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Simple Harmonic Oscillation of Ferromagnetic Vortex Core

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Here we report a theoretical description of ferromagnetic vortex dynamics. Based on Thiele's formulation of the Landau-Lifshitz-Gilbert equation, the motion of the vortex core could be described by a function of the vortex core position. Under a parabolic potential generated in the circular magnetic patterns, the vortex core showed a circular rotation–namely the gyrotropic motion, which could be described by a 2-dimensional simple harmonic oscillator. The gyrotropic frequency and apparent damping constant were predicted and compared with the values obtained micromagnetic calculation.

Keywords : ferromagnetic vortex, vortex dynamics, micromagnetic calculation

1. Introduction

The ferromagnetic vortex structure–curling in-plane magnetization around the core of out-of-plane magnetization–is one of the common magnetic structures usually appearing in typical size of the magnetic application device elements, in particular in materials with low crystalline anisotropy. Despite of their technological importance in spintronic devices, experimental characterization on their detailed properties has begun only recently by means of either the advanced magnetic imaging technologies [1, 2] or the picosecond-range time-resolved dynamics probing technologies [3, 4]. The most interesting features of the vortex dynamics are their low frequency nature with a long lifetime–the frequency in the range of sub-GHz with the lifetime in the range of tens of nanoseconds. The low frequency mode basically comes from the slow translation speed of the vortices (~100 m/s) over the traveling path length (~ a few hundred nanometers). The vortex translation is basically similar to the group velocity of the spin wave packet, which is much slower than the phase velocity especially for small wave packet structures. On the other hand, the long lifetime is ascribed to the small volume of the vortex core where the damping occurs in the motion.

The vortex motion has been explained by the Thiele's formulation [5], which provides a force equation as a function of the vortex core position. In his formulation, Thiele assumed a rigid vortex structure, which could be clearly parameterized by only one variable–the position of vortex core. However, in the patterned magnetic structure, the boundary might disturb the vortex structure and thus, the rigid vortex assumption does not hold. Alternative approach is the "surface-charge-free" spin distribution model [6]. From the FMR measurements, Novad *et al.* confirmed this model's validity experimentally [7]. In this paper, using "surface-charge-free" spin distribution model, we propose a modified Thiele's equation, which incorporates with the circular boundary condition.

2. Theoretical Description

An alternative description of the Landau-Lifshitz-Gilbert (LLG) equation was proposed by Thiele. His derivation can be briefly summarized as follows. The LLG equation is originally given by

$$\frac{d\vec{M}}{dt}=-|\gamma|(\vec{M}\times\vec{H}^{r})+\frac{\alpha}{M_S}\vec{M}\times\frac{d\vec{M}}{dt}. \quad (1)$$

The time-derivative term in the left side can be written by use of vector algebra as

$$\vec{M}\times\left(\vec{M}\times\frac{d\vec{M}}{dt}\right)=\vec{M}\left(\vec{M}\cdot\frac{d\vec{M}}{dt}\right)-\frac{d\vec{M}}{dt}(\vec{M}\cdot\vec{M})=-M_S^2\frac{d\vec{M}}{dt}. \quad (2)$$

Here the uniform magnitude of the magnetization was assumed i.e.

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$$\begin{cases} \vec{M}\cdot\vec{M}=M_S^2 \\ \vec{M}\cdot\dfrac{d\vec{M}}{dt}=\dfrac{1}{2}\dfrac{d(\vec{M}\cdot\vec{M})}{dt}=\dfrac{1}{2}\dfrac{dM_S^2}{dt}=0 \end{cases} . \tag{3}$$

Then, the LLG equation becomes

$$-\frac{1}{M_S^2}\vec{M}\times\left(\vec{M}\times\frac{d\vec{M}}{dt}\right)=-|\gamma|(\vec{M}\times\vec{H}^r)+\frac{\alpha}{M_S}\vec{M}\times\frac{d\vec{M}}{dt}. \tag{4}$$

This equation can be rewritten as

$$|\gamma|\vec{M}\times(\vec{H}^g+\vec{H}^r+\vec{H}^\alpha)=0, \tag{5}$$

after one defines the magnetic field terms in the equation as

$$\begin{cases} \vec{H}^g=-\dfrac{1}{|\gamma|M_S^2}\vec{M}\times\dfrac{d\vec{M}}{dt} \\ \vec{H}^\alpha=-\dfrac{\alpha}{|\gamma|M_S}\dfrac{d\vec{M}}{dt} \end{cases}, \tag{6}$$

where $\vec{H}^g$ is the gyrotropic field and $\vec{H}^\alpha$ is damping field. $\vec{H}^r$ is the effective field originally appearing in the LLG equation. Eq. (5) holds when

$$\vec{H}^g+\vec{H}^r+\vec{H}^\alpha=0. \tag{7}$$

This is the alternative formulation of the LLG equation.

