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Comparison of Fiber-Based Frequency Comb and Ti:Sapphire-Based Frequency Comb

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For the first time we compare two kinds of optical frequency combs, one of which is based on a Ti:sapphire femtosecond laser and the other is based on a mode-locked erbium-doped fiber laser. The comparison is performed by measuring an optical frequency standard with these two combs simultaneously. The two frequency measurements agree within 1.8 Hz (3.8×10^{-15}) with the uncertainty of 17.2 Hz (3.6×10^{-14}), from which it can be concluded that the Ti:sapphire-based frequency comb and the fiber-based frequency comb have no systematic discrepancy at this level of uncertainty.

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I. INTRODUCTION

The advent of the octave-spanning femtosecond optical frequency combs has revolutionized the optical frequency metrology by making the one-step absolute optical frequency measurement possible without the employment of the complicated traditional frequency chains [1,2]. The optical frequency combs now play a key role in a variety of applications including absolute optical frequency measurement [1], optical atomic clocks [3], direct comb spectroscopy [4], and time variation of fundamental constants [5]. As these applications demand an extreme accuracy level, there have been continuing efforts to investigate the practical and fundamental accuracy limits of the optical frequency comb technique [6-13]. These accuracy tests were performed by the frequency comparisons between optical frequency combs, which will be introduced and analyzed in detail in section II.

Most frequency comb systems have been based on Ti:sapphire femtosecond lasers. Ti:sapphire femtosecond lasers, the spectra of which are broadened by photonic crystal fibers, are undoubtedly the most promising devices for optical frequency synthesizers with reliable and robust operation. Recently, systems based on femtosecond erbium-doped fiber lasers were developed and are tracing the technological achievements of the Ti:Sapphire-based systems [14-16]. Fiber combs have advantages of trouble-free long term operation over weeks, ease of use, cost-effectiveness, compact setup, and possibility of frequency dissemination via existing fiber networks.

Some experimental results of Ti:Sapphire-based comb comparison [6-8, 10-13] and fiber-based comb comparison [9] have been reported. Here we present the comparison between a Ti:sapphire-based comb and a fiber-based comb for the first time to our knowledge. The comparison is performed by measuring an optical frequency standard with these two combs simultaneously. The two frequency measurements agree within 1.8 Hz (3.8×10^{-15}) with the uncertainty of 17.2 Hz (3.6×10^{-14}). We can conclude by this result that the frequency comb technique introduces no systematic discrepancy to this level of uncertainty, even though the Ti:sapphire-based combs and the fiber-based combs have quite different configuration and are based on different mode-locking mechanisms.

II. ANALYSIS OF COMB COMPARISON SCHEMES

The accuracy limit for the comb comparison is basically determined by two factors; the stability of the reference frequency for the comb stabilization and the data acquisition time. If the reference frequency stability is better, shorter averaging time is needed to reach a specified uncertainty level. Although a long averaging time is needed for the lower uncertainty, there is a practical limit on the data acquisition time due to the

TABLE 1. Analysis of comb comparison schemes. The relative uncertainty was chosen at the gate time of 1 s for clear comparison. ((a) fiber-delivered H-maser frequency reference, (b) 100-s gate time data, (c) diode laser of which frequency is locked on a supercavity)

Ref. No.	Frequency reference	Allan deviation of frequency reference at 1 s	Allan deviation of frequency difference at 1 s	Total averaging time in s	Relative uncertainty
[6]	quartz oscillator	1.5×10^{-13}	3.3×10^{-13}	4310	5.1×10^{-15}
[7]	H-maser[(a)]	5.4×10^{-13}	not available	1000	1.2×10^{-14} [(b)]
[8]	H-maser	3×10^{-13}	3×10^{-13}	18300	1.2×10^{-15}
[9]	H-maser	order of 10^{-13}	3×10^{-13}	39467	1.3×10^{-15}
[10]	optical[(c)]	$\leq 4\times10^{-15}$	9×10^{-16}	1186	6.3×10^{-16}
[13]	optical[(c)]	order of 10^{-15}	2×10^{-17}	76585	7.8×10^{-20}

possibility of phase slips and unstable counting. The comb comparison results with different schemes are summarized in Table 1.

