore, we conclude that the transverse magnetic field has strong destabilizing effect while the axial field is stabilizing for all short and long wavelengths.

In addition to that the streaming has a strong destabilizing effect. So we are unable to decide whether the model is stable or not. Consequently, we have to make numerical discussions.

In comparing these results with those of ordinary hollow jet as $q \to \infty$, we see that the geometric factor q (which is the ratio of the fluid radius to the gas radius) has a stabilizing influence on the MHD hollow jet. This refers to that the present model is more realistic and can be realized than the open hollow jet, i.e., a gas jet surrounded by an infinite fluid, and so its stability states are more clear than in the other open model.

7. Numerical discussions

The main aim of the numerical analysis that we may identify exactly where are the stable and unstable domains of the model at hand. Also we find out the effect of the acting forces on the model.

In order to perform numerical discussions we have to write down the stability criterion (4.12) in dimensionless form in the most important mode $m = 0$ of perturbation as $U = 0$. Now since the natural quantity $\sqrt{\frac{T}{\rho R_0^3}}$ has unit of $(\text{time})^{-1}$ we have $\sigma^* = \left[\frac{\sigma}{\sqrt{\frac{T}{\rho R_0^3}}}\right]$ is the dimensionless temporal amplification and $H_s = \sqrt{\frac{T}{\mu R_0}}$ has a unit of magnetic field. Consequently, equation (4.12) becomes

$$\sigma^{*2} = (1 - x^2)F_0(x,y) + \left(\frac{H_0}{H_s}\right)^2 \left[-x^2 + \left(\alpha^2 x \frac{I_0(x)}{I_1(x)} - \beta^2\right)F_0(x,y)\right]. \tag{7.1}$$

The dispersion relation (7.1) has been computed in the computer for all short and long wavelengths $0 \le x \le 5$.
The values of σ^* corresponding to the unstable domains and those of $\omega^* = \frac{\omega}{\sqrt{\frac{T}{\rho R_0^3}}}$ corresponding to the stable domains are collected, tabulated and represented graphically. Such calculations have been elaborated for different values of (α, β) for various values of q and $\left(\frac{H_0}{H_1}\right)$ for regular values of $0 \le x \le 5$.

The numerical data are plotted graphically in figures 1-4 and tabulated in tables from which we deduce the following.

1. Corresponding to $(\alpha,\beta) = (0,2),(0,3)$ and $(1,3)$ for $q = 1.3$ and $\left(\frac{H_0}{H_1}\right) = 1$, it is found that the unstable domains are $0 < x < 1.383$, $0 < x < 2.546$ and $0 < x < 1.97$. While the stable domains for $(\alpha,\beta) = (0,1),(0,2),(0,3),(1,0),(3,0)$ and $(1,3)$ are $0 < x < \infty$, $1.383 \le x < \infty$, $2.546 \le x < \infty$, $0 < x < \infty$, $0 < x < \infty$ and $1.97 \le x < \infty$. Where the equalities are associated with the marginal stability states. See figure 1.
2. Corresponding to $(\alpha,\beta) = (0,1),(0,2),(0,3),(1,0),(3,0)$ and $(1,3)$ for $q = 1.5$ and $\left(\frac{H_0}{H_1}\right) = 0$, it is found only one unstable domain which is $0 < x < 1$ the capillary unstable domain since there is no magnetic field influence. The stable domain is $1 \le x < \infty$, where $x_c = 1$ the transition from stable domain to the unstable one. See figure 2.
3. Corresponding to $(\alpha,\beta) = (0,1),(0,2),(0,3)$ and $(1,3)$ for $q = 3$ and $\left(\frac{H_0}{H_1}\right) =$

1.8, it is found that the unstable domains are $0 < x < 1.181$, $0 < x < 2.606$, $0 < x < 4.225$ and $0 < x < 3.062$. While the stable domains for $(\alpha, \beta) = (0,1), (0,2), (0,3), (1,0), (3,0)$ and $(1,3)$ are $1.181 \leq x < \infty$, $2.606 \leq x < \infty$, $4.255 \leq x < \infty$, $0 < x < \infty$, $0 < x < \infty$ and $3.062 \leq x < \infty$. Where the equalities are associated with the marginal stable states. See figure 3.

