Hybrid Diversity-Beamforming Technique for Outage Probability Minimization in Spatially Correlated Channels

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Abstract: **In this paper, we present a hybrid multi-antenna technique that can minimize the outage probability by combining the diversity and beamforming techniques. The hybrid technique clusters the transmission antennas into multiple groups and exploit diversity among different groups and beamforming within each group. We analyze the performance of the resulting hybrid technique for an arbitrary correlation among the transmission antennas. Through the performance analysis, we derive a closed-form expression of the outage probability for the hybrid technique. This enables to optimize the antenna grouping for the given spatial correlation. We show through numerical results that the hybrid technique can balance the trade-offs between diversity and beamforming according to the spatial correlation and that the optimally designed hybrid technique yields a much lower outage probability than the diversity or beamforming technique does in partially correlated fading channels.**

Index Terms: **Beamforming, diversity, hybrid technique, outage probability, spatial correlation.**

I. INTRODUCTION

The multi-antenna systems that use multiple transmit antennas at the base station have recently emerged as an attractive topic for research. For such systems, two representative transmission techniques have been developed for an efficient utilization of the antennas: One is the transmit diversity technique that enables more reliable transmission by transmitting signals through multiple channels and the other is the transmit beamforming technique that increases the received signal power by steering the antenna beam-pattern to a particular direction. These two techniques are known to have somewhat opposite but complementary characteristics. According to Friedlander and Scherzer [1], beamforming performs better than diversity in terms of outage capacity and transmission power when fadings at different antennas are highly correlated, whereas diversity outperforms beamforming in the case of independent fading channels.

In order to take advantage of the desirable features of both diversity and beamforming, Soni *et al.* [2] proposed a new hybrid antenna architecture that realized diversity by spacing two different groups of antennas far apart and realized beamforming as well by spacing the antennas within each group closely together. Nezafat and Kaveh [3] analyzed the performance of this hybrid technique for the special case where fadings at different groups are independent while those at different antennas within a single group are fully correlated.

In this paper, we investigate the hybrid diversity-beamforming technique in more general context. We consider a flexible hybrid technique that clusters multiple transmission antennas into several groups by grouping certain number of adjacent antennas. Similarly to [2], we employ the diversity technique among different groups and the beamforming technique within each group. In this technique, we balance the trade-offs between the diversity and beamforming techniques by controlling the number of groups. We choose the optimal number of groups such that the outage probability be minimized. The performance of the hybrid technique depends mainly on the spatial correlation, which in turn is determined by the antenna spacing and angular spread. So we present in this paper a generalized performance analysis of the hybrid technique that is applicable to the cases with arbitrary antenna spacing and angular spread. Through the performance analysis, we derive a closed-form expression of the outage probability. This enables to optimize the antenna grouping of the hybrid technique for any given environment. As will be become clear in Section V, the optimally designed hybrid technique yields a much lower outage probability than the diversity or beamforming technique does in partially correlated fading channels.

The rest of the paper is organized as follows: Section II describes the system model under consideration. Section III introduces the performance analyses of the conventional diversity and beamforming techniques. Section IV presents the architecture of the hybrid technique and discusses the performance analysis and the outage minimization of the hybrid technique. Finally, Section V compares the performances of various techniques through numerical results, thereby confirming the properties of the hybrid technique.

II. SYSTEM MODEL

We consider a point-to-point communication link with M transmit antennas and a single receive antenna. We denote by h_m the channel gain between transmit antenna m, $m = 1, 2, \cdots, M$, and the receive antenna. For Rayleigh fading channels, h_m's are complex Gaussian random variables. We denote by $\mathbf{h}$ the vector of the channel gains, i.e., $\mathbf{h} = [h_1, \cdots, h_M]^T$. Without loss of generality, we can normalize the channel gain such that $E[|h_m|^2] = 1$. Then, for the total transmission power P and the noise power σ^2, the average input SNR at the receiver takes the form $\rho \equiv P/\sigma^2$.

For the multi-antenna system, the correlation among the channel gains at different antennas depends on both the multipath angular distribution of the signal and the antenna spacing. When the transmit antennas are positioned at a higher elevation than their surroundings, the correlation between the channel gains at

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antennas a and b, for a, $b = 1, 2, \cdots, M$, can be modeled by [4]

$$R_{ab} = \int_{-\pi}^{\pi} f_\theta(\theta) \exp\left(j\frac{2\pi \sin(\theta)}{l} D(a,b)\right) d\theta \qquad (1)$$

where l is the carrier wavelength, θ the angular direction of arrival, $f_\theta(\theta)$ the probability density function of the angular distribution, and $D(a, b)$ the distance between antennas a and b.[1] For a M-transmit antenna and 1-receive antenna system, the correlation matrix of the channel gain vector is written by

$$\mathbf{R} = [R_{ab}]_{M\times M} \qquad (2)$$

an $M \times M$ matrix whose (a, b) element is given by R_{ab}. If we assume that the signal is uniformly spread within $[-\Delta, \Delta]$, centered at θ_0, the probability density function of the angular distribution is given by [5]

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\Delta}, & -\Delta + \theta_0 \le \theta \le \Delta + \theta_0, \\ 0, & \text{elsewhere.} \end{cases} \qquad (3)$$

In this case, the angular spread θ_s, or the standard deviation of the truncated uniform distribution, becomes $\Delta/\sqrt{3}$. In general, the channel correlation among the antennas decreases as the antenna spacing or the angular spread increases [6].