For a rigid vortex, the magnetization distribution can be described by the vortex core position $\vec{X}$. Thiele used the Cartesian coordinate under an assumption of rigid vortex in the linear translation. The magnetization is then written as

$$\vec{M}(\vec{x})=\vec{M}_0(\vec{x}-\vec{X}), \tag{8}$$

where $\vec{x}$ is the space coordinate and $\vec{M}_0$ is the magnetization distribution when the vortex core locates at origin.

The work density done in magnetization under a magnetic field is $dw=\vec{H}\cdot d\vec{M}$. Since the magnetization is a function of position, the equation can be written for the Cartesian coordinate as

$$dw=\vec{H}\cdot\frac{\partial\vec{M}}{\partial X_i}dX=f_i dX_i, \tag{9}$$

for summation over $i = x, y, z$, where the force density f_i is given by

$$f_i=\vec{H}\cdot\frac{\partial\vec{M}}{\partial X_i}=-\vec{H}\cdot\frac{\partial\vec{M}}{\partial x_i}. \tag{10}$$

The gyrotropic force density then becomes

$$f_i^g=-\vec{H}^g\cdot\frac{\partial\vec{M}}{\partial x_i}=-\frac{1}{|\gamma|M_S^2}\varepsilon_{lmn}M_l\frac{\partial M_m}{\partial x_i}\frac{dM_n}{dt}, \tag{11}$$

and the damping force density is

$$f_i^\alpha=-\vec{H}^\alpha\cdot\frac{\partial\vec{M}}{\partial x_i}=\frac{\alpha}{|\gamma|M_S}\frac{dM_n}{dt}\frac{\partial M_n}{\partial x_i}. \tag{12}$$

By use of the rigid vortex assumption,

$$\frac{dM_n}{dt}=\frac{\partial M_n}{\partial X_j}\frac{dX_j}{dt}=\frac{\partial M_n}{\partial X_j}v_j=-\frac{\partial M_n}{\partial x_j}v_j. \tag{13}$$

Substituting Eq. (13) into Eq. (11), the gyrotropic force density becomes

$$f_i^g=\frac{1}{|\gamma|M_S^2}\varepsilon_{lmn}M_l\frac{\partial M_m}{\partial x_i}\frac{\partial M_n}{\partial x_j}v_j=-\varepsilon_{ijk}v_j g_k \tag{14}$$

with the gyrocoupling vector

$$g_k=-\frac{1}{2|\gamma|M_S^2}\varepsilon_{ijk}\varepsilon_{lmn}M_l\frac{\partial M_m}{\partial x_i}\frac{\partial M_n}{\partial x_j}, \tag{15}$$

and the dissipation force density becomes

$$f_i^\alpha=-\frac{\alpha}{|\gamma|M_S}\frac{\partial M_n}{\partial x_j}\frac{\partial M_n}{\partial x_i}v_j=d_{ij}v_j, \tag{16}$$

with the dissipation tensor

$$d_{ij}=-\frac{\alpha}{|\gamma|M_S}\frac{\partial M_l}{\partial x_j}\frac{\partial M_l}{\partial x_i}. \tag{17}$$

The total force F_i exerting on the vortex core position is then given by integrating the force density f_i over the volume V. Thus, the total force terms become

$$\begin{cases} F_i^g=\int f_i^g dV=-\varepsilon_{ijk}G_k v_j \\ F_i^\alpha=\int f_i^d dV=D_{ij}v_j, \end{cases} \tag{18}$$

where the total gyrocoupling vector and the total dissipation tensor are

$$\begin{cases} G_k=\int g_k dV=\dfrac{1}{|\gamma|M_S^2}\varepsilon_{ijk}\varepsilon_{lmn}\int M_l\dfrac{\partial M_m}{\partial x_i}\dfrac{\partial M_n}{\partial x_j}dV \\ D_{ij}=\int d_{ij}dV=-\dfrac{\alpha}{|\gamma|M_S}\dfrac{\partial M_l}{\partial x_j}\dfrac{\partial M_l}{\partial x_i}dV. \end{cases} \tag{19}$$