4. Corresponding to $(\alpha, \beta) = (0,1), (0,2), (0,3)$ and $(1,3)$ for $q = 5$ and $\left(\frac{H_0}{H_1}\right) = 2$, it is found that the unstable domains are $0 < x < 1.207$, $0 < x < 2.753$, $0 < x < 4.537$ and $0 < x < 3.204$. While the stable domain corresponding to $(\alpha, \beta) = (0,1), (0,2), (0,3), (1,0), (3,0)$ and $(1,3)$ are $1.207 \leq x < \infty$, $2.753 \leq x < \infty$, $4.5377 \leq x < \infty$, $0 < x < \infty$, $0 < x < \infty$ and $3.204 \leq x < \infty$. See figure 4.

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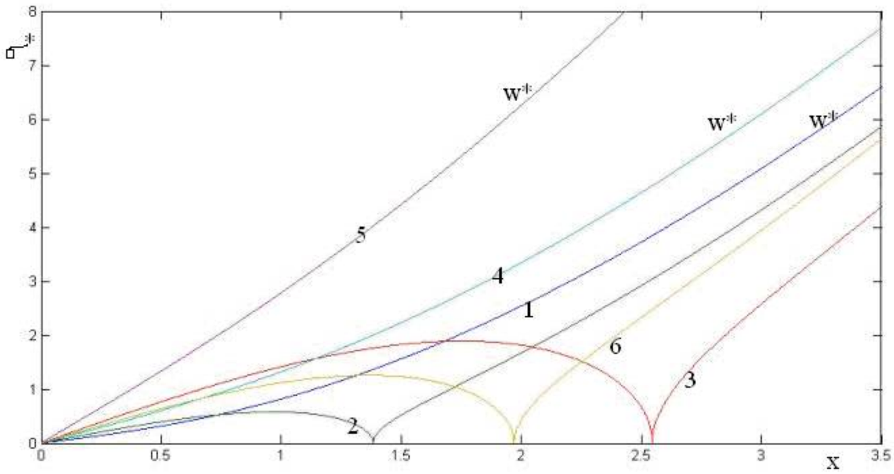

Figure 1: MHD stability of a gas jet in bounded fluid penetrated by non-uniform field for $q = 1.3$ and $\left(\frac{H_0}{H_1}\right) = 1$

(α, β)	$(0, 1)$	$(0, 2)$	$(0, 3)$	$(1, 0)$	$(3, 0)$	$(1, 3)$
line	1	2	3	4	5	6

Table 1: See the Figure 1.

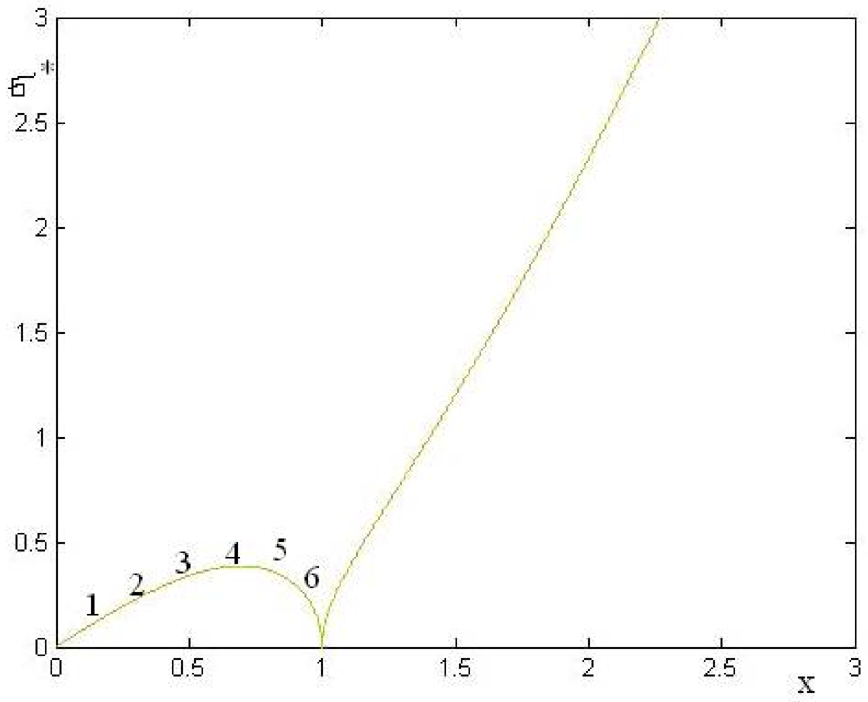

Figure 2: MHD stability of a gas jet in bounded fluid penetrated by non-uniform field for $q = 1.5$ and $\left(\frac{H_0}{H_1}\right) = 0$

(α, β)	$(0,1)$	$(0,2)$	$(0,3)$	$(1,0)$	$(3,0)$	$(1,3)$
line	1	2	3	4	5	6

Table 2: See the Figure 2.

