

## Predictive Control for a Fin Stabilizer

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**Abstract :** A predictive controller can solve a control problem related to a disturbance-dominant system such as roll stabilization of a ship in waves. In this paper, a predictive controller is developed for a fin stabilizer. Future wave-induced moment is modeled simply using two typical regular wave components for which six parameters are identified by the recursive Fourier transform and the least squares method using the past time series of the roll motion. After predicting the future wave-induced moment, optimal control theory is applied to discover the most effective command fin angle that will stabilize the roll motion. In the results, wave prediction performance is investigated, and the effectiveness of the predictive controller is compared to a conventional PD controller.

**Key words :** Predictive control, Fin stabilizer, Wave-induced moment, Optimal control theory, Roll reduction

### 1. Introduction

A ship is continuously in motion while operating in waves. The main reason for such motion is due to the wave exciting force and moment acting on a ship (Bhattacharyya, 1976). Among various motion modes such as surge, sway, heave, roll, pitch, and yaw, the roll of a conventional displacement-type vessel influences mainly the safety of the cargo and the comfort of the passengers. In order to reduce the roll of ship in waves, anti-rolling stabilizers are used. An anti-rolling tank, a moving weight stabilizer, and a fin stabilizer are typical anti-rolling devices (Lewis, 1989; Lloyd, 1989; Martin, 1994). Among these devices, the fin stabilizer is most effective for a ship running at normal speed (Sellars and Martin, 1992).

As a rule, the command fin angle of a fin stabilizer is calculated using a PD controller (Åström and Hägglund, 1995; Silva et al., 2006) using the measured roll angle and rate of a ship. However, such a roll angle and rate respond with some delay on the wave-induced moment. For this reason, a conventional PD controller may not be the most feasible approach for a fin stabilizer. In addition, if the wave-induced moment is considered as an external random disturbance, an anti-rolling control problem may lead to problems with the regulator. In this case, as the external disturbance is dominant on roll mode, it will be more efficient if the one part of control moment is used to remove the external disturbance and the other part

regulates the roll motion of the ship. To do this, it is necessary to predict the wave-induced moment based on the knowledge of the ocean wave characteristics (Bhattacharyya, 1976). The prediction result can then be used to formulate a control algorithm.

A predictive control method can be a solution for this type of problem which commonly referred to as a disturbance-dominant problem. The technique consists of two parts. The first part is the prediction of the wave-induced moment and the second is the determination of the optimal command fin angle. Future wave-induced moment is predicted by the wave parameters that are determined from the motion data of the past. The prediction results are then included in a one-dimensional roll equation of motion. Finally, command fin angle for the current time is calculated using optimal control theory (Kirk, 1970).

Fig. 1 shows a flowchart of the predictive control algorithm developed in this paper for a fin stabilizer. The effectiveness of the proposed algorithm is confirmed by a comparison of the results of the roll reduction performances depending on the control method.

### 2. Equations of Motion

#### 2.1 Coordinate system

The motion of a ship in waves is generally described with respect to a body-fixed coordinate system and an

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earth-fixed coordinate system in order to consider six degrees of freedom motion. However, a one-dimensional simplified roll equation is sufficient to design a control algorithm for a fin stabilizer. For this reason, one inertial coordinate system moving at constant speed of a ship is used to describe the roll motion.

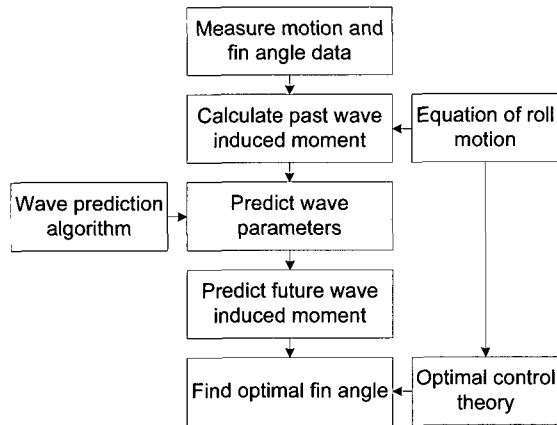


Fig. 1 Flowchart of the overall procedure

## 2.2 Roll equation

The simplified one-dimensional roll equation is described as follows:

$$(I_x + J_x)\ddot{\phi} + B_e\dot{\phi} + \Delta GM\phi = M_w + M_c \quad (1)$$

where,  $I_x, J_x, B_e, \Delta, GM, \phi$  and  $M$  represent a ship's axial mass moment of inertia, roll added mass moment of inertia, equivalent linear roll damping coefficient, weight, metacentric height, roll angle and external moment, respectively. Subscripts  $w$  and  $c$  denote wave and control moments, respectively. The dot over the roll angle denotes its time derivative, i.e. its roll rate.