III. PERFORMANCE ANALYSIS OF DIVERSITY AND BEAMFORMING TECHNIQUES

A. Diversity Technique

We first consider a technique in which the M transmit antennas are all utilized solely for achieving diversity and no channel state information from the receiver is fed back to the transmitter. Though there are various possible schemes to realize transmit diversity technique, we do not adhere to any specific scheme but rather consider an idealized transmit diversity technique that can achieve the maximum diversity gain without channel state information at the transmitter.[2] Specifically, we assume that the idealized transmit diversity technique provides the same diversity gain as the receive diversity with *maximal ratio combining* (MRC) does, but does not yield the array gain of the receive diversity (i.e., no M-fold increase in the received SNR).

For the idealized transmit diversity technique, the instantaneous output SNR at the receiver for a diversity technique is given by

$$\text{SNR}_d = \frac{\rho \gamma_d}{M} \qquad (4a)$$

for

$$\gamma_d = \sum_{m=1}^{M} |h_m|^2. \qquad (4b)$$

Note that the term M in the denominator represents that the transmit diversity technique does not provide the array gain.

In the special case when h_m's are mutually independent, γ_d has a chi-square distribution with $2M$ degree of freedom and the mean M [11]. In this case, the diversity technique achieves full diversity gain. In other cases, the distribution of γ_d is determined as follows [1]: For the channel correlation matrix $\mathbf{R}$, the channel gain vector is expressed by $\mathbf{h} = \mathbf{R}^{1/2}\mathbf{g}$, where $\mathbf{g}$ is a vector of complex, zero-mean, unit variance, independent Gaussian random variable. Then, γ_d takes the expression

$$\gamma_d = \mathbf{g}^H \mathbf{R} \mathbf{g} \qquad (5)$$

where $[\cdot]^H$ denotes the Hermitian transpose. We decompose $\mathbf{R}$ to the form $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{\Lambda} = \text{diag}(\lambda_1, \cdots, \lambda_M)$ for the eigenvalues λ_m, $m = 1, 2, \cdots, M$. Then, (5) takes the form $\gamma_d = \bar{\mathbf{g}}^H \mathbf{\Lambda} \bar{\mathbf{g}}$ for the transformed vector $\bar{\mathbf{g}} = \mathbf{U}^H \mathbf{g}$, which is also a vector of complex, zero-mean, unit variance, independent Gaussian random variable. Therefore, we get the expression

$$\gamma_d = \sum_{m=1}^{M} \lambda_m |\bar{g}_m|^2 = \sum_{m=1}^{M} \lambda_m (|r_m|^2 + |i_m|^2) \qquad (6)$$

where $\bar{g}_m = r_m + j i_m$ for Gaussian random variables r_m and i_m having zero-mean and variance $1/2$. By using the form of (6), we can get the probability density function of γ_d [12], which is given by

$$f_d(\gamma) = \sum_{m=1}^{M} (a_m/\lambda_m) e^{-\gamma/\lambda_m} \qquad (7)$$

for

$$a_m = \lambda_m^{M-1} \prod_{n=1, n\neq m}^{M} (\lambda_m - \lambda_n)^{-1}. \qquad (8)$$

When the spatial correlation is high, there exists a dominant eigenvalue which is much larger than other eigenvalues [6]. So the distribution of γ_d approximates to an exponential distribution. In contrast, as the spatial correlation decreases, the amplitudes of smaller eigenvalues become comparable to that of the largest one. As an extreme case, if all the eigenvalues have the same amplitude, γ_d has a chi-square distribution.

Using the distribution of γ_d obtained above, we can calculate the outage probability (i.e., the probability that the instantaneous capacity is less than a given value C_{out}) by

$$\begin{aligned} P_{out-d} &= \int_0^{(2^{C_{out}}-1)M/\rho} f_d(\gamma) d\gamma \\ &= \sum_{m=1}^{M} a_m \left(1 - e^{-\frac{(2^{C_{out}}-1)M}{\rho \lambda_m}}\right). \end{aligned} \qquad (9)$$

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[1] The model is appropriate for a macrocell deployment in a suburban environment where the antennas at base station are elevated above urban clutter and far away from the scatters [4].