R. Holzwarth *et al.* [6] compared two Ti:sapphire-based frequency combs referenced to an rf signal provided by a quartz oscillator with an uncertainty of 5.1×10^{-15} by measuring the frequency difference at the gate time of 1 s. J. Ye *et al.* [7] measured the 1064 nm Nd:YAG laser quasi-simultaneously at JILA and at NIST by using an optical fiber network connecting these two institutes. The two measurements agreed in 0.74 Hz with an uncertainty of 3.4 Hz (1.2×10^{-14}) at 100-s gate time. The frequency reference for the JILA comb was provided by a fiber-delivered H-maser signal. L.-S. Ma *et al.* [8] made the first international comparison of femtosecond laser combs using a H-maser as the frequency reference. The agreement among the three combs that participated in that comparison was found to be on the subhertz level at 563 THz (1.2×10^{-15}). P. Kubina *et al.* [9] compared two mode-locked erbium-doped fiber lasers referenced to a H-maser. The two measurements agree within 1.3×10^{-15}.

The above comparisons of frequency combs referenced to rf sources typically show Allan deviations of an order of 10^{-13} at 1 s and reach the uncertainty level of $10^{-14}\sim10^{-15}$ in averaging time of 1000 s$\sim$10000 s. To decrease the upper limit for the comparison accuracy level, an optical clock configuration may be adopted utilizing the excellent stability (order of 10^{-15}) of a cw laser at optical frequency. Referencing to this optical standard provides improved stability allowing shorter averaging times, leading to lower uncertainty. S. A Diddams *et al.* [10] compared two octave-spanning Ti:sapphire-based combs that have the same repetition rate and are phase-locked to a low-frequency-noise diode laser. They demonstrated the intrinsic fractional frequency noise of a comb is $\leq6.3\times10^{-16}$ in 1 s of averaging. L.-S. Ma *et al.* [11-13] compared optically referenced Ti:sapphire laser frequency combs at the relative frequency uncertainty level as low as 8×10^{-20}. To reach this accuracy level, they averaged the data over 76000 s, enclosed the beam path, and arranged the optical path of two combs to have better common path rejection, because the mechanical and thermal fluctuations along the light path begin to play a role at this level of accuracy.

In our research, the Ti:sapphire-based frequency comb and the fiber-based frequency comb were referenced to a H-maser. Thus, the comb frequency comparison result is expected to have the uncertainty level of $10^{-14}\sim10^{-15}$.

III. OPTICAL FREQUENCY COMBS

The frequency of the optical frequency combs can be described by [1]

$$f = N\times f_{rep} + f_{ceo} \tag{1}$$

where N is the mode number of the comb, f_{rep} is the pulse repetition rate corresponding to the spacing of the comb lines, and f_{ceo} is the carrier-envelope-offset frequency, which is due to the difference of the phase velocity and the group velocity of the femtosecond pulse inside the laser cavity. f_{rep} and f_{ceo} are in the rf regime and can be detected and be controlled with well-developed rf techniques. The absolute frequency of an optical radiation can be determined by measuring the beat frequency with a frequency comb.

The Ti:sapphire frequency comb used in this research, which will be named "Comb-T" in this article, is based on a Kerr-lens mode-locked femtosecond Ti:sapphire laser (Menlosystems GmbH, FC8004). The laser resonator consists of ten mirrors. The intra-cavity group delay dispersion is compensated by chirped mirrors. It is pumped by a 4.5 W cw Nd:YAG Laser (532 nm). The spectrum is centered at 800 nm with spectral width (full width at the half maximum) of 57 nm. The f_{rep} is about 200 MHz and the pulse width is about 15 fs. Its output is coupled to a photonic crystal fiber (PCF) to obtain the octave-spanning spectrum for the f_{ceo}

detection and for the frequency measurement in the visible wavelength region. The f_{rep} can be phase-locked to an rf signal by feeding back the phase error signal to the PZT, which is attached to one of the mirrors constituting the Ti:sapphire laser cavity. The f_{ceo}, also, can be phase-locked to an rf signal by using an acousto-optic modulator for an actuator of the pump laser power.