The wave-related moment can be divided into three components of the radiation, diffraction, and Froude-Krilov moments. It is difficult to consider the radiation and diffraction moments in a linearized equation, therefore, it is assumed that the Froude-Krilov moment is the most dominant of the three components, and only the Froude-Krilov moment is considered as the wave exciting moment. Although the roll damping coefficient is the function of the roll, the fin geometry and the wave characteristics, it is assumed that the coefficient can be represented by the constant equivalent damping coefficient. In addition, while the slope of the restoring moment arm is generally decreased as the roll angle increases, the restoring moment term is simply modeled as in Eq.(1) with the assumption that the roll angle would not be large enough.

The control moment is described as follows:

$$M_c = \rho S V^2 l C_{L_\alpha} \delta \quad (2)$$

where,  $\rho, S, V, l, C_{L_\alpha}$ , and  $\delta$  are the fluid density, fin projected area, ship's speed, moment arm, lift curve slope, and fin deflection angle, respectively.  $M_c$  is the sum of the moments generated by the starboard and port fins.

It is convenient to convert Eq.(1) into the following standard form by dividing Eq.(1) by the virtual mass moment of inertia.

$$\ddot{\phi} + 2\zeta\omega_n\dot{\phi} + \omega_n^2\phi = m_w + m_c \quad (3)$$

In Eq.(3),  $\zeta$  and  $\omega_n$  are the standard roll damping ratio and the natural frequency.  $m_w$  and  $m_c$  are the values obtained by dividing  $M_w$  and  $M_c$ , respectively, by the virtual mass moment of inertia.

## 2.3 Actuator dynamics

Hydraulic devices drive fin shafts to the desired command deflection angles in the water. As the maximum deflection rate of a fin is limited, it is impossible for the fin to come up to the desired position instantly. For this reason, actuator dynamics are modeled as follows;

$$\dot{\delta} = K(\delta_c - \delta) \quad (4)$$

where  $K$  is the bandwidth of the hydraulic device and  $\delta_c$  is the command fin deflection angle calculated by a control law.

## 3. Wave Prediction

### 3.1 Wave-induced moment model

The elevation of an ocean wave is described as the sum of a number of sine components with different frequencies, amplitudes and phase angles due to its periodic characteristics. Eq.(5) is a description of wave elevation ( $\zeta$ ),

$$\zeta = \sum_i \zeta_i \cos(\omega_i t - k_i y - \delta_i) \quad (5)$$

where  $\zeta_i, \omega_i, k_i$  and  $\delta_i$  are the amplitude, frequency, wave number and phase angle of  $i$ -th wave component with respect to ship's motion, respectively.  $y$  is the lateral coordinate of the body-fixed coordinate system.

There are two wave-induced moment models used in this paper. The first is a model that estimates the dominant frequency and amplitude components. The second is a

model that accurately predicts the wave-induced moment during a short interval, even when the performance of the long-term prediction is not good.

Many frequency components have to be considered in the former wave-induced moment model in order to select two dominant wave components. The wave-induced moment component can be approximately modeled using the effective wave slope coefficient ( $r_i(k_i, B)$ ), which is the constant number representing the function of the wave number and the ship's breadth ( $B$ ). After summing the components, the wave-induced moment can be described by the Fourier series, as follows;

$$m_w = \omega_n^2 \sum_i r_i(k_i, B) \zeta_i \cos(\omega_i t - \delta_i - \gamma_i) \quad (6)$$

$$= \sum_i m_{wi} \cos(\omega_i t - \beta_i)$$

where  $\gamma_i$  and  $\beta_i$  are the  $i$ -th phase angles between the wave-induced moment and the wave elevation, and between the wave-induced moment and the ship's roll motion, respectively.