[2] In [7], Alamouti presented a space-time transmit diversity scheme that by utilizing two transmit antennas and one receive antenna achieves the same diversity gain as the maximal ratio combining with one transmit antenna and two receive antennas. In [8], Tharokh *et al.* proposed more general schemes, referred to as *orthogonal space-time block code* (OSTBC), that holds the same property as Alamouti's scheme for any number of transmit antennas. In addition, another type of transmit diversity technique, or a delay transmit diversity scheme, was shown to achieve almost the same diversity benefit as the receive diversity does in independent fading channels [9, 10].

B. Beamforming Technique

Secondly, we examine the technique in which the M transmit antennas are all utilized solely for beamforming. We consider a statistical beamforming technique that adjusts the beam pattern only based on the second-order channel statistics. We denote by $\mathbf{w} = [w_1, \cdots, w_M]^T$ the weight vector of the transmit antennas. Then, the instantaneous SNR for a beamforming technique is given by

$$\text{SNR}_b = \rho\gamma_b \tag{10a}$$

for

$$\gamma_b = \mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w} = \left|\sum_{m=1}^{M} w_m h_m\right|^2. \tag{10b}$$

We determine the weight vector $\mathbf{w}$ to maximize the average SNR, i.e., $E[\rho\gamma_b]$. It is well known that the average SNR is maximized when $\mathbf{w}$ is set to the the normalized eigenvector associated with the largest engenvalue of $\mathbf{R}$, λ_{max} [13].[3] For the optimal weight vector, the average SNR is given by

$$E[\rho\gamma_b] = \rho\mathbf{w}^H\mathbf{R}\mathbf{w} = \rho\lambda_{max}. \tag{11}$$

This indicates that the array gain is given by the largest eigenvalue of $\mathbf{R}$, λ_{max}.[4]

Since $X \equiv \sum_{m=1}^{M} w_m h_m$ is a complex Gaussian random variable with zero-mean and the variance is $\sigma_X^2 = E[|\sum_{m=1}^{M} w_m h_m|^2] = \lambda_{max}$, γ_b has an exponential distribution with mean λ_{max}. In this case, the probability density function of γ_b is given by

$$f_b(\gamma) = \frac{1}{\lambda_{max}} e^{-\gamma/\lambda_{max}}. \tag{12}$$

Since the beamforming technique provides no diversity gain, γ_b always has an exponential distribution, as γ_d in (7) does for the fully correlated case. In addition, as mentioned above, the beamforming technique achieves an λ_{max}-fold increase in the average SNR. So the spatial correlation affects only the average SNR: When the channel gains are fully correlated, we get $\lambda_{max} = M$, the full array gain. On the other hand, when the channel gains are mutually independent, we get no array gain, i.e., $\lambda_{max} = 1$.

The outage probability of the beamforming technique is given by

$$\begin{aligned} P_{out-h} &= \int_0^{(2^{C_{out}}-1)/\rho} f_b(\gamma)d\gamma \\ &= 1 - e^{-\frac{(2^{C_{out}}-1)}{\rho\lambda_{max}}}. \end{aligned} \tag{13}$$

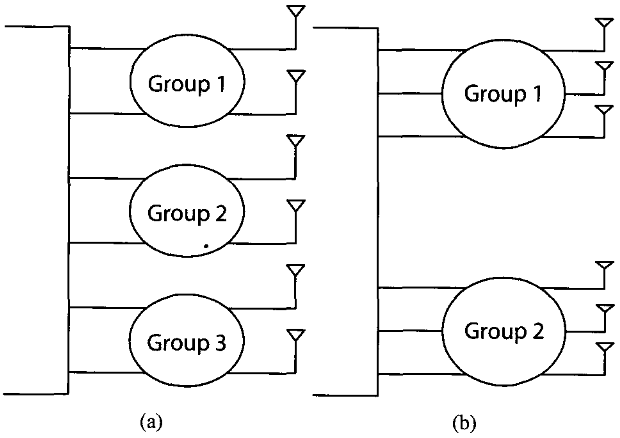

Fig. 1. Examples of antenna groupings for 6-element antenna system (i.e., $M = 6$): (a) (2,2,2), (b) (3,3).