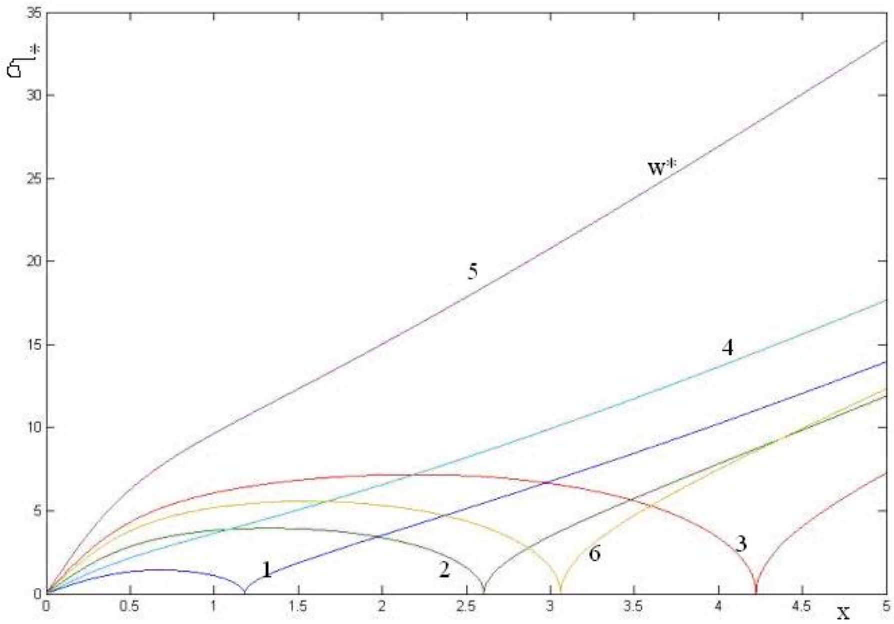

Figure 3: MHD stability of a gas jet in bounded fluid penetrated by non-uniform field for $q = 3$ and $\left(\frac{H_0}{H_1}\right) = 1.8$

(α, β)	$(0, 1)$	$(0, 2)$	$(0, 3)$	$(1, 0)$	$(3, 0)$	$(1, 3)$
line	1	2	3	4	5	6

Table 3: See the Figure 3.

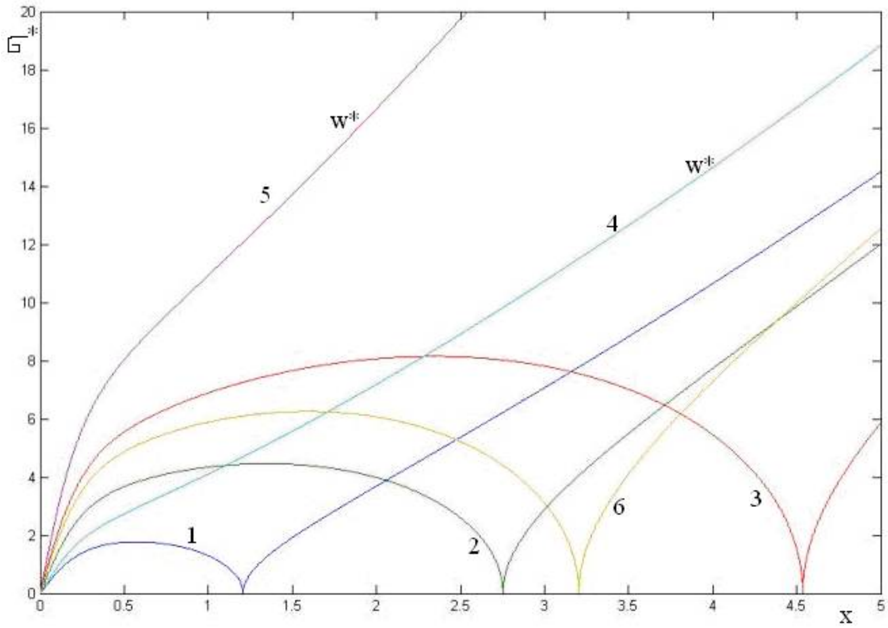

Figure 4: MHD stability of a gas jet in bounded fluid penetrated by non-uniform field for $q = 5$ and $\left(\frac{H_0}{H_1}\right) = 2$

(α, β)	$(0,1)$	$(0,2)$	$(0,3)$	$(1,0)$	$(3,0)$	$(1,3)$
line	1	2	3	4	5	6

Table 4: See the Figure 4.