The force from the effective field $\vec{H}^r$ originally appearing in the LLG equation can be simply given by the magnetic potential as

$$\vec{F}^r=-\vec{\nabla}_{\vec{X}}U(\vec{X}), \tag{20}$$

where the magnetic potential includes the exchange, anisotropy, and the magnetostatic energy and is given by the function of the vortex position $\vec{X}$. The magnetic field equation given in Eq. (6) is then converted into the force equation as

$$\vec{F}^g+\vec{F}^r+\vec{F}^\alpha=\vec{G}\times\dot{\vec{v}}-\vec{\nabla}U+\tilde{D}\cdot\dot{\vec{v}}=0\,. \tag{21}$$

This is the Thiele's equation originally described in his paper [5]. Note that all terms are in the dimension of force and given by a function of the vortex position.

For the thin film geometry the total gyrocoupling vector $\vec{G}$ is perpendicular to the film plane, i.e. $\vec{G}=G\hat{z}$ if we put the film on (x, y) plane. The gyroconstant is $G=2\pi qpt_f M_s/\gamma$, t_f is thickness. The topological charge $q=\pm 1, \pm 2,...$ determines direction of core rotation and its frequency. The vortex polarization p is determined by M_z. It is ascribed to the assumption of the uniform magnetization across the film thickness i.e. $\partial\vec{M}/\partial z=0$. From this assumption, the z-index components of total dissipation tensor $\tilde{D}$ vanish i.e. $D_{xz}=D_{yz}=D_{zz}=D_{zx}=D_{zy}=0$. Together with this, for the vortex structure, the off-diagonal components of total dissipation tensor vanish i.e. $D_{xy}=D_{yx}=0$ and the diagonal terms have the unique value i.e. $D_{xx}=D_{yy}=D$ due to the symmetry. Thus, the total dissipation force can be written as $\vec{F}^\alpha=\tilde{D}\cdot\dot{\vec{v}}=D\dot{\vec{v}}$ where $\dot{\vec{v}}$ lies in (x, y) plane. The magnetic potential is an even function of the vortex position around the equilibrium position. For a small displacement $\vec{X}$ from the equilibrium position, it can be thus given by a parabolic function of the vortex position i.e. $U(\vec{X})=1/2kX^2$, where the restoring force constant k is related to the initial susceptibility [6].

The gyrocoupling coefficient G and the dissipation coefficient D are constant irrespective to the vortex position as we will discuss later by means of micromagnetic evaluation. Then, Eq. (19) can be written with the vortex position $\vec{X}=(x, y, 0)$ as

$$G(\dot{x}\hat{j}-\dot{y}\hat{i})-k(x\hat{i}-y\hat{j})+D(\dot{x}\hat{i}-\dot{y}\hat{j})=0\,. \tag{22}$$

Decoupling the $\hat{i}$ and $\hat{j}$ components results in

$$\begin{cases} -G\dot{y}+D\dot{x}-kx=0 \\ +G\dot{x}+D\dot{y}-ky=0 \end{cases}. \tag{23}$$

After eliminating y, the equation is equivalent to the damped harmonic oscillation equation as

$$\ddot{x}+2\gamma\dot{x}+\omega_0^2 x=0\,, \tag{24}$$

where the dissipation coefficient and the natural frequency corresponds to

$$\begin{cases} \gamma=-\dfrac{kD}{(G^2+D^2)} \\ \omega_0^2=\dfrac{k^2}{(G^2+D^2)} \end{cases}. \tag{25}$$

Thus, the experimental gyrotropic vortex motion of the circular (or spiral) rotation can be explained.

3. Numerical Calculation

To check the validity of the analytic description, a micromagnetic simulation was carried out by use of OOMMF [8]. The assumptions made in the model were the invariance of the dynamics parameters G and D, together with the parabolic dependence of the internal magnetic energy. Once all these assumptions are turned out to be valid, the model can be used in prediction the vortex dynamics. Here we present the micromagnetic evaluation of the assumptions.