The fiber comb, which will be named "Comb-F", is a fiber femtosecond ring laser mode-locked through nonlinear polarization rotation (Menlosystems GmbH, FC1500). The fiber laser spectrum is centered at 1550 nm. The f_{rep} is about 250 MHz and the pulse width is less than 150 fs. A part of this laser output was amplified by an erbium-doped fiber amplifier (EDFA) and was spectrally broadened to the wavelength range from 1050 nm to 2100 nm using a highly nonlinear fiber (HNF) in order to measure and to stabilize the f_{ceo}. Another part of the fiber fs laser output was amplified by another EDFA, and was frequency-doubled by a periodically poled lithium niobate (PPNL) crystal in a temperature-controlled oven to produce a short pulse around 775 nm. This near IR comb was spectrally broadened further by a PCF to cover the wavelength range from 500 nm to 1000 nm. The f_{rep} can be locked to an rf signal by a feedback to the PZT located at the free-space part of the fiber laser ring cavity. The f_{ceo} can be phase-locked to an rf-signal by controlling the pump power.

IV. EXPERIMENTAL RESULTS

The experimental setup for the comb comparison is shown in Fig. 1. The comb frequency comparison was performed by measuring an offset-locked iodine-stabilized He-Ne laser (474 THz) with Comb-T and Comb-F simultaneously. The He-Ne laser is offset-locked to the offset value of 256 MHz from another He-Ne laser stabilized on the e-component of the $^{127}I_2$ 11-5, R(127) line. It has high output power (1 mW) and has no frequency modulation. Its frequency stability in terms of Allan deviation is 5.6×10^{-12} (2.7 kHz) at 1 s of averaging time. The He-Ne laser output was delivered to the optical table, on which Comb-T and Comb-F are installed, by a 5-m-long single-mode fiber. The fiber-delivered output of this offset-locked He-Ne laser was split into two parts by using a 90:10 single-mode fiber coupler. 10% of the He-Ne laser output was used to obtain the heterodyne beat with Comb-F. The signal-to-noise ratio (SNR) of this beat was about 30 dB in a resolution bandwidth (RBW) of 400 kHz, which is sufficient for the correct frequency counting. And 90% of the He-Ne laser output was used to produce the heterodyne beat signal with Comb-T. By adopting the SNR enhancing method, which was introduced in Ref. [17], we could obtain the SNR of about 35 dB in a RBW of 400 kHz. f_{rep} (202 MHz) and f_{ceo} (20 MHz) of Comb-T were phase-locked to a reference 10 MHz signal from a hydrogen maser, which is linked to the SI second via a global positioning system. f_{rep} (250 MHz) and f_{ceo} (20 MHz) of Comb-F were phase-locked to a reference 10 MHz signal from the same hydrogen maser. All the frequency counters were referenced to this hydrogen maser.

The optical heterodyne beat frequency between the He-Ne laser and Comb-T and that between the He-Ne laser and Comb-F were detected by respective photodiodes (PD) and were measured by respective counters. Two counters operate synchronously without dead time. We tested the counter limits by measuring a frequency modulated rf signal (modulation frequency; 3 kHz, modulation span; 400 kHz) simultaneously with two counters used in this work at the gate time of 1 s. The standard deviation of the difference of two counter readings was 0.0069 Hz, which is well below the expected comb comparison uncertainty level. Using the phase-locked f_{rep}, f_{ceo} and the measured beat frequency,