The second wave-induced moment model for predicting the short-term future moment is identical to the first model described in Eq.(6) except that there are only two frequency components, and that the parameters representing the model are obtained by the least squares method applied to the recent wave-induced moment data. The second wave-induced moment model is described as:

$$m_w = \sum_{i=1}^2 A_i \cos(\omega_i \bar{t} - \bar{\beta}_i) \quad (7)$$

where  $\bar{t}$  and  $\bar{\beta}_i$  are time elapsed since the current time and total phase at the current time  $t$  which is calculated by subtracting the  $\omega_i t$  from  $\beta_i$  in Eq.(6).  $\omega_i$  is selected by the first wave moment model and  $A_i$  and  $\bar{\beta}_i$  are determined by the least squares method.

### 3.2 Prediction procedure

The prediction procedure to determine the final wave-induced moment model described in Eq.(7) consists of three stages.

In the first stage, the wave moment at the current time is identified using the measured motion data and the roll equation described in Eq.(3). Given that the estimated external moment also includes the simultaneous control moment, it must be excluded.

Next, a Fourier transform is performed to determine the  $\omega_1$  and  $\omega_2$  frequencies of the dominant wave components in

Eq.(7), which have large amplitudes compared to the other components. In order to implement this algorithm onto real hardware, a recursive algorithm with a forgetting factor is used for the transform. The forgetting factor is determined considering the significant frequency range of ocean wave components.

After estimating the past wave-induced moment and selecting dominant frequencies, the parameters in Eq.(7), which will be used to predict the wave-induced moment when the optimal control command input is determined, are identified by the least squares method. The performance index to be minimized is;

$$F = \sum_k [m_w(t_k) - A_1 \cos(\omega_1 t_k - \beta_1) - A_2 \cos(\omega_2 t_k - \beta_2)]^2 \quad (8)$$

where  $t_k$  is the past time at which the estimated wave moment is recorded.

## 4. Predictive Control

As described above, the control algorithm for the fin stabilizer must work well in a disturbance-dominant environment like the sea. In this case, the wave-induced moment is the disturbance. It can exert stabilizing or destabilizing moment on a rolling ship. For this reason, it is helpful to consider the wave-induced moment when the command fin deflection angle is calculated.

Therefore, in order to determine the current command fin angle, the predictive control method is applied, which uses the system model in Eq.(3) and the optimal control theory. In this case, the estimated wave-induced moment and the maximum fin deflection angle are considered automatically.

After determining the time series of the optimal command fin angle, the first value is used for the current command. This procedure is carried out every time step for which a command fin angle is calculated.

### 4.1 Mathematical formulation

In order to determine the control input, which is the command fin angle in this paper, using the optimal control theory, it is necessary to formulate a physical problem mathematically as a two-point boundary value problem.

The state variable vector, of which the components are included in Eq.(3) and Eq.(4), is defined as follows;

$$\underline{x}(t) = [\phi(t) \quad \dot{\phi}(t) \quad \delta(t)]^T \quad (9)$$

where superscript  $T$  of a vector denotes its transpose. As the control variable is only the command fin

deflection angle, the control variable vector is represented by the scalar as follows;

$$u(t) = \delta_c(t) \quad (10)$$

There are two types of constraint in this problem. The first is related to the dynamics of the ship and the second is related to the operational limits of the fin deflection angle. Constraints are described mathematically, as follows;

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), u(t), t), \quad \text{and} \quad (11)$$

$$|\delta(t)| \leq \delta_{\max},$$

where,

$$\underline{f}(\underline{x}(t), u(t), t) = \begin{bmatrix} \dot{\phi}(t) \\ -2\zeta\omega_n\dot{\phi}(t) - \omega_n^2\phi(t) + m_w(t) + m_c(t) \\ K(\delta_c - \delta) \end{bmatrix} \quad (12)$$

Boundary conditions, which are the initial and final conditions of state variables, and final time are described as follows;

$$\underline{x}(t_0) = \underline{x}_0 = [\phi_0 \quad \dot{\phi}_0 \quad \delta_0]^T, \quad (13)$$

$$\underline{x}(t_f) = [\text{unspec} \quad \text{unspec} \quad \text{unspec}]^T, \quad \text{and}$$

$t_f$  fixed,

where 'unspec' denotes "unspecified".