IV. OUTAGE PROBABILITY OF HYBRID DIVERSITY-BEAMFORMING TECHNIQUE

Now, we consider the hybrid diversity-beamforming technique that exploits both diversity and beamforming gains by dividing M transmit antennas into N groups. We conduct the antenna grouping in such a way that each group consists of adjacent multiple antennas. We then apply the beamforming technique to the antennas in each group, while applying the diversity technique among different groups. Then the hybrid technique can be specified simply by $(L_1, \cdots, L_N)$ where L_n is the number of antennas belonging to group n. Fig. 1 illustrates the two examples of antenna grouping, $(2, 2, 2)$ and $(3, 3)$, for a 6-element antenna technique. We denote by $\mathbb{L}_N$ the set of all the possible antenna partitioning when the number of antenna groups is N, i.e., $\mathbb{L}_N \equiv \{(L_1, \cdots, L_N)| \sum_{n=1}^{N} L_n = M\}$. We also denote by $A_n = \{a_1^n, \cdots, a_{L_n}^n\}$ the index set of antennas belonging to group n and by $\mathbf{w}_n = [w_1^n, \cdots, w_{L_n}^n]^T$ the weight vector allocated to group n, respectively. Fig. 2 depicts the hybrid technique with N antenna groups. In the figure, we employ the delay transmit diversity scheme as an example of the diversity technique. The diversity and beamforming techniques in the previous subsections correspond to two special cases of this hybrid technique with antenna grouping of $(1, 1, 1, 1, 1, 1)$ and (6), respectively.

In antenna group n of the hybrid technique, the channel gain vector is given by

$$\mathbf{h}_n = [h_{a_1^n}, \cdots, h_{a_{L_n}^n}]^T. \tag{14}$$

The optimum weight vector $\mathbf{w}_n$ is the normalized eigenvector associated with the largest eigenvalue of $\mathbf{R}_n$, the correlation matrix of $\mathbf{h}_n$.

The instantaneous SNR of the received signal from antenna group n is given by $\frac{\rho}{N}\mathbf{h}_n^H\mathbf{w}_n\mathbf{w}_n^H\mathbf{h}_n$. After applying the diversity technique among all the antenna groups, the instantaneous SNR for the total received signal is given by the sum of those for each antenna group, i.e.,

$$\text{SNR}_h = \frac{\rho\gamma_h}{N} \tag{15a}$$

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[3] On the contrary to the diversity technique, the beamforming technique requires a low-rate feedback channel to send the information of the correlation matrix or the eigenvector corresponding to the maximum eigenvalue.

[4] The array gain is defined to be the ratio of the average received SNR of the multi-antenna system to that of the single antenna system.

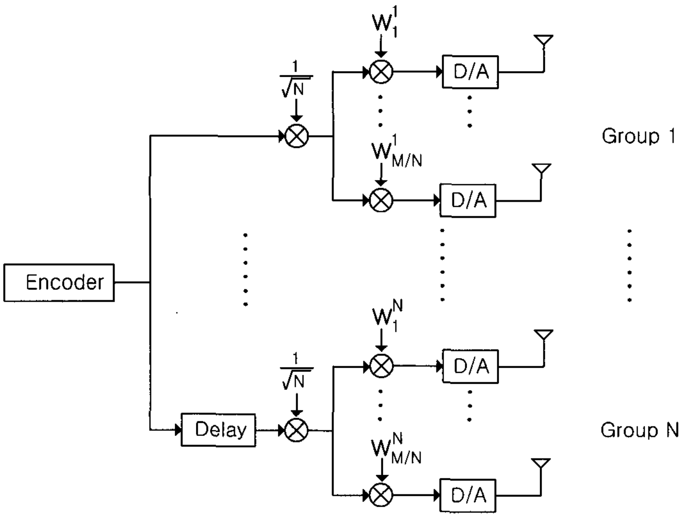

Fig. 2. Configuration of the hybrid technique (composed of M transmit antennas, divided into N groups).

for

$$\gamma_h = \sum_{n=1}^{N} \mathbf{h}_n^H \mathbf{w}_n \mathbf{w}_n^H \mathbf{h}_n. \tag{15b}$$

By comparing (14) with (4), we may interpret the hybrid technique as a diversity technique that has N virtual transmit antennas with the channel gain

$$\mathbf{h}^v = [\mathbf{w}_1^H \mathbf{h}_1, \cdots, \mathbf{w}_N^H \mathbf{h}_N]^T. \tag{16}$$

The correlation between the channel gains of two virtual antennas p and q, for $p, q = 1, 2, \cdots, M$, is given by

$$R_{pq}^v = \sum_{i=1}^{L_p} \sum_{j=1}^{L_q} w_i^p w_j^q R_{a_i^p a_j^q} \tag{17}$$

and the correlation matrix can be written by

$$\mathbf{R}^v = [R_{pq}^v]_{N \times N} \tag{18}$$

where R_{pq}^v is the (p, q) element of $\mathbf{R}^v$. By rearranging (15b) as in Section III-A, we get

$$\gamma_h = \sum_{n=1}^{N} \lambda_n^v |\bar{g}_n|^2 \tag{19}$$

where λ_n^v's are the eigenvalues of $\mathbf{R}^v$ and $\bar{g}_n$'s are complex, zero-mean, unit variance, independent Gaussian random variables. Then the average SNR is given by

$$E[\gamma_h] = \frac{\rho}{N} \sum_{n=1}^{N} \lambda_n^v. \tag{20}$$

By using γ_h in (19), we can determine the probability density function of SNR $f_h(\gamma)$ and the outage probability P_{out-h} in a way similar to that in Section III-A. Then we can derive the following closed-form expression of the outage probability:

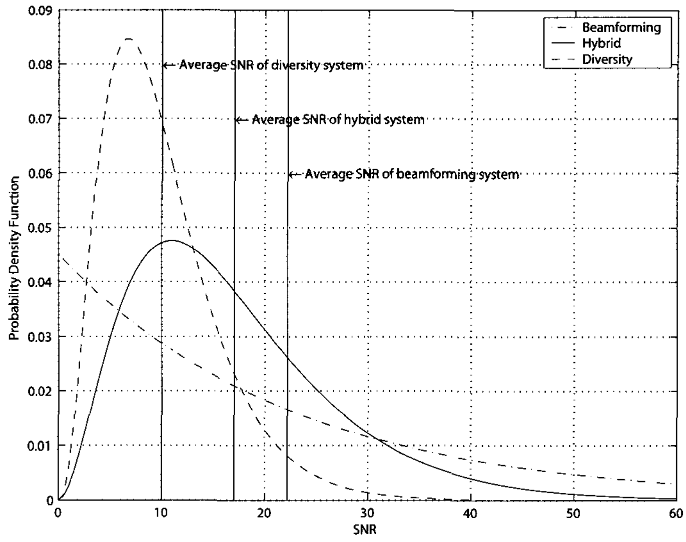

Fig. 3. Probability density function of the received SNR and its average SNR ($M = 6$, $\theta_s = 30°$, and $\rho = 10$ dB).

$$\begin{aligned} P_{out-h} &= \int_0^{(2^{C_{out}}-1)N/\rho} f_h(\gamma) d\gamma \\ &= \sum_{n=1}^{N} a_n^v \left(1 - e^{-\frac{(2^{C_{out}}-1)N}{\rho \lambda_n^v}}\right) \end{aligned} \tag{21}$$

for

$$a_n^v = (\lambda_n^v)^{N-1} \prod_{l=1, l \neq n}^{N} (\lambda_n^v - \lambda_l^v)^{-1}. \tag{22}$$

Now, we can optimize the antenna grouping to minimize the outage probability. The optimization problem is formulated by

$$\min_{N \in \{1, \cdots, M\}} \left[\min_{(L_1, \cdots, L_N) \in \mathbb{L}_N} P_{out-h}(L_1, \cdots, L_N) \right]. \tag{23}$$

This optimization requires computing the outage probability of all possible antenna groupings for each number of antenna groups. The total number of antenna groupings is $\sum_{N=1}^{M} \binom{M-1}{M-N}$. The computation complexity of searching the optimal antenna grouping may be high, but the search space can be reduced by pruning the cases where weakly correlated antennas belong to the same group.

V. NUMERICAL RESULTS

For numerical example, we consider a 6-element uniformly linear array of antennas with the antenna spacing $d = 0.5\lambda$. We consider the Rayleigh fading channel and use the truncated uniform distribution model with $\theta_0 = 0$.

A. Impact of Antenna Grouping on the Performance of Hybrid Technique

We first investigate how the antenna grouping affects the performance of the hybrid technique. Fig. 3 shows the probability density function of the received SNR of the diversity, beamforming and hybrid techniques when $\theta_s = 30°$ and $\rho = 10$ dB. In the case of the hybrid technique we divide the transmission antennas equally into 3 groups, which was found to minimize the outage probability for the given parameters. We observe in the figure that the SNR distribution of the hybrid technique concentrates around the mean, which is similar to that of the diversity technique. At the same time, the average SNR of the hybrid technique increases above that of the diversity technique. Since the channel gains at different antennas are partially correlated, the array gain that can be achieved by beamforming is small.[5] Consequently, there exists small difference between the average SNR of the hybrid technique and that of the beamforming technique. This indicates that the hyrid technique can get both diversity gain and array gain, with the diversity gain obtained from the antennas spaced far apart and the array gain obtained from the adjacent antennas, under partially correlated channel. In essence, the hybrid technique turns out to operate in such a way that the probability of having low SNR be minimized. This leads to a minimized outage probability.

B. Performance Comparisons among Diversity, Beamforming, and Hybrid Techniques

We now compare the performances of the diversity, beamforming, and hybrid techniques. Fig. 4 plots the outage probability of the hybrid technique with various antenna groupings,[6] including the optimally designed hybrid technique and the other two techniques with respect to angular spread θ_s for $\rho = 10$ dB and $C_{out} = 1.5$ bps/Hz. As expected, the beamforming technique yields the lowest outage probability when the angular spread is extremely small, while the diversity technique exhibits the lowest outage probability when the angular spread is extremely large. In the case of the hybrid technique, it yields the lowest level of outage probability when the angular spread is in the middle range where channel gains at different antennas are partially correlated. We observe that it is advantageous to use a larger number of groups as the angular spread increases. Since the antenna system has the uniform architecture, it is optimal to distribute the transmission antennas to each group such that all the groups have the same number of antennas or number of antennas within each group differs at most by one.