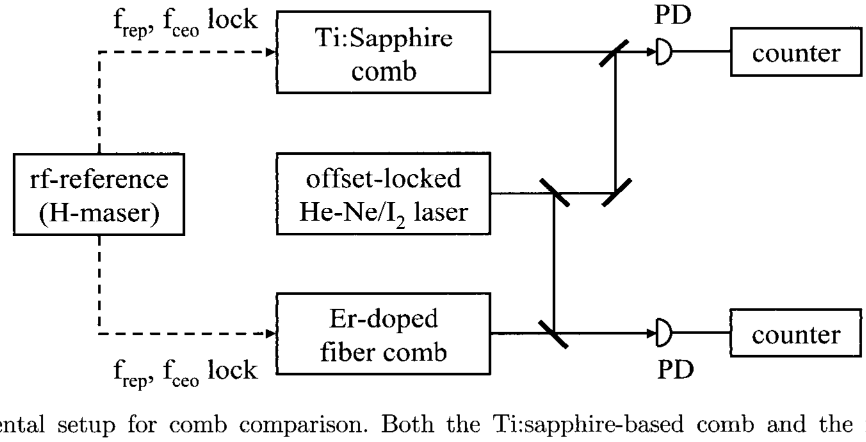

FIG. 1. Experimental setup for comb comparison. Both the Ti:sapphire-based comb and the fiber-based comb are phase-locked to common H-maser rf reference. (PD; photodiode).

we could determine the absolute frequency of the offset-locked iodine-stabilized He-Ne laser. Fig. 2 shows the absolute frequency of the stabilized He-Ne laser measured by Comb-T (filled triangles) and that measured by Comb-F (empty squares). The zero frequency represent the offset frequency (255 997 kHz) of the He-Ne laser plus the frequency of the e-component of the $^{127}I_2$ 11-5, R(127) line (473 612 366 967 kHz) recommended by the International Committee of Weights and Measures (CIPM) [18]. The frequency fluctuation is due to the stability of the offset-locked He-Ne laser. Because we are interested in the comb frequency comparison, we calculated the difference between the absolute frequency measured by Comb-T and that measured by Comb-F. Fig. 3 shows the result of this difference. In Fig. 2 and Fig. 3, the gate time was 1 s and the frequency was measured for about 500 s. Although the respective frequency measurements in Fig. 2 shows a frequency fluctuation of 2.7 kHz in terms of the standard deviation, the difference frequency fluctuation was much reduced that the standard deviation was 647 Hz and the standard error was 29 Hz by virtue of the simultaneous frequency counting. Fig. 4 shows the stability of the measured frequency difference. The Allan deviation is 1.5×10^{-12} at 1 s averaging time and decreases as approximately $1/\tau$.

This difference frequency was measured four times in total acquisition time of 1203 s and the result is summarized in Fig. 5. The data points represent the means of the frequency difference and the error bars represent

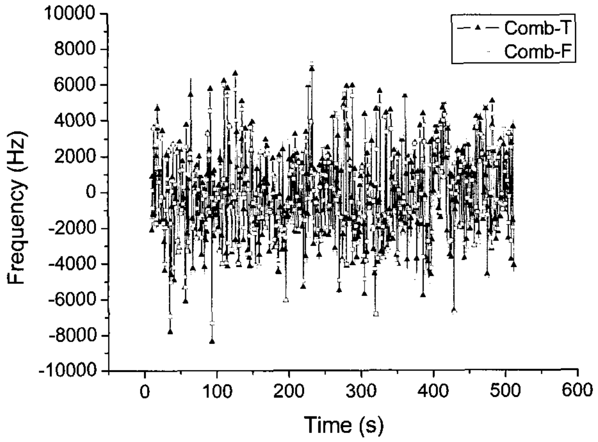

FIG. 2. The absolute frequency of the stabilized He-Ne laser measured by Comb-T (filled triangles) and that measured by Comb-F (empty squares). The zero frequency represents the offset frequency (256 MHz) of the He-Ne laser plus the frequency of the e-component of the $^{127}I_2$ 11-5, R(127) line.