At this point, the performance index is defined in order to apply the optimality concept. The performance index can be translated into the requirement or the objective of control. It is necessary for the roll angle and rate to be small during the pre-determined interval, which here is from  $t_0$  to  $t_f$ . In addition, it is better for the control energy to be as small as possible. The performance index considered in this paper is;

$$J = \int_{t_0}^{t_f} L(\underline{x}(t), u(t), t) dt \quad (14)$$

where

$$L(\underline{x}(t), u(t), t) = \alpha_1\phi^2 + \alpha_2\dot{\phi}^2 + \beta\delta_c^2 + \mu \left\{ \begin{aligned} &(\delta + \delta_{\max})^2 1(-\delta - \delta_{\max}) \\ &+ (\delta - \delta_{\max})^2 1(\delta - \delta_{\max}) \end{aligned} \right\}. \quad (15)$$

In Eq.(15), very large number  $\mu$  was used, as was a Heaviside step function denoted as  $1(\cdot)$  to consider the inequality constraint of the limitation of the maximum fin angle of Eq.(7).  $\alpha_1$ ,  $\alpha_2$ ,  $\beta$ , and  $\mu$  are design parameters that are positive constants.

## 4.2 Optimization

If the dynamic constraint and performance index are determined by Eq.(11) and Eq.(14), respectively, the Hamiltonian (Kirk 1970) can be defined by

$$H = L(\underline{x}(t), u(t), t) + \underline{\lambda}(t)^T \underline{f}(\underline{x}(t), u(t), t) \quad (16)$$

where  $\underline{\lambda}(t)$  is Lagrange multiplier or a costate vector.

When the Hamiltonian is defined as in Eq.(16), a two-point boundary value problem with the optimality condition for the control input can be formulated simply, as follows;

$$\begin{cases} \dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), u(t), t), \\ \dot{\underline{\lambda}} = -\frac{\partial H}{\partial \underline{x}}(\underline{x}(t), u(t), t), \\ \frac{\partial H}{\partial u}(\underline{x}(t), u(t), t) = 0, \end{cases} \quad (17)$$

$$\begin{cases} \underline{x}(t_0) = [\phi_0 \quad \dot{\phi}_0 \quad \delta_0]^T, \quad \text{and} \\ \underline{\lambda}(t_f) = \underline{0} \end{cases}$$

## 4.3 Numerical method

In order to calculate the nonlinear two-point boundary value problem described by Eq.(17), a first-order gradient method was used. The iterative procedure is displayed in Fig. 2.

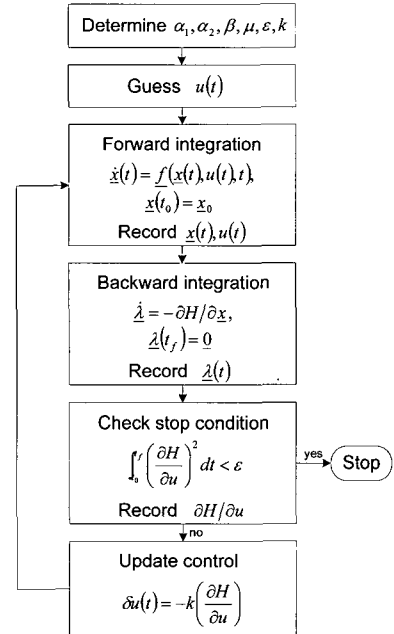


Fig. 2 Flowchart to calculate the optimal control input by using the first-order gradient method

It is necessary to estimate the initial time history of the control input well in order to reduce the iteration time.

Therefore, the initial values were determined by using a conventional PD controller considering its saturation.

### 5. Results and Discussion

In order to confirm the effectiveness of the proposed predictive control algorithm, a simulation study was performed. A target ship was selected; its principal dimensions are listed in Table 1.

Table 1 Principal dimensions of the target ship

Items	Values
Displacement	572 ton
Length between perpendiculars	55.2 m
Breadth	8.2 m
Draft	2.5 m
Natural roll period	5.35 second
Fin area	2.34 m <sup>2</sup>

#### 5.1 Wave prediction result

In order to confirm the established procedure for predicting the wave-induced moment, a time series of irregular wave-induced moment was generated with fifteen frequency components and random phases.