We also compare the three techniques in terms of the ergodic capacity in Fig. 5. For a given SNR, the erdogic capacity is given by

$$C = E\left[\log_2\left(1 + \mathrm{SNR}\right)\right]. \tag{24}$$

We calculated the ergodic capacity numerically by using the distribution of SNR, which was derived for each technique in the previous sections. As expected, the ergodic capacity of the beamforming technique is higher than that of the diversity technique due to the array gain and the difference between them decreases as the angular spread increases. We observe that the hybrid technique can provide an ergodic capacity comparable to the beamforming techiniqe when θ_s lies in the $15 \sim 25°$ range. In addition, in the middle or low range of spatial correlation (i.e., when θ_s is larger than 25°), the ergodic capacity of the hybrid technique can be even higher than that of the beamforming technique. This happens because the capacity is not a linear function of SNR. If the beamforming vector is set to maximize the ergodic capacity, rather than to maximize the average SNR, the beamforming technique would yield the highest ergodic capacity regardless of the spatial correlation. Note that the hybrid technique designed for the outage probability minimization does not always maximize the ergodic capacity.

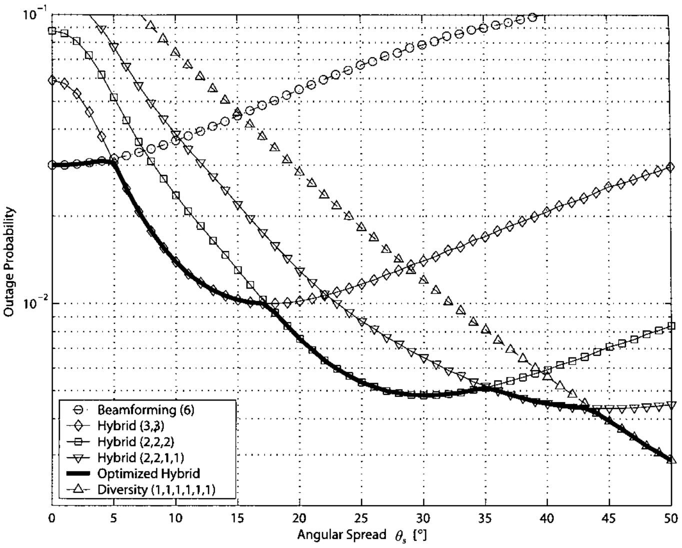

Fig. 4. Outage probability of the hybrid technique with respect to angular spread ($M = 6$, $\rho = 10$ dB, and $C_{out} = 1.5$ bps/Hz).

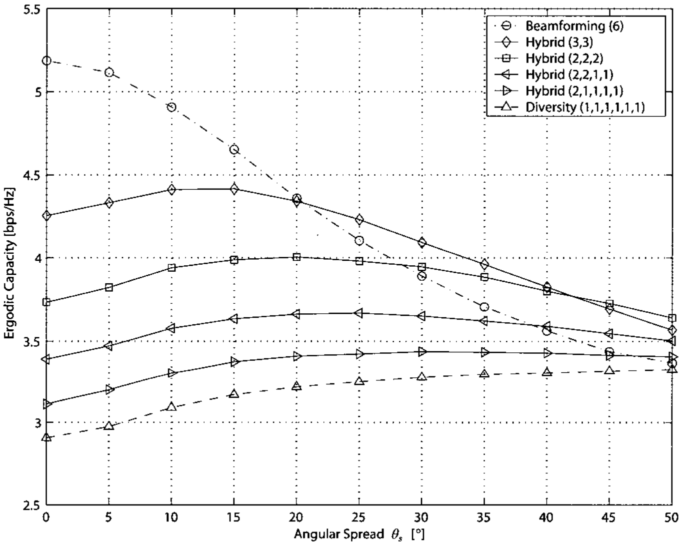

Fig. 5. Ergodic capacity of the hybrid technique with respect to angular spread ($M = 6$ and $\rho = 10$ dB).

Fig. 6 plots the outage probability of the three techniques with respect to the average SNR ρ when $\theta_s = 30°$ and $C_{out} = 1.5$ bps/Hz. We observe that the optimally designed hybrid technique minimizes the required average SNR for a given outage probability. Table 1 lists the optimal antenna groupings determined by applying the analysis in Section IV to different

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[5] The array gain of the beamforming technique is only about 2.2, which is much smaller than the maximum array gain of 6.

[6] Any permutations of $(L_1, \cdots, L_N)$ exhibit the same outage probability since the uniform antenna system is symmetric.