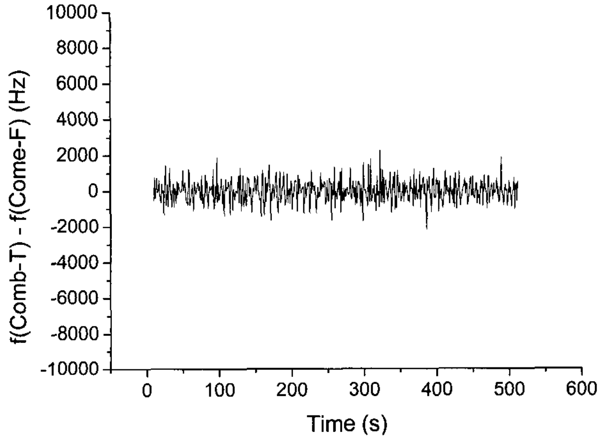

FIG. 3. The difference between the absolute frequency measured by Comb-T and that measured by Comb-F.

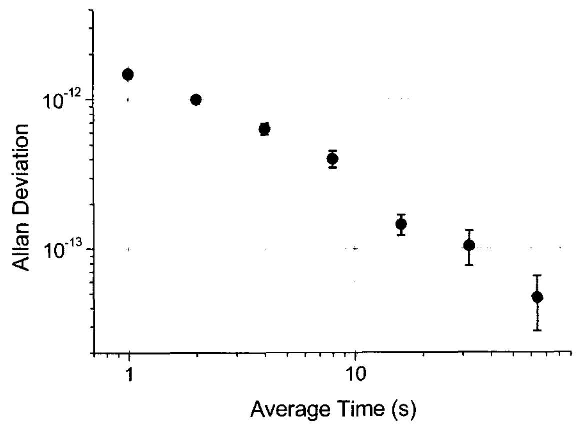

FIG. 4. Allan deviation of the measured frequency difference.

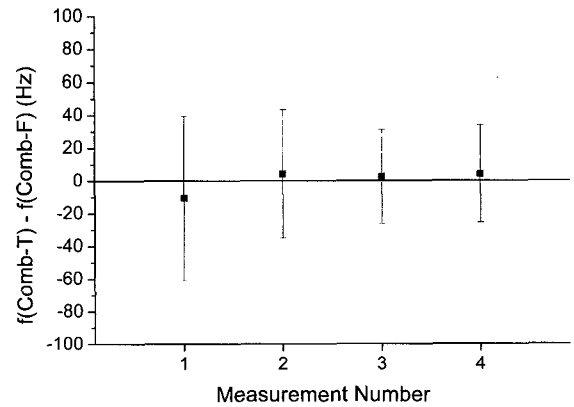

FIG. 5. Whole set of measurements of frequency difference between Comb-T and Comb-F. The data points represent the mean frequencies of the difference and the error bars represent the standard errors.

the standard errors which are dependent on the total measurement time (100~500 s). We calculated the weighted mean of the whole data sets to find the final average of the difference frequency concluding that the Ti:Sapphire comb and the fiber comb agree within 1.8 Hz (3.8×10^{-15}) with an uncertainty of 17.2 Hz (3.6×10^{-14}). This comparison result has larger uncertainty than the previously reported comparisons between frequency combs referenced to rf sources [6-9] approximately by an order. We attribute this to the relatively poor frequency stability (5.6×10^{-12}) of the optical frequency standard used in this work. However, our result has the significance of being the first comparison between a fiber-based frequency comb and a Ti:sapphire-based frequency comb.

V. CONCLUSIONS

The frequencies of a Ti:sapphire-based comb and a fiber-based comb are compared for the first time to our knowledge. The comparison is performed by measuring simultaneously an offset-locked iodine-stabilized He-Ne laser with these two combs. Both combs are phase-locked to an rf signal provide by a H-maser. The two frequency measurements agree within 1.8 Hz (3.8×10^{-15}) with the uncertainty of 17.2 Hz (3.6×10^{-14}). It is notable that the Ti:sapphire-based frequency combs and the fiber-based frequency combs show no systematic discrepancy to this level of uncertainty, even though these two combs have completely different configuration and are based on different mode-locking mechanisms. This result assists in ensuring that the absolute frequency measurement using optical frequency combs is trustable and consistent.

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