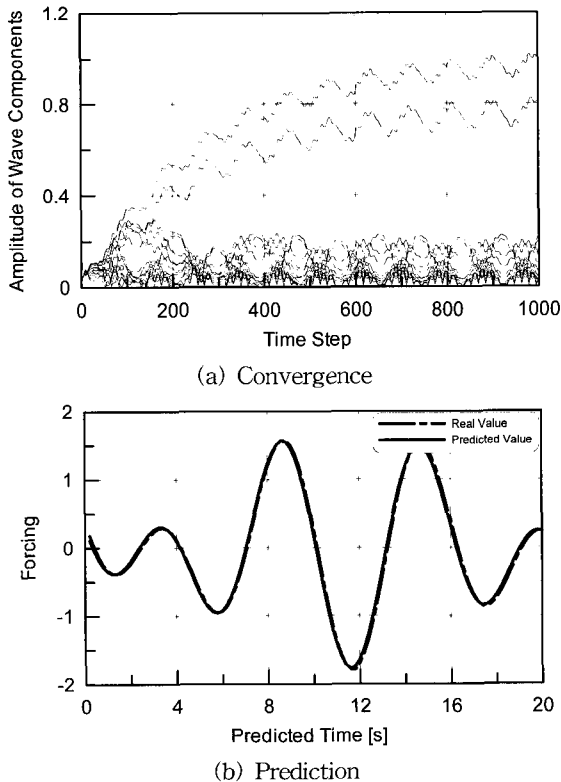


Fig. 3 Wave prediction result (Two dominant wave components)

Fig. 3 and Fig. 4 show the results of the estimated wave-induced moment components and the predicted wave-induced moments. In Fig. 3, the amplitudes of two dominant components are 1.0 and 0.7. The frequencies are 1.0 and 0.7 times the natural frequency of the ship. In Fig. 4, the amplitudes of the three dominant components are 1.0, 0.5 and 0.2. The frequencies are 1.0, 0.7 and 1.3 times the natural frequency of the ship.

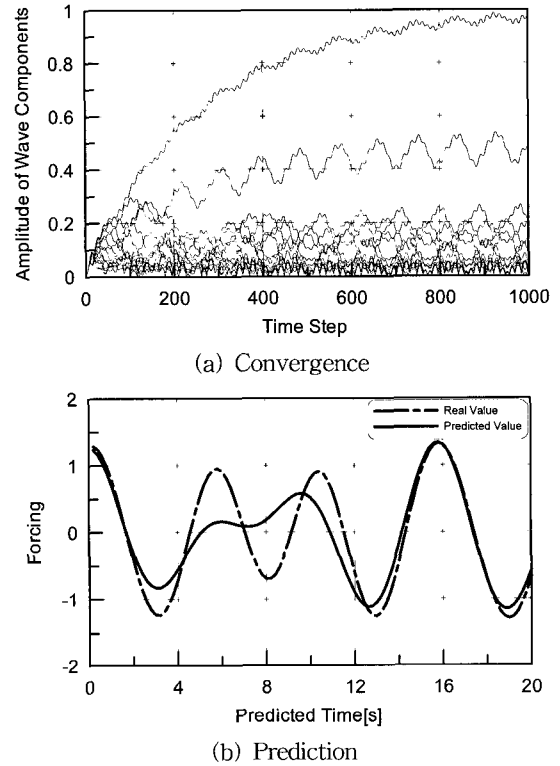


Fig. 4 Wave prediction result (Three dominant wave components)

As shown in Fig. 3(a) and Fig. 4(a), the dominant components are estimated well after nearly six hundred time steps, despite the fact that a recursive algorithm was used. As the wave prediction model described in Eq.(7) has two frequency components, even a lengthy predicted value is feasible, as shown in Fig. 3(b). However, for three wave components, the prediction result is only valid up to a small interval. When the wave prediction result is included in the predictive control algorithm, only a brief predicted value is used as an optimal control command is also found for such a short duration. For this reason, the half natural period of the ship listed in Table 1 was selected as the prediction time interval.