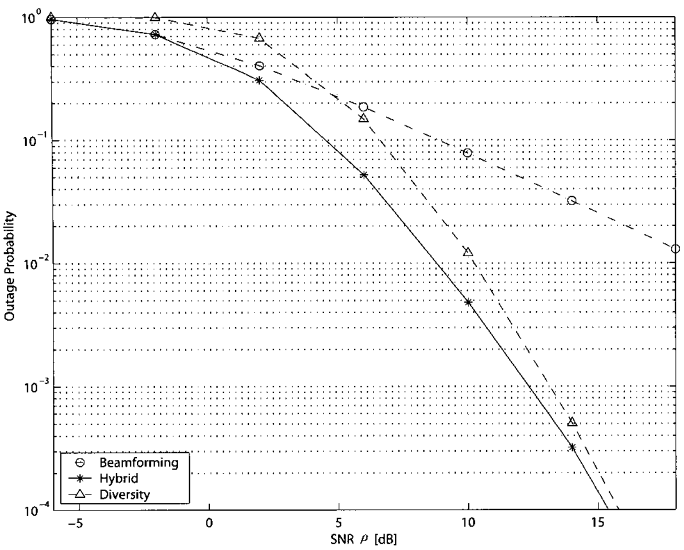

Fig. 6. Outage probability of the hybrid technique with respect to ρ ($M = 6$, $\theta_s = 30°$, and $C_{out} = 1.5$ bps/Hz).

Table 1. Optimal antenna grouping with respect to ρ.

ρ [dB]	Optimal antenna grouping $(L_1, \cdots, L_N)$
-6	(6)
-2	(6, 0)
2	(3, 3)
$6 \sim 10$	(2, 2, 2)
$14 \sim 18$	(2, 2, 1, 1)

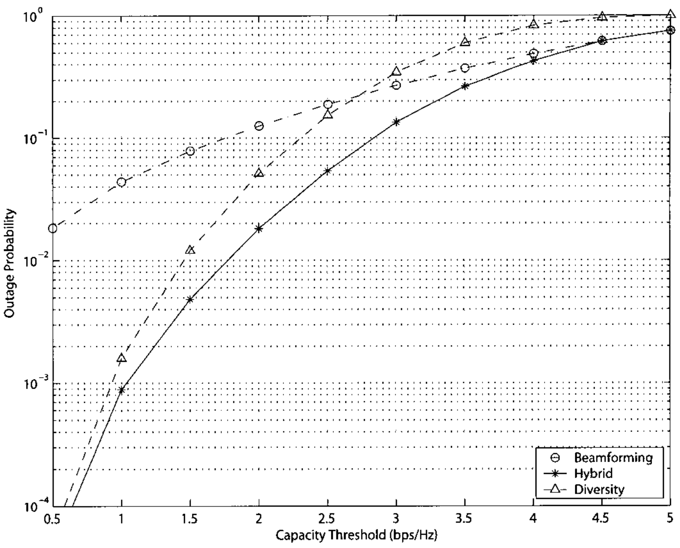

Fig. 7. Outage probability of the hybrid technique with respect to capacity threshold C_{out} ($M = 6$, $\theta_s = 30°$, and $\rho = 10$ dB).

Table 2. Optimal antenna grouping with respect to capacity threshold C_{out}.

C_{out} (bps/Hz)	Optimal antenna grouping $(L_1, \cdots, L_N)$
0.5	(2, 2, 1, 1)
$1 \sim 3$	(2, 2, 2)
$3 \sim 4.5$	(3, 3)
5	(6)

values of average SNR. We observe that the optimal number of groups tends to increase as the average SNR increases. This happens because it is advantageous to exploit more diversity gain when the average SNR increases to the level where array gain cannot be exploited any further.

Fig. 7 plots the outage probability of the three techniques with respect to the capacity threshold C_{out} when $\theta_s = 30°$ and $\rho = 10$ dB. We observe that the outage probability increases as the capacity threshold increases for all the three techniques, and that the optimally designed hybrid technique minimizes the outage probability. Table 2 lists the optimal antenna groupings for different values of capacity threshold. We observe that, as the capacity threshold increases, it is favorable to make the number of groups smaller, thereby exploiting more array gain than diversity gain.

C. *Balancing Diversity Gain and Array Gain*

According to (19), in the case of the hybrid technique with N transmit antenna groups, there are N independent channels with each channel gain weighted by λ_n^v, the eigenvalue of the correlation matrix $\mathbf{R}^v$. The diversity gain is small if the corresponding eigenvalue is small, so the number of dominant eigenvalues represents the diversity order. On the other hand, according to (20), the average of the eigenvalues represents the array gain.