Figure 1 shows a time evolution of the vortex core position of permalloy disk with 200 nm in diameter and 10 nm in thickness. The material parameters of permalloy was chosen as the default values in OOMMF i.e. the saturation magnetization $M_S=8.6\times10^5$ A/m and the exchange stiffness $A_x=1.3\times10^{-11}$ J/m with zero anisotropy. The Gilbert damping coefficient was 0.05 and the lateral size of the unit cell was 2.5 nm. The vortex was initially excited under an external magnetic field 10 mT and then, released by sudden removal of the magnetic field. Figure 1(a) shows the spiral motion of the vortex core position–so-called gytropic motion. Fig. 1(b) shows the displacement of the vortex in the x axis with time, which shows the damped harmonic oscillation as predicted in Eq. (24). Damped oscillating motion was first obtained by Guslienko *et al.* [6], and we reconfirmed spiral and harmonic oscillation motion of vortex.

Thiele's gyrotropic parameters G and D were evaluated

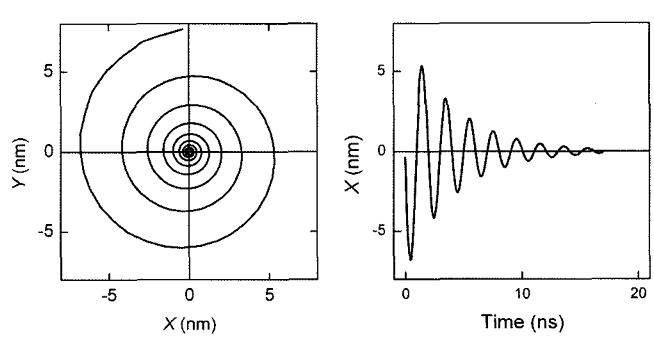

Fig. 1. (a) The vortex core trajectory in XY plane. (b) The displacement in the x axis with time.

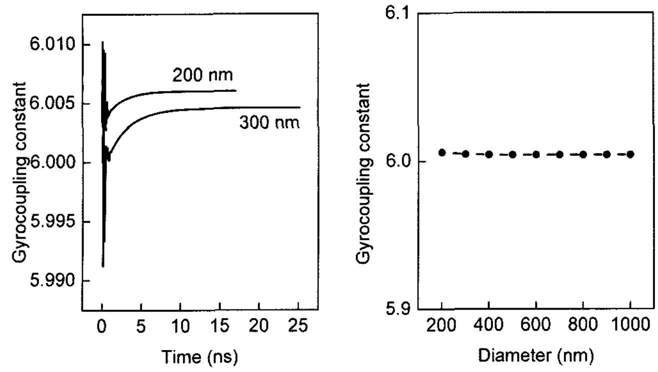

Fig. 2. (a) The gyrocoupling coefficient G of 200 nm and 300 nm disks with respect to time. (b) The gyrocoupling coefficient with respect to the disk diameter.

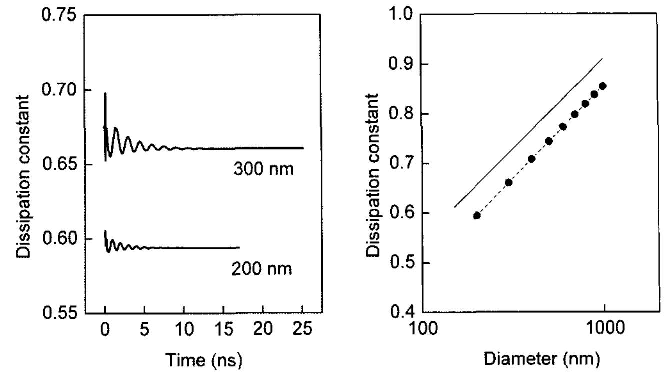

Fig. 3. (a) The reduced damping coefficient d of 200 nm and 300 nm disks with time. (b) Reduced damping coefficient with respect to the disk diameter. Circle dots are simulated results, solid line is calculated value.