#### 5.2 Control algorithm

While a PD controller is generally used for the fin stabilizer, the optimal control algorithm was used to determine

the time series of control inputs for a short time in the near future. In order to compare the optimal control algorithm with the conventional PD controller for a fin stabilizer, two virtual situations were generated, and the roll reduction performances were then investigated. When the optimal control algorithm was applied, a wave prediction was not carried out in order to compare only the control algorithms. For simplicity, it was assumed that the wave-induced moment of a single component was acting on the ship.

First, as the external disturbance was large, the control input was saturated, as shown in Fig. 5. Changing the fin angle to the opposite side more rapidly, which is calculated automatically in the optimal control algorithm, increases the reduction performance by nearly 30 %, as the most effective way is determined at every time step.

Next, if the resultant fin angle is changed slowly on the command angle on account of slow actuator dynamics, the reduction performance by the PD controller is diminished if the actuator dynamics is not considered when determining the control gains. In Fig. 6, reduction performances are compared for a system with slow actuator dynamics. Although the magnitude of the control input generated by the optimal control algorithm is smaller than the PD controller, its performance is better. If control gains of the PD controller are determined considering the actuator dynamics, the difference between two control algorithms may be small. However, it is easier to consider such actuator dynamics in the optimal control algorithm than in the PD controller.

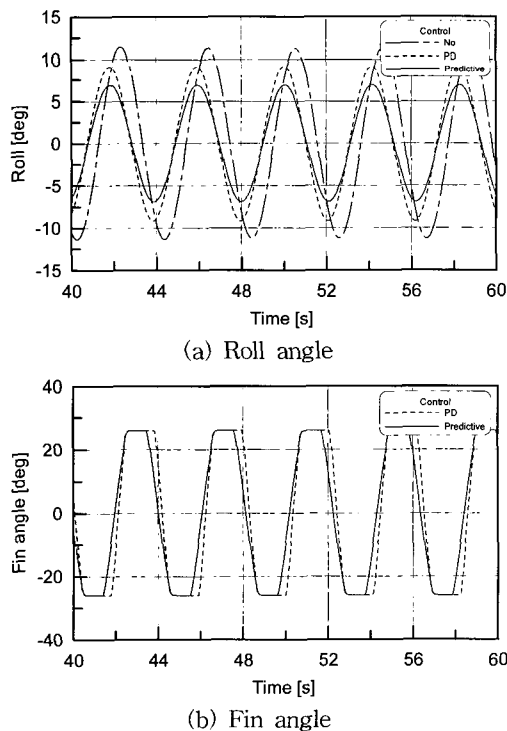


Fig. 5 Comparison of control algorithms when the external disturbance is large

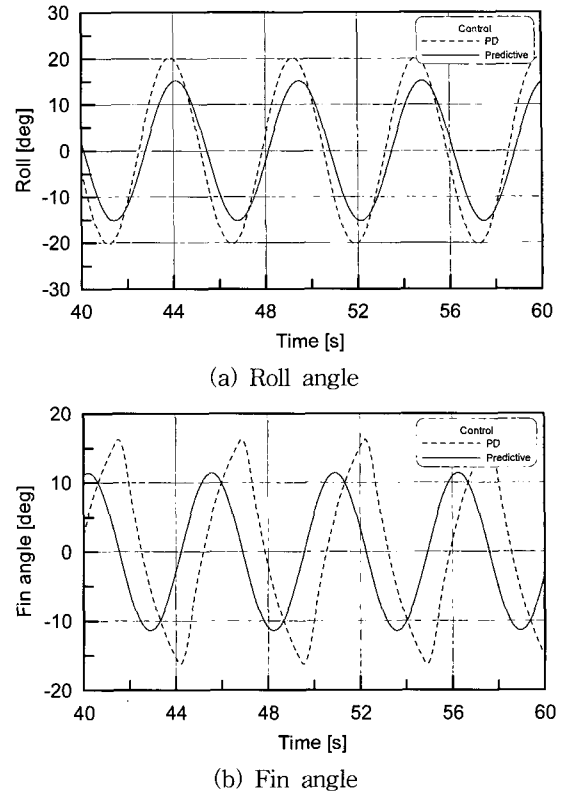


Fig. 6 Comparison of control algorithms when an actuator dynamics is slow

### 5.3 Roll reduction performance

Ship motion in waves was calculated by the five degrees of freedom equations of motion without yaw (Fossen, 1994). For simplicity, it was assumed that the ship's autopilot keeps her on course. The wave-induced moment was calculated based on the linear gravitational wave theory and strip theory (Lee, 2004). In this case, a long-crested irregular wave was generated by the ITTC wave spectrum (Fossen, 1994). Ten wave components were selected and a beam sea condition was assumed. The measure representing the roll reduction performance was selected as the root mean square (RMS) roll angle during a predefined duration.