Fig. 8 plots the minimum and the average of the eigenvalues with respect to the angular spread for the same parameters as used for Fig. 4. The division of the four regions is done according to the outage probability curve in Fig. 4. For simplicity, we specify the hybrid technique only by the number of antenna groups.[7] We observe from Fig. 8(a) that the optimal number of groups increases as the angular spread increases. At the boundary where the optimal number of antenna groups changes, the corresponding minimum eigenvalue becomes larger than a certain threshold, which indicates that the number of dominant eigenvalues increases by 1. This implies that the hybrid technique gets more diversity gain as the achievable diversity order increases. In addition, we observe from Fig. 8(b) that the average value becomes small when the angular spread is large. In this case, mode change occurs even when the minimum eigenvalue increases slightly. This implies that the hybrid technique can exploit more diversity gain when the achievable array gain is small. Therefore, in the hybrid technique, it is possible to balance the diversity gain and the array gain effectively by adjusting the number of antenna groups depending on the spatial correlation.

VI. CONCLUSIONS

In this paper, we have analyzed the hybrid technique that combines the diversity and beamforming techniques in spatially correlated fading channel. We have shown that the hybrid tech-

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[7] As mentioned earlier, the uniform distribution of transmission antennas is optimal for a given number of antenna groups.

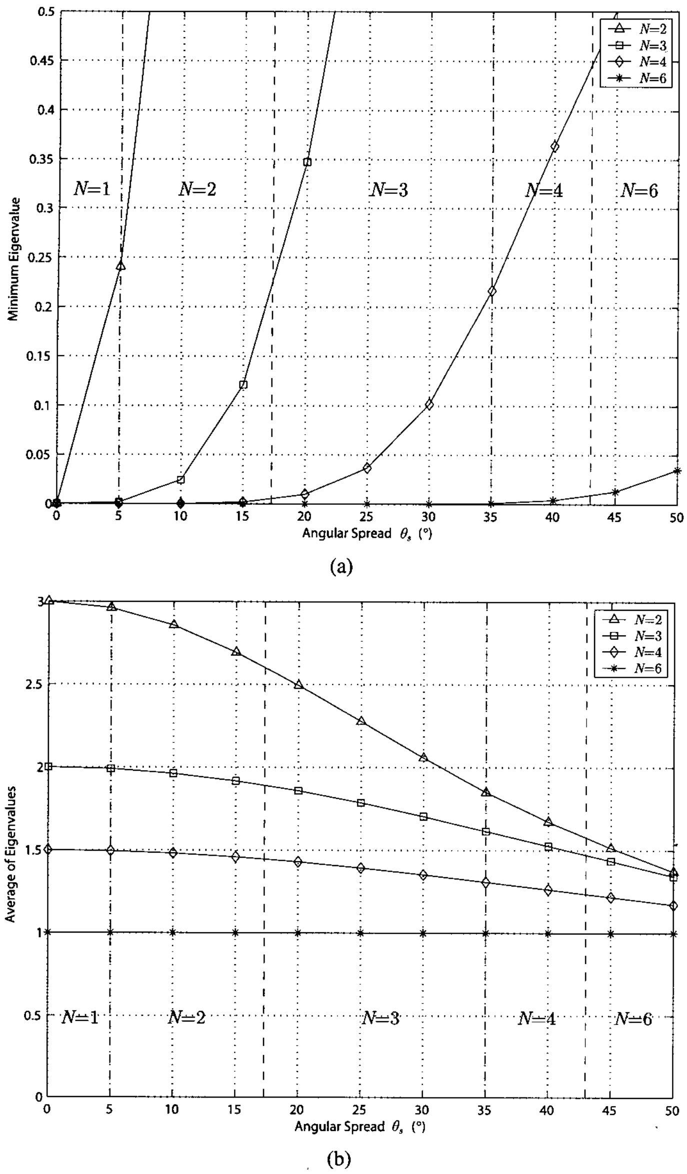

Fig. 8. Eigenvalues of $\mathbf{R}^v$ of the hybrid technique with respect to angular spread ($M = 6$ and $\rho = 10$ dB): (a) Minimum value, (b) average value.

nique can control the distribution of the received SNR by controlling the number of antenna groups and this enables balancing the trade-offs between the diversity and beamforming depending on the spatial correlation. We determined the probability density function of SNR of the hybrid technique for arbitrary antenna spacing and angular spacing, and based on this, we derived the closed form of the outage probability of the hybrid technique. This made it possible to adjust the antenna grouping of the hybrid technique to maximize the performance for the given environment. Numerical results revealed that whereas the conventional diversity and beamfomrming techniques exhibit good performance only in the extreme cases, i.e., fully correlated and independent fading channels, respectively, the optimally designed hybrid diversity-beamforming technique can reduce the outage probability significantly in partially correlated channel.

The multi-antenna technology will surely emerge as a key element of the next-generation communication systems, and the various multi-antenna techniques that have been reported to date are targeted at different performance criteria and different channel environments. For a given performance metric and channel condition, we may either select one particular technique that possibly maximizes the performance or design a hybrid technique that combines the multiple techniques together. The proposed hybrid diversity-beamforming technique belongs to the latter category that combines the diversity and beamforming techniques targeting at outage probability minimization in the partially correlated channel.

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