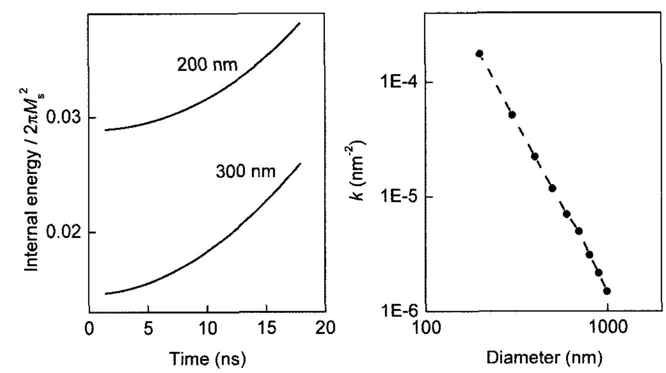

Fig. 4. (a) The internal magnetic energy with respect to the vortex position. The energy was normalized by the magnetostatic energy i.e. $2\pi M_S^2$. (b) The restoring force constant k with respect to the sample diameter. The constant was in the dimension of nm^{-2}.

from the micromagnetic results. In Fig. 2(a), the gyrocoupling coefficient G initially shows large fluctuation originated from the sudden removal of the magnetic field, which generated noisy broadband spin waves. Such spin waves diminished quickly within one nanosecond and then, a stable vortex configuration came out. The value of the gyrocoupling coefficient changed a little bit with time, but variation was less than 0.1% to the mean value. Such a small variation would be ignorable compared with the accuracy provided by the experimental measurement technique and thus, it is reasonable to consider the gyrocoupling coefficient unchanged during the motion.

Fig. 2(b) shows the gyrocoupling coefficient with respect to the disk size. With increasing the disk size, the gyrocoupling coefficient converged to a certain value. It can be understood since the gyrocoupling vector given in Eq. (15) has the finite values only in the vicinity of the vortex core and thus, a disk certainly larger than the vortex core size has the unique gyrocoupling coefficient irrespective to the disk size. Even for the smallest disk the variation was again quite less than 0.1% and thus, it is reasonable to assume that the universal value of the gyrocoupling coefficient with is

$$G=6.004 t_f \frac{M_S}{|\gamma|},$$

where t_f is the thickness of the film. The analytical model predicts gyrocoupling coefficient 6.283, and this value agreed well experimentally [7]. But analytical prediction and simulation result for gyrocoupling coefficient has only a few differences probably come from finite thickness effect.

Fig. 3(a) shows the reduced damping coefficient d with time. d is $d=-D\gamma/M_s t_f$. The fluctuation came from the numerical error generated at the discrete disk boundary, since the circular disk in the simulation was discretized based on the square cells. It is interesting to note that the mean value of the reduced damping coefficient was almost unchanged within a sample, but largely different between samples. Fig. 3(b) shows the simulated and calculated damping coefficient with respect to the disk size. Note that the disk size is in logarithmic scale and the reduced damping coefficient shows clear logarithmic dependence to the disk size. In soliton model of the magnetic vortex, calculation yields $D = -\alpha\pi M_s t_f[2+\ln(R/R_c)]/\gamma$. Here α stands for the Gilbert damping constant. The thickness dependent vortex R_c is $R_c = 0.68L_e(t_f/L_e)^{1/3}$ at $t_f \geq L_e$, and exchange length L_e is $L_e=(2A/M_s^2)^{1/2}$ [9]. The simulated result is similar to calculated value.

The internal magnetic energy with respect to the vortex position is shown in Fig. 4(a). As discussed earlier, it shows an even function of the vortex position around the equilibrium position. It is clear that the function could be well fit by a parabolic function of the vortex position i.e.

$U(\vec{X})=1/2kX^2$ for a small displacement $\vec{X}$ from the equilibrium position. The restoring force constant k with respect to the sample diameter is shown in Fig. 4(b).

Summary

We developed an analytic model of the simple harmonic oscillatory behavior in the vortex dynamics. The validity of the model was confirmed by a numerical calculation based on a micromagnetic solver – OOMMF.

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References

[1] T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto, and T. Ono, Science **289**, 930 (2000).
[2] A. Wachowiak *et al.*, Science **298**, 577 (2002).
[3] S.-B. Choe *et al.*, Science **304**, 420 (2004).
[4] J. P. Park, P. Eames, D. M. Engebretson, J. Berezovsky, and P. A. Crowell, Phys. Rev. B **67**, 020403 (2003).
[5] A. A. Thiele, Phys. Rev. Lett. **30**, 230 (1973).
[6] K. Yu. Guslienko *et al.*, J. of Appl. Phys. **91**, 8037 (2002).
[7] V. Novosad *et al.*, Phys. Rev. B **72**, 024455 (2005).
[8] http://math.nist.gov/oommf/
[9] K. Yu. Guslienko, Appl. Phys. Lett. **89**, 022510 (2006).