The simulation was performed assuming that a ship should stay in non-controlled mode for one hundred seconds, and then would be controlled for the next one hundred seconds. The prediction interval was selected as the half natural period of a ship.

In Fig. 7, the roll is shown to be reduced after stabilizing commences. The RMS roll angles under no control, PD control, and predictive control are 7.41, 2.29, and 1.36 degrees, respectively. Greater than a 40% reduction performance is obtained when adopting the predictive control algorithm instead of a PD controller. However, the RMS fin angle used for stabilizing under predictive control

is 12.97 degrees this value is 5.57 degrees under PD control. For the predictive control, a percentage of the total control fin angle is used to overcome the predicted wave-induced moment, and the remaining percentage is used for regulating. This is possible because the wave-induced moment is predicted.

The control gains of PD controller were determined by the pole placement technique, in order for the feedback system to become 1.2 times of ship natural roll frequency and have 0.3 of standard damping ratio. The most effective control gains of PD controller has to be changed depending on the wave conditions. However, in the predictive controller, wave prediction algorithm considers this effect and the optimal control inputs determined automatically following the case by case.

## 6. Conclusions

In this paper, a predictive control algorithm for a type of fin stabilizer generally used for roll stabilization of a ship is developed. To do this, the wave-induced moment was predicted using a simple model identified by measured motion

data. This procedure is utilized to determine the optimal control input command.

Although the model describing the wave-induced moment has only two frequency components, it was confirmed that the prediction results are valid for a short time. The optimal control algorithm was superior when the disturbance was large and a number of sub-system dynamics existed.

Through the motion simulation of a ship in irregular beam waves, it was confirmed that the roll reduction performance could be increased if the predictive control algorithm is applied in place of a conventional PD controller which has constant control gains. Predictive controller maintains good performance in the various wave conditions, because wave prediction algorithm responds automatically on such conditions.

In the implement of the developed algorithm on actual hardware, the calculation time is a stumbling block. Therefore, a more efficient calculation procedure should be sought. In addition, as the dynamic model of a ship and wave prediction model are important factors for the performance of the controller, reliable identification methods for these models are needed.

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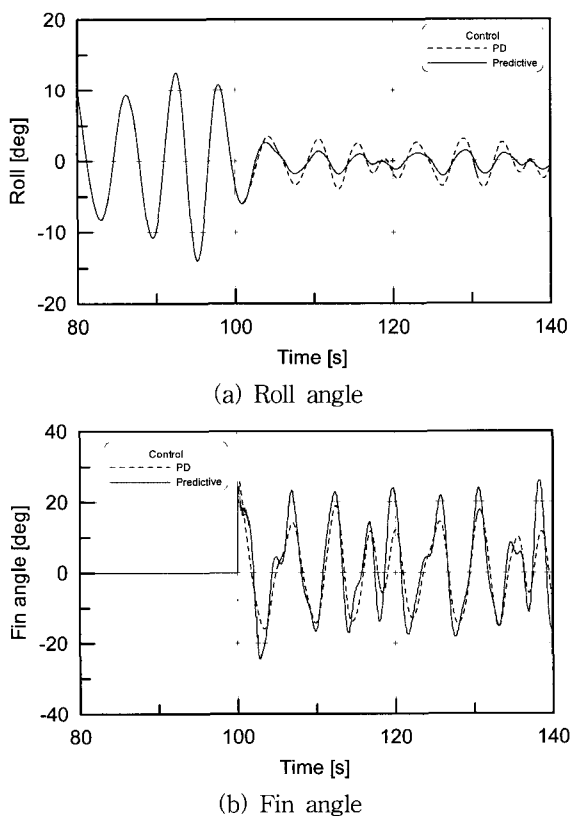


Fig. 7 Roll reduction performance depending on the control algorithms applied to a ship running in irregular